Using mean reversion as a measure of persistence[*]

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Abstract

This paper suggests a new scalar measure of persistence together with a companion estimator, which has the advantage of not requiring the specification and estimation of a model for the series under investigation. The statistical properties of the companion estimator are established, which allow tests of hypotheses to be performed, under very general conditions. The use of the new measure is illustrated by re-evaluating persistence of inflation for the United States and the Euro Area. The conclusions for the United States do not differ significantly from what has been found in previous empirical studies. However, for the Euro Area we find evidence of a significant break occurring in 2000/2001, such that persistence becomes virtually nil for the period that follows the launch of the euro and the implementation of a common monetary policy by the European Central Bank.

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1 Introduction

Understanding inflation persistence is crucial for the central bank, because it may have strong implications for the design and implementation of monetary policy. In particular, it may be argued that the appropriate response to shocks hitting the economy depends on the degree to which their effect on inflation is persistent. Furthermore, the horizon at which monetary policy should aim for price stability also depends on the persistence of inflation: with less persistence, inflation can be stabilised in shorter time following a shock, so that the degree of inflation persistence may also be seen as an important factor determining the medium-term orientation of monetary policy. On the other hand, it mays also be claimed that persistence of inflation is a major determinant of the economic costs of disinflation[1]. No wonder thus, that inflation persistence has been, over the last decade, one of the most intensely investigated topics in macroeconomics. Issues such as whether inflation is highly persistent or not and its implications for monetary policy strategy, whether it has changed over time or remained constant, whether it is structural or rather may vary according to the specific monetary policy regime, are examples of relevant questions that have been addressed in the literature.

In a different context some literature has also found important to investigate persistence of other macroeconomic variables, such as aggregate output or the deviations of the economy from purchasing power parity (PPP) conditions.

Alogoskoufis and Smith (1991), Edwards (1999), Burdekin and Siklos (1999), Stock (2001), Bleaney (2001), Willis (2003), Levin and Piger (2004), Gadzinski and Orlandi (2004), Cogley and Sargent (2001, 2007) and Pivetta and Reis (2007) are examples of contributions in the literature aiming at evaluating persistence of inflation, while Rogoff (1996), Frankel and Rose (1996), Cheung and Lai (2000) and Murray and Papell (2002),

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[1] Buiter and Jwett (1981) explain costly disinflation using a sticky-inflation model. Fuhrer and Moore (1995) find consistent theoretical and empirical results and Fuhrer (1995), through simulation, shows that when inflation is persistent, the output loss asociated with disinflation is larger than when there is no persistence. Bordo and Haurich (2004) find that the key factor in the yield curve's hability to predict ouput growth is the persistence of inflation.

to name but a few, are examples of contributions to the measurement of persistence of PPP deviations from equilibrium.

In most of the papers that try to evaluate persistence of inflation the "sum of the autoregressive coefficients" emerges as the most popular scalar measure of persistence, while the "half-life" is very popular in the literature that investigates persistence of PPP deviations. Other scalar measures of persistence also used in the literature include, for instance, the "largest autoregressive root" (see Stock, 1991, 2001) and the "spectrum a zero frequency" (see, for instance, Andrews and Chen, 1994). The usefulness of scalar measures of persistence stems from the fact that they are summary measures of the information contained in the impulse response functions of the estimated models. Of course the use of such scalar measures of persistence may be criticised exactly on the grounds that they are not capable of retaining all the potentially relevant features of the underlying impulse response functions. However, by itself, the impulse response function being an infinite-length vector is not very useful as a measure of persistence, especially so if the purpose is to quantify and compare the degree of persistence across different time series. Thus, despite their limitations, the scalar measures of persistence remain a useful way of quantifying persistence of time series data in empirical applications.

All the above cited measures of persistence share the common feature that they are parametric in the sense that they are defined and computed by usually estimating a times series model (usually an autoregressive process) for the data under investigation. This paper contributes for this strand of the literature by suggesting a new measure of persistence, which is broader in scope than the widely used "sum of the autoregressive coefficients" and has the advantage of not requiring the specification and estimation of a model for the data. This new measure of persistence, denoted in the paper by γ, relies on the idea that there is a relationship between persistence and mean reversion, and is defined as the unconditional probability of a stationary stochastic process not crossing its mean in period t. A non-parametric estimator of γ, denoted by $\widehat{\gamma}$, is also suggested and its theoretical distributional properties investigated.

In particular, it is shown that $\widehat{\gamma}$ is an unbiased estimator of γ, when the mean of the time series process is known and a consistent estimator of γ when the mean is unknown. Inference on γ may be conducted resorting to the conventional approach in which a consistent kernel estimator for the asymptotic variance of $\widehat{\gamma}$, σ^2_γ, is used, or following the recent approach suggested in Kiefer and Vogelsang (2002), in which a non-consistent kernel estimator for σ^2_γ is used to construct a statistic with a non-standard distribution.

The relationship between γ and other measures of persistence with a particular focus on the "sum of the coefficients" in a pure autoregressive process, which we denote by ρ, is also investigated. It is shown that there is a monotonic relationship between ρ and γ (and some other scalar measures of persistence) when the data are generated by an AR(1) process, but such a monotonic relationship ceases to exist once higher order autoregressive processes are considered.

The finite sample performance of $\widehat{\gamma}$ and $\widehat{\rho}$, the OLS estimator of ρ, is compared using some Monte Carlo experiments. It is seen that $\widehat{\gamma}$, which has the nice property of being immune to potential model misspecifications, is not significantly affected by the presence of outliers in the data, and the coverage ratio of their empirical confidence intervals is on par with the ones obtained for $\widehat{\rho}$, when one uses the method proposed by Kiefer and Vogelsang (2002).

Finally, the use of the new measure of persistence is illustrated by evaluating inflation persistence in the United States and the Euro Area. We find that, conditional on a break in the mean of inflation, the U.S. and the E. A. do not differ significantly, as far as inflation persistence for the period 1984-2006 is concerned. Both countries exhibit low levels of persistence and there is no significant evidence that the degree of persistence has changed over time with the average level of inflation. This evidence is basically in line with the results in Gadzinski and Orlandi (2004) for the U.S. and E.A. and in Levin and Piger (2004) for the U.S.. However, when we look for changes in persistence not related to changes on average inflation, the use of γ allows us to uncover a significant reduction of inflation persistence in the Euro Area, occurring after 2000/2001, coinciding with the launch of the euro and the implementation of a common monetary policy by the European Central Bank. This reduction, which will have remained undetected if we stuck to ρ as the single measure of persistence, is such that persistence in the Euro Area becomes virtually nil for the period 2000-2006.

The rest of the paper is organized as follows. Section 2 introduces γ and its non-parametric estimator $\widehat{\gamma}$ and derives the distributional properties of $\widehat{\gamma}$. Section 3 discusses the relationship between γ and alternative measures of persistence in the context of the AR(1) and AR(2) models. Using Monte Carlo simulations, section 4 investigates the finite sample properties of $\widehat{\gamma}$ and $\widehat{\rho}$. Section 5 illustrates the use of the new measure of persistence by evaluating the persistence of inflation in the United States and the Euro Area, and section 6 concludes.

2 An alternative measure of persistence

In general terms, persistence of a stationary time series, y_t, may be defined as the speed with which y_t converges to its equilibrium or long-run level after a shock[2]. In the context of the so-called univariate approach, persistence is investigated by looking at the univariate time series representation of the data. For that purpose it is usually assumed that the data are generated by a stationary autoregressive process of order p, (AR(p)), which may be written as

$$y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \varepsilon_t \tag{1}$$

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[2] This definition is similar to other definitions in the literature under the assumption of stationary processses (see, for instance, Andrews and Chen, 1994, Willis, 2003, or Pivetta and Reis, 2007). For a different definition of persistence, especially suited for I(1) processes see Jaeger and Kunst (1990).

and reparameterised as:

$$\Delta y_t = \alpha + \sum_{j=1}^{p-1} \delta_j \Delta y_{t-j} + (\rho - 1) y_{t-1} + \varepsilon_t \tag{2}$$

where

$$\rho = \sum_{j=1}^{p} \beta_j \tag{3}$$

and $\delta_j = -\sum_{i=1+j}^{p} \beta_i$. In the context of model (1), y_t is said to be (highly) persistent if, following a shock to the disturbance term ε_t, y_t converges slowly to its mean (which in the context of such a model may be seen as representing the equilibrium level of y_t). Thus, in the context of this parametric representation of y_t, the concept of persistence appears as intimately linked to the impulse response function (IRF) of the AR(p) process.

It has been argued (see, Andrews and Chen, 1994) that the cumulative impulse response (CIR) is generally a good way of summarizing the information contained in the impulse response function (IRF) and as such a good scalar measure of persistence. In a simple AR(p) process the cumulative impulse response is given by $CIR = 1/(1-\rho)$ where ρ is the "sum of the autoregressive coefficients", as defined in (3). As there is a monotonic relationship between the CIR and ρ it follows that, under the above assumption, one can simply rely on the "sum of the autoregressive coefficients" as a measure of persistence. This explains why ρ may be viewed as a measure of persistence. Using the CIR or simply ρ as a measure of persistence amounts to measuring persistence as the sum of the disequilibria (deviations from equilibrium) generated during the whole convergence period. The larger the ρ, the larger the cumulative impact of the shock will be.

We now highlight the relationship between persistence and mean reversion as this would allow us to better understand the intuition behind the new measure of persistence to be suggested below. To that end we may start by further reparameterise model (2) as:

$$\Delta y_t = \sum_{j=1}^{p-1} \delta_j \Delta y_{t-j} + (\rho - 1)[y_{t-1} - \mu] + \varepsilon_t \tag{4}$$

where $\mu = \alpha/(1-\rho)$ is the unconditional mean of the series. Now, it is well known that one identifying characteristic of any stationary process is that it must exhibit mean-reversion. In equation (4) the presence of mean reversion is reflected in the term $(\rho - 1)[y_{t-1} - \mu]$. This implies that if in period $(t-1)$ the series y is above (below) the mean, the deviation $[y_{t-1} - \mu]$ will contribute as a driving force to a negative (positive) change of the series in the following period, through the coefficient $(\rho - 1)$, thus bringing it closer to the mean. Of course, everything else constant, mean reversion will increase when the coefficient $(\rho - 1)$ increases (in absolute terms). Given that we can measure persistence by ρ and mean reversion by $(\rho - 1)$, we conclude that mean reversion and persistence are inversely related: high persistence implies low mean reversion and vice-versa. This correspondence between persistence and mean reversion allows us to carry out a simple preliminary evaluation of persistence by visual inspection of two different series: in a graph with two stationary series the one that crosses the mean less frequently, is expected to be the one exhibiting higher persistence and vice-versa.

2.1 The new measure of persistence

We may now introduce a new measure of persistence, which we denote by γ, and define as the *unconditional probability of a stationary stochastic process* y_t *not crossing its mean in period* t. By noticing that y_t does not cross the mean, μ, in period t if and only if $(y_t - \mu).(y_{t-1} - \mu) > 0$, we may formally define γ as

$$\gamma = P\left\{[(y_t - \mu) > 0 \wedge (y_{t-1} - \mu) > 0] \vee [(y_t - \mu) < 0 \wedge (y_{t-1} - \mu) < 0]\right\} \tag{5}$$

From definition (5) it is clear that in order to compute the value of γ we need to know the joint probability density function (p.d.f.) of y_t and y_{t-1}, which can be very difficult to obtain if y_t and y_{t-1} are not independent. However, γ may very easily be obtained if we assume that y_t follows an AR(p) process with normal innovations. As it is well known, in this case, the joint p.d.f. of y_t and y_{t-1} is the bivariate normal distribution

$$\begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix} \sim N\left[\begin{pmatrix} \mu \\ \mu \end{pmatrix}; \begin{pmatrix} Var(y_t) & Cov(y_t, y_{t-1}) \\ Cov(y_t, y_{t-1}) & Var(y_t) \end{pmatrix}\right] \tag{6}$$

Denoting the cumulative distribution function of y_t and y_{t-1} by $F(y_t^*, y_{t-1}^*) = P(y_t \leq y_t^* \wedge y_{t-1} \leq y_{t-1}^*)$ and using the fact that the bivariate normal distribution is symmetric relative to any line that crosses the mean and divides the Cartesian plane evenly, we have $\gamma = 2 \times F(0;0)$ which may be easily computed by resorting to some standard software packages. As a simple illustration, let us consider an AR(2) process $y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t$. In this case $Cov(y_t, y_{t-1}) = \rho_1/(1-\rho_2)$ and, without loss of generality, we may assume $Var(y_t) = 1$. Therefore (6) reduces to

$$\begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 & \frac{\rho_1}{1-\rho_2} \\ \frac{\rho_1}{1-\rho_2} & 1 \end{pmatrix}\right] \tag{7}$$

For instance, for $\rho_1 = 0.5$ and $\rho_2 = 0.2$ the corresponding γ is 0.7149. Obtaining the values of γ for a general ARMA model is straightforward provided normality of the innovations is assumed[3].

Intuitively, the use of γ as a measure of persistence may be justified as a simple implication following directly from the very definition of persistence. If a persistent series is the one which converges slowly to its equilibrium level (i.e., the mean) after a shock, then such a series, by definition, must exhibit a low level of mean reversion, i.e., must cross its mean only infrequently. Similarly, a non-persistent series must revert to

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[3] Difficulties may arise under a different assumption for the distribution of the innovations. But from an empirical point of view this is not a serious problem. Below we suggest an estimator of γ which is an unbiased estimator when the mean of the process is known, so that a fairly good approximation to γ may be obtained using Monte Carlo simulations for any assumed distribution of the innovations.

its mean very frequently. And γ simply measures how infrequently a given time series crosses its mean.

In contrast to ρ, which requires the data generating process (DGP) to follow a pure autoregressive process, γ is defined independently of the specific underlying DGP, provided stationarity is assumed. In this sense γ as a measure of persistence is broader in scope than ρ. To see that just notice that in the simplest case of an ARMA(1,1) process, $y_t = \rho y_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1}$, the parameter ρ (the sum of autoregressive coefficients) is no longer the parameter of interest as it ceases to measure persistence of the y_t series. In fact, we have just seen that ρ is used as a measure of persistence in the context of the pure autoregressive model because it displays a monotonic relationship with the CIR of the model. However, for model ARMA (1,1) we have $CIR = (1-\theta)/(1-\rho)$ so that the one-to-one relationship between ρ and the CIR is lost. But, of course, the fact that the data is generated by an ARMA process does not prevent using γ as a measure of persistence.

2.2 A nonparametric estimator for γ

In empirical applications the DGP is not known so that γ is also not known. In theory an estimate of γ could be computed from an estimated autoregressive model following the steps presented above. But this is not a practical procedure as it would require the specification and estimation of a model and moreover would make the properties of the estimator of γ dependent upon the properties of the estimator of the model. As an alternative we suggest estimating γ by

$$\widehat{\gamma} = 1 - \frac{n}{T} \tag{8}$$

where n stands for the number of times the series y_t crosses the mean during a time interval with $T+1$ observations. The ratio n/T is usually denoted in the literature as the mean crossing rate (MCR) so that we may equivalently write $\widehat{\gamma} = 1 - MCR$.

Since $\widehat{\gamma}$ is computed by counting the number of mean crossings irrespective of the true underlying DGP, γ may be used as a measure of persistence without requiring the researcher to specify and estimate a model for the y_t series. In empirical applications this may imply an important advantage in the use of γ over alternative scalar measures of persistence such as ρ, the half-life, etc. In fact, by construction, $\widehat{\gamma}$ is immune to potential model misspecifications and, given its non-parametric nature, it can be expected to be robust against outliers in the data. We shall investigate such a claim below in section 5.

Note that γ, by definition, and $\widehat{\gamma}$, by construction, are always between zero and one. In the next section, where the values of γ are computed for different stationary processes, it is seen that for a symmetric zero mean white noise process (zero persistence process) we have $\gamma = 0.5$, so that values of $\widehat{\gamma}$ close to 0.5 signal the absence of any significant persistence while figures significantly above 0.5 signal significant persistence. On the other hand, figures below 0.5 signal negative long-run autocorrelation.

In order to investigate the asymptotic distribution of the $\widehat{\gamma}$ statistic let us assume that we have a sample of data with T+1 observations, denoted as y_0, y_1,..., y_T, generated by a stationary and ergodic process with a known mean, μ.

A useful way to proceed is to think of $\widehat{\gamma}$ as being equal to one minus $\overline{x}$, where $\overline{x}$ is the sample mean of a series x_t (t=1, 2, ...T) which equals 1 if the series y_t crosses the mean in period t and is zero otherwise. From here it follows that all the results available in the literature concerning consistence and asymptotic distribution of the sample mean of x_t apply directly to the $\widehat{\gamma}$ statistic. In particular we can invoke the law of large numbers (LLN) and the central limit theorem (CLT).

As a first step, we can start by noticing that x_t can be seen as a binomial random variable, which equals one with probability $(1-\gamma)$ and zero with probability γ. In general, x_t is not independent of x_{t-1}, x_{t-2},..., etc., but the structure of time dependency is determined by the autocorrelation structure of the assumed underlying stationary and ergodic process for y_t. In fact we know that if y_t is a stationary and ergodic process x_t is also a stationary and ergodic process, because it is a measurable function of the current and past values of y_t[4]. Thus it follows from the law of large numbers that $\widehat{\gamma} = 1 - \overline{x}$ is a consistent estimator of γ[5].

But in our case we can go a little further by thinking of the conditions under which $\widehat{\gamma}$ is not only a consistent but also an unbiased estimator of γ. In fact, an alternative way of looking at the x_t series is to think of x_t as a covariance stationary process, which, by definition, meets the conditions i) $E(x_t) = 1 - \gamma$, ii) $E[x_t - (1-\gamma)][x_{t-j} - (1-\gamma)] = r_j$ and iii) $\sum_{j=0}^{\infty} |r_j| < \infty$. In such a case $\widehat{\gamma} = 1 - \overline{x}$ is an unbiased estimator of γ[6].

However, unbiasedness of $\widehat{\gamma}$, under the above conditions, can only be guaranteed when the mean of y_t is known[7]. When the mean of y_t is unknown (and $\overline{y}$, the sample mean is used instead of μ) it follows that the true x_t series is also unknown. What we know is the x_t^* series, which differs from x_t to the extent that the use of $\overline{y}$ instead of μ may imply some additional mean crossings in x_t^* which are not present in x_t. However when $T \to \infty$ we know that $\overline{y}_T \to \mu$ so that $x_t^* \to x_t$ and consistency of $\widehat{\gamma}$ follows.

Under the assumption that x_t (or x_t^*) meets the above conditions for a covariance stationary process it follows immediately by the CLT that $\widehat{\gamma}$ is asymptotically normal distributed[8], i.e.,

$$\frac{\sqrt{T}(\widehat{\gamma} - \gamma)}{\sqrt{\sigma^2_{\widehat{\gamma}}}} \xrightarrow{d} N[0,1] \tag{9}$$

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[4] See White (1984), Theorem 3.35.

[5] See, for instance, White (1984), Theorem 3.34.

[6] See, for instance, Hamilton (1994), Chap.7, section 7.2.

[7] Below we discuss the conditions under which the mean of y_t can be assumed as known or must be assumed as unknown.

[8] See Hamilton (1994) Chap.7, section 7.2.

where $\sigma^2_{\widehat{\gamma}}$ the asymptotic variance of $\sqrt{T}(\widehat{\gamma}-\gamma)$ is given by

$$\sigma^2_{\widehat{\gamma}} = \lim_{T\to\infty} T.E(\widehat{\gamma}-\gamma)^2 = \sum_{j=-\infty}^{\infty} r_j = r_0 + 2\sum_{1}^{\infty} r_j \tag{10}$$

with $r_j = cov(x_t, x_{t-j})$.

In the special case in which y_t follows a symmetric zero mean white noise process (zero persistence) (9) reduces to:

$$\frac{\sqrt{T}\,(\widehat{\gamma}-0.5)}{\sqrt{0.5}} \xrightarrow{d} N(0,1) \tag{11}$$

which allows carrying out some simple tests on the statistical significance of the estimated persistence (i.e., $\gamma = 0.5$).

In order to implement (9) in practice a consistent estimator for $\sigma^2_{\widehat{\gamma}}$ can be obtained following a nonparametric estimator of the form

$$s^2_{\widehat{\gamma}}\,(m) = \widehat{r}_0 + 2\sum_{j=1}^{T-1} w_m(j)\ \widehat{r}_j \tag{12}$$

where $\widehat{r}_j = T^{-1}\sum_{t=j+1}^{T}(x_t-\overline{x})(x_{t-j}-\overline{x})$ is the estimate for the j-th order autocovariance of x_t, and $w_m(.)$ is a lag-window or kernel function depending on a bandwidth parameter m. Examples of conventional kernels often used in the literature are the Bartlett, the Parzen and the Quadratic Spectral kernel (for a discussion see, for instance, Andrews, 1991, or den Haan and Levin, 1997). More recently Politis and Romano (1995) suggested the use of the flat-top lag-window, which according to the authors, may be expected to yield better results in finite samples (see also Politis, 2007)[9].

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[9] The properties of $s^2_{\widehat{\gamma}}\,(m)$ in finite samples depend both on the kernel as well as on the bandwidth parameter used. For this reason, some authors have suggested the use of data-dependent procedures to estimate the optimal bandwidth parameter. This approach was explored, for instance, by Andrews (1991) and Newey and West (1994) for the Bartlett and Quadratic Spectral kernels and by Politis (2003) for the flat-top lag-window. For a discussion on the sources of bias in the kernel-based spectral estimators see den Haan and Levin (1997).

An alternative approach to get an estimate of $\sigma^2_{\widehat{\gamma}}$ is the use of bootstrap time series techniques, especially designed for estimating the variance of the sample mean of a stationary process. Examples are the moving blocks bootstrap, the circular bootstrap, the stationary bootstrap and the tapered block bootstrap (see, Künsch, 1989, Liu and Singh 1992, Politis and Romano, 1992, 1994 and Paparoditis and Politis 2001).

In a recent paper, Kiefer and Vogelsang (2002), denoted as KV(2002) hereafter, proposed an alternative approach to make inference on the parameters of a model, when the residuals are serially correlated. Their results also allow to conduct inference on γ using the estimator $\widehat{\gamma}$. The method is easy to implement and it basically requires to estimate $\sigma^2_{\widehat{\gamma}}$ by some kernel method using the bandwidth parameter equal to sample size in (12). Although this estimator is not consistent for $\sigma^2_{\widehat{\gamma}}$, the authors show that the t-ratio that is built based on this estimator converges to some non-standard distribution that does not depend on nuisance parameters. In particular, when the Bartlett window is used we have the following result for $\widehat{\gamma}$:

$$t^* = \frac{\sqrt{T}(\widehat{\gamma} - \gamma)}{\sqrt{s^2_{\widehat{\gamma}}(T)}} \xrightarrow{d} \frac{W(1)}{\sqrt{2\int_0^1 B_1(r)^2\, dr}}. \tag{13}$$

where $W(1)$ denotes a standard Brownian motion, $B_1(r)$ denotes a standard Brownian bridge and $s^2_{\widehat{\gamma}}(T)$ corresponds to (12) with a bandwidth equal to T[10]. Below resorting to Monte Carlo techniques, we investigate the finite sample properties of $\widehat{\gamma}$ including the performance of (9) and (13) when the Bartlett window is used.

2.3 Testing for changes in persistence using γ

Testing for changes in persistence as measured by γ, in empirical applications, is straightforward. By noticing that $\widehat{\gamma}$ can be obtained as the sample mean of a stationary series,

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[10] The critical values of t^* (which has a symmetric distribution around 0) are reported in KV(2002), Table 1. In particular, for the Bartlett kernel these are 3.764 and 4.771 for the 95 and 97.5 percentiles, respectively.

x_t, testing for changes in γ using $\widehat{\gamma}$ is equivalent to test for changes in the mean of x_t using the sample mean as its estimator. This implies that all the tests available in the literature that allow testing for breaks in the mean of stationary processes may be used to test for changes in γ. Examples include Andrews (1993), Bai (1994, 1997), Bai and Perron (1998) and Altissimo and Corradi (2003). The above tests have the advantage of allowing testing for breaks at unknown points in the sample. But of course if one wants to test for a single change in persistence in a specific (exogenously determined) date we may directly use (9) or (13). Suppose we are investigating persistence for the period $t = 1, 2, ...T$ and we want to test whether there is a change in persistence occurring in period $t = s$, such that persistence for the sub-period $t = 1, 2, ...s$ differs from persistence for the sub-period $t = s + 1, ...T$. By noting that $\widehat{\gamma}$ can be computed by regressing x_t on a constant, i.e., by estimating the model $x_t = \alpha + v_t$ by OLS, from which we get $\widehat{\alpha} = \overline{x} = 1 - \widehat{\gamma}$, to test for a change in persistence it suffices to estimate the model

$$x_t = \alpha_1 + \alpha_2 d_t + u_t \tag{14}$$

where d_t is a dummy variable which is zero until the date of the break ($t \leq s$) and equals 1 thereafter ($t > s$). In (14) we have $\alpha_1 = 1 - \gamma_1$ and $\alpha_2 = \gamma_1 - \gamma_2$ where γ_1 and γ_2 are the measures of persistence for the first and second sub-period, respectively. Thus, testing whether persistence has changed from the first to the second sub-period amounts to test whether α_2 is significantly different from zero in (14). This test can be done by either using (9) or (13), properly adapted to this specific situation. In the case of (9), the variance of $\widehat{\alpha}_2$ must be estimated by some heteroskedasticity autocorrelation consistent estimator, like the ones mentioned in the previous subsection, while in the case of (13) one may use the Newey-West estimator with a bandwidth equal to T.

2.4 The literature on the frequency of mean crossings

It is important to note that $\widehat{\gamma}$ the estimator of our measure of persistence has a strong bearing with some other statistics suggested in the literature in areas involving level-crossing problems, runs tests and unit root tests. The so-called zero-crossing problem, which involves determining the distribution of the length of time between the zeros of a zero-mean stationary random process and computing parameters such as the expected number of zero crossings per unit of time and its variance, has long ago been identified as an important issue in Mathematics/Statistics and related applied fields. Examples of significant contributions that address this problem are Rice (1945), Helstrom (1957), Longuet-Higgins (1962), Itô (1964), Ylvisaker (1965), Wong (1966), Blake and Lindsay (1973), Marcus (1977) and Abrahams (1982, 1986). In a discrete context, the behaviour of the zero crossing rate, which in a zero mean stationary process corresponds exactly to $1-\widehat{\gamma}$, has also been investigated in order to establish important properties of the underlying processes. See, for instance, Kedem and Slud (1982), Kedem (1986), He and Kedem (1990) and Cheng *et al.* (1997).

The estimator $\widehat{\gamma}$ is also closely related to the runs test, which has been used in the literature as a way of testing for the absence of serial correlation. The runs test traces at least to David (1947) (see also Goodman, 1958, or David and Barten, 1962) and may be formulated using the total number of runs in an ordered sequence, the length of the longest run, or the distribution of lengths of runs, a run being an interval in which you are always on one side of the mean, the median or any arbitrary constant. If we take the total number of runs defined with reference to the mean it is immediate to realize that the number of mean crossings equals the total number of runs less one, i.e., $n = R - 1$, where R stands for the total number of runs. Granger (1963) discusses the use of the statistic $S = (2R/T) - 1$ suggested in David (1947), as a way of testing for the absence of serial correlation. Given that $n = R - 1$, we have $S = 1 - 2(T-1)\widehat{\gamma}/T$ so that for large T we get $S \approx 1 - 2\widehat{\gamma}$. Under the assumption that the distribution of the process is symmetric, the David runs test is such that for sufficiently large T the distribution of $R/T \approx 1-\widehat{\gamma}$ can be well approximated by $N(1/2, 1/2\sqrt{T})$ so that we get the result (11) above. Thus, using the runs test R/T is equivalent to using $\widehat{\gamma}$ to test for the absence of serial correlation under the assumption of a symmetric white noise process.

The use of the frequency of mean crossings to test for a unit root in a discrete time series has also been suggested in the literature (see Burridge and Guerre, 1996 and García and Sansó, 2006). Under the assumption that the series, y_t, is generated by a pure random walk process with no drift ($y_t = y_{t-1} + \varepsilon_t$) and initial value equal to zero ($y_0 = 0$) Burridge and Guerre (1996) demonstrated that the statistic

$$K_T^*(0) = \frac{\sqrt{\frac{\sum(\Delta y_t)^2}{T}}}{\frac{\sum|\Delta y_t|}{T}} \frac{n}{\sqrt{T}} = \omega \frac{n}{\sqrt{T}} \tag{15}$$

is such that $K_T^*(0) \xrightarrow{d} |Z|$, where Z is a standard normal N(0,1) and n is the number of sign changes of y_t during a time interval with T observations. It is straightforward to show that $K_T^*(0)$ can be rewritten in terms of the $\widehat{\gamma}$ statistic as $K_T^*(0) = \omega\frac{(T+1)}{\sqrt{T}}(1-\widehat{\gamma})$. Once $\widehat{\gamma}$ converges to 1 as ρ goes to 1, this statistic could in principle be used to test the null of $\gamma = 1$. However, there seems not to be much to be gained in proceeding in such a direction. On the one hand, statistic (15) is valid only under the assumption that the data were generated by a random walk without drift with $y_0 = 0$ and, on the other hand, Burridge and Guerre (1996) evaluated the potential use of $K_T^*(0)$ to test for a unit root and concluded that it has lower power compared to the conventional Dickey-Fuller τ-test. In turn, García and Sansó (2006), which generalised the Burridge and Guerre statistic for processes with a deterministic trend and more general innovations, also concluded that their generalization has lower power than the DF^{GLS} test proposed by Elliot et al. (1996). Thus, if testing for a unit root in the data is considered an appropriate first step, using the unit root tests available in the literature is recommended, before embarking in a persistence evaluation exercise.

3 The relationship between γ and alternative measures of persistence

In this section we compare γ with alternative measures of persistence, in the context of different stationary AR(p) models.

Column (2) of Table 1 presents the values of γ corresponding to the value of ρ in column (1) assuming that the data are generated by the AR(1) process $y_t = \rho y_{t-1} + \varepsilon_t$ with normal innovations, ε_t. For instance, for the AR(1) process with $\rho = 0$ (white noise process) γ is 0.50 while, for the AR(1) process with $\rho = 0.60$ the corresponding γ is 0.705. Table 1 shows that, in the context of the AR(1) process, there is a one-to-one correspondence between these two measures of persistence.

Table 1 also reports the half-life, h, defined as the number of periods for which the effect of a unit shock remains above 0.5, as well as, m_{50}, m_{95} and m_{99}, which denote the number of periods required for the accumulated effect of a unit shock to be equal to 50%, 90% and 99% of the total effect, respectively. We see these measures of persistence as being useful to evaluate how fast the series "approaches" the equilibrium[11].

Looking at columns (3) and (4) we realise that h and m_{50} assume exactly the same values for the different values of ρ, as one would expect under an AR(1) process. However, it is apparent from Table (1) that these two measures are not very informative for values of $\rho \leq 0.70$ as they do not allow discriminating among the different models.

As to the remaining two measures of persistence m_{95} and m_{99} they exhibit a monotonic relationship with ρ and γ (specially so for m_{99}). Thus, in the context of the AR(1) model, ρ, γ and m_{99} (leaving aside, for the moment, potential estimation problems) appear as giving the same message about the degree of persistence. This

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[11] We compute these four measures of persistence directly from the IRF. Moreover, in contrast to some literature that computes the half-life on a continuous basis, in Tables 1 and 2 we assume a discrete range of variation for this measure of persistence (as well as for m_{50}, m_{95} and m_{99}). In our view, keeping the half-life on a discrete domain seems more in line with its definition because "the number of periods" is a discrete variable..

result stems from the fact that the speed of convergence to the equilibrium in the AR(1) model is constant throughout the adjustment period[12].

In order to investigate whether this relationship among the different measures of persistence carries over to higher order autoregressive models we now consider the AR(2) process, $y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_t$. Given the wide range of possibilities for the two autoregressive coefficients we set the sum of the autoregressive coefficients equal to 0.80 (i.e., $\rho = \rho_1 + \rho_2 = 0.80$). Thus models, (1) to (9) in Table 2 correspond to different combinations of the two AR coefficients that meet the conditions: i) $\rho_1 \geq 0$, ii) $\rho_1 + \rho_2 = 0.80$ and iii) the model is stationary. In addition to the measures of persistence considered in Table 1, Table 2 also includes the "largest autoregressive root", lar, as an alternative measure of persistence[13].

In Table 2 models are listed according to the values of ρ_1 in descending order. If we stick to the "sum of the autoregressive coefficients" as the single measure of persistence all the 9 models would be seen as equally persistent with $\rho = 0.80$. However, as we move from model (1) to model (9) we see that the value of γ and of the half-life, h, decrease monotonically while, in strong contrast, the values of lar, m_{50}, m_{95} and m_{99} increase monotonically. Thus, if instead we stick to one of these alternative six measures the 9 models would appear as essentially different in terms of persistence. In other words, the 6 alternative measures of persistence behave quite independently from the value of ρ[14].

This is a very important result as it shows that the conclusion obtained above for the AR(1) model that the alternative measures of persistence exhibit a monotonic relation among them (and with ρ) and thus all convey the same message, does not carry out to more general autoregressive processes. By investigating the IRF's of the 9 AR(2)

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[12] In other words, in the AR(1) case the disequilibrium in period k, ρ^k, is a fixed proportion, ρ, of the disequilibrium in period $k-1$, ρ^{k-1}.

[13] On the use of the largest autoregressive root as a measure of persistence see Stock (1991, 2001).

[14] Notice that for models (1), (2) and (3), which have complex roots, the IRF is oscillatory around zero with the consequence that the partial sums of the IRF are also oscillatory. This implies that the number of periods required for a given proportion of the total adjustment to take place is not uniquely defined. Thus, in order to obtain the values of h, m_{50}, m_{95} and m_{99} in Table 2 we took the number of periods required for the values of these statistics to be observed for the first time.

processes in Table 2, we conclude that it is the non-constant speed of convergence in the IRF which gives rise to the diverging results for the different measures of persistence.

The idea that the scalar measures of persistence could in some specific situations be very misleading about the true level of persistence is, of course, not new[15]. What seems to be new (at least for the authors) is the extension of the problem. More than saying that in some special cases the CIR (and thus all the measures based on the CIR as ρ, m_{50}, m_{95} or m_{99}) is not a good summary measure of the information contained in the IRF (and thus is not a good measure of persistence) it seems more appropriate to state that with the exception of the very special case of the AR(1) model, the scalar measures of persistence for the general autoregressive model may be very misleading, either because they may suggest the presence of a strong degree of persistence when it is absent or the absence of significant persistence when it is present.

We may push the argument a little further by looking at specific realizations of some of the AR(2) processes. For that purpose Graph No.1 displays a realization of models (1), (5) and (9) with 80 observations[16]. On the one hand it seems difficult to argue that the three models, despite having the same $\rho = 0.80$, should be seen as equally persistent, given the disparate values for the 6 measures of persistence in Table 2. On the other hand if we think of persistence as the frequency of mean reversion we see

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[15] Andrews and Chen (1994) discuss several situations in which the CIR and thus also ρ might not be sufficient to fully capture the existence of different shapes in the impulse response function. In particular, ρ and thus also the CIR will not be able to distinguish between two series in which one exhibits a large initial increase and then a subsequent quick decrease in the IRF while the other exhibits a relatively small initial increase followed by a subsequent slow decrease in the IRF.

In turn, Pivetta and Reis (2007) list the main limitations of the "half-life". In particular, they notice that the "half-life" may not be a good measure of persistence, if the IRF is oscillating and, even for monotonically decaying processes, it is not adequte to compare two different series, if one exhibits a faster initial decrease and then a subsequent slower decrease in the IRF than the other. Finally, for highly persistent processes the half-life is always very large and thus makes it difficult to distinguish changes in persistence over time.

The use of the largest autoregressive root as a measure of persistence is criticised both in Andrews and Chen (1994) and in Pivetta and Reis (2007). The main point against this statistic is that it is a very poor summary measure of the IRF because the shape of this function depends also on the other roots and not only on the largest one.

[16] The three series in Graph No.1 were generated using the same series of residuals generated from the N(0,1) distribution.

that mean reversion in model (5) and model (9) is clearly higher than in model (1) suggesting that persistence could be lower in those two models. Moreover, we also see that mean reversion in model (9) is higher than in model (5). From Table 2 we can see that γ, starting with model (1), decreases monotonically from a value as high as 0.853 (signalling a very persistent process) to a figure as low as 0.50, which signals a model with zero persistence.

We expect γ to be equal to 0.50 when a white noise process generates the data, but by simply eyeballing the series we see that model (9) in Graph No.1 does not behave like a white noise. Rather it seems to display a kind of cyclical behaviour, which makes the process to cross the mean at irregular intervals, but such that on average it crosses the mean as often as if it were a symmetric white noise process. This means that γ does not distinguish between a process with a low ρ (close to a white noise behaviour) and a process with a cyclical pattern such that, on average, during a given time interval the two processes cross the mean the same number of times. In other words γ, in contrast to ρ, does not see the cyclical pattern of the process as relevant persistence.

Given the evidence in Table2 that shows that for higher order processes the different scalar measures of persistence may deliver conflicting views on the degree of persistence, it seems wise, in empirical applications, not to rely on a single measure of persistence. In this regard, using γ and ρ as companion measures of persistence seems to be a good strategy that may allow uncovering important features of persistence, that would not be identifiable if we stuck to a single measure. In fact, we have just seen that a high value of ρ accompanied by a low value of γ could be a sign of a cyclical pattern in the DGP. We also shall see below that an estimate of ρ clearly below to what could be expected given the value of $\widehat{\gamma}$ may be a signal of significant downward biases in $\widehat{\rho}$ stemming from the presence of outliers in the data.

4 Some Monte Carlo evidence on the finite sample properties of $\widehat{\gamma}$ and $\widehat{\rho}$.

In this section we use some Monte Carlo experiments in order to investigate the properties of $\widehat{\gamma}$ and $\widehat{\rho}$ regarding i) unbiasedness, ii) robustness to outliers and iii) coverage ratio of empirical confidence intervals. These properties are investigated in the context of the AR(1) and AR(2) processes considered in section 4.

From the discussion in section 3 we may expect the information about the mean of the process to be statistically relevant for persistence evaluation. For this reason, below we distinguish the situation in which the mean is known from the situation in which the mean is unknown. For example, in the context of an inflation persistence evaluation exercise, assuming that the mean is known may be realistic for those countries for which an inflation targeting monetary policy was implemented and an explicit inflation target was announced. In this case, the true mean of the series can be computed exogenously to realised inflation and is given by the publicly announced inflation target. However, for most countries, the exact (implicit) inflation target used by the central bank when setting monetary policy is unknown. In theses cases the mean must be computed from realised inflation. As we shall see below this may have noticeable consequences for the process of persistence evaluation if the sample is not very large.

4.1 Unbiasedness

Let us start by assuming that the true mean of the process is known and define an experiment that constructs the data to follow an AR(1) process (with no intercept) given by $y_t = \rho y_{t-1} + \varepsilon_t$, for ρ ranging between 0 and 1 and where the errors are serially uncorrelated standard normal variables. Samples of size $T = 50$, 75, 100, 150, 250, 500 and 1000 are used in the experiments[17].

The output of the experiment for T=100 is displayed in Table 3[18]. For values of ρ ranging from 0 to 0.95, column (2) reports the values of γ (taken from Table 1), column (3) reports the average value for the Monte Carlo OLS estimates of ρ ($\overline{\rho}$) and column (5) the average value of the Monte Carlo estimates of γ ($\overline{\gamma}$). From columns (1) and (3) we can see that the OLS estimator of ρ is slightly (mean) downward biased and that the absolute bias increases as ρ increases, as expected (see, Sawa, 1978, Phillips, 1977, Evans and Savin, 1981, Andrews, 1993, Andrews and Chen, 1994).

As regards the $\widehat{\gamma}$ statistic we see that the values of γ and $\overline{\gamma}$ in columns (2) and (5) are not statistically different i.e., $\widehat{\gamma}$ behaves as an unbiased estimator of γ, as expected, given the discussion in section 3 [19].

Let us now assume that the mean of the process is unknown and thus has to be estimated from the data. As discussed above, in most situations, this may be a more realistic assumption. The results of this new Monte Carlo experiment are in columns (6)-(9) of Table 3[20]. Looking at column (7) we see that the downward bias of OLS estimator has now significant damaging consequences on the expected estimates of ρ. In fact, the bias has now more than doubled vis-à-vis the situation in column (4), confirming the claim in Sawa (1978) and Andrews (1993) that the bias of $\widehat{\rho}$ is more acute when one assumes that the mean of the process is unknown and has to be estimated from the data

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[17] As the sampling distribution $\widehat{\rho}$ depends on the initial value of the process, y_0 (see, for instance, Evans and Savin,1981) in our simulations we create $T + 100$ observations and discard the first 100 observations in order to remove the effect of initial conditions.

All the experiments are replicated $10,000$ times with the data generated by setting $y_{-100} = 0$. The replications were carried out using TSP 5.0.

[18] The output of the simulations for other values of T is available from the authors upon request

[19] Note that the sampling variability allows $\overline{\gamma}$ (and $\overline{\rho}$) to change for a fixed T. As the standard error of $\overline{\gamma}$ may be approximated by $(0.5/\sqrt{T})/\sqrt{10000}$ and $\overline{\gamma}$ has a Normal distribution, a 95% confidence interval for $\overline{\gamma}$ is given by $\overline{\gamma} \pm 1.96 * (0.5/\sqrt{T})/\sqrt{10000}$, which reduces to $\overline{\gamma} \pm 0.001$ for $T = 100$. From the output of the simulations we see that the values of $\overline{\gamma}$ do not differ from γ by more than 0.001, so that the difference can be attributed to sampling variability.

[20] Figures in Table 3 were obtained by generating the model $y_t = 0.01 + z_t$ with $z_t = \rho z_{t-1} + \varepsilon_t$. We note that the sampling distributions of $\widehat{\rho}$ and $\widehat{\gamma}$ do not depend on the value of the mean, but simply on the fact that the mean is unknown and has to be estimated.

using a model with an intercept. In empirical applications this downward bias of $\widehat{\rho}$ will naturally translate into all measures of persistence that are computed using an estimate of ρ (h, m_{50}, m_{95} and m_{99}, in Table 1). This is the case for which it might be worth using the "approximated median unbiased estimator" suggested in Andrews and Chen (1994).

As to the $\widehat{\gamma}$ statistic we see that it also appears slightly downward biased as expected given the discussion in section 3. This result is intuitive, as the estimator of the mean (the sample average) is expected to increase mean reversion vis-à-vis the situation with the true mean, and thus reduce the estimated γ.

Let us now take a look at the AR(2) process. In our Monte Carlo experiment we consider as our DGP's the same 9 models of section 4 for which $\rho = \rho_1 + \rho_2 = 0.80$. The output of the experiment is in Table 4. To facilitate comparisons, column (6) reports the values of γ taken from Table 2.

By looking at columns (2) and (3), we see that the biases of $\widehat{\rho}$ increase monotonically as we move from model (1) to model (9). This is an interesting result because it shows that for higher order processes the bias of $\widehat{\rho}$ depends not only on ρ but also on the specific combination of ρ_1 and ρ_2. In strong contrast, by comparing columns (6) and (7) we see that once again, as expected, $\widehat{\gamma}$ behaves as an unbiased estimator of γ.

If we assume that the mean of the process is unknown and estimate a model with an intercept we find that, as expected, the downward biases of the OLS estimator of ρ increase significantly (columns (4) and (5)). As regards the $\widehat{\gamma}$ statistic we find that, similarly to the AR(1) case, a small downward bias emerges (columns (8) and (9)) but it is always smaller than the bias displayed by $\widehat{\rho}$.

Thus, from the preceding analysis, we conclude that in the process of persistence evaluation it may be worth distinguishing between two different possibilities. When the mean is known we may expect to be able to estimate persistence with no (expected) bias if $\widehat{\gamma}$ is used or, with a small downward bias if $\widehat{\rho}$ is used (in this latter case, assuming also that the order of the process is known). However, when the mean is estimated from the data, which corresponds to the common practice in the literature, we may expect such a fact to introduce an additional downward bias into the conventional measures of persistence. This bias might be particularly significant if OLS estimators are used to get an estimate of ρ. The $\widehat{\gamma}$ statistic is very much less affected in such a situation.

4.2 Robustness to outliers

In order to evaluate the robustness of $\widehat{\gamma}$ and $\widehat{\rho}$ to the presence of outliers in the data we define the DGP as corresponding to the AR(1) model without an intercept used in sub-section 5.1, with the addition of 5% of observations drawn from the $N(0,5^2)$ distribution[21]. Table 5 reports the results for $T = 100$. Column (4) presents the estimated bias for $\widehat{\rho}$ measured as a percentage deviation from the estimated values obtained in the absence of outliers (see column (3) in Table 3). This way we are measuring only the bias due to outliers.

The first important comment is that the presence of additive outliers in the data has a devastating effect on the OLS estimators. For instance, when $\rho = 0.60$ the average estimated $\widehat{\rho}$ ($\overline{\rho}$ in column (3)) is as low as 0.359 (it is equal to 0.589 when no outliers are present) which corresponds to a downward bias of 39.07%. The effect of outliers decreases as ρ increases, but even for values of ρ as large as 0.80 the average $\widehat{\rho}$ is only 0.56.

As regards the $\widehat{\gamma}$ statistic, the estimated bias (measured as a percentage deviation from the estimated values obtained in the absence of outliers in column (5) of Table 3) is reported in column (7). We can see that there is some downward bias as expected (given that some outliers will imply an additional crossing of the mean), but it is quite

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[21] The type of outliers we consider are those that correspond to shocks that affect observations in isolation due to some non-repetitive events, which may occur as a result of measurement errors or special events (changes in VAT rates or union strikes, for instance). This type of outliers, usually referred to in the literature as additive outliers, is known to have strong impacts on the parameters of estimated models (see, for instance, Lucas, 1995).

small. For instance, for the model with $\rho = 0.60$ the average $\widehat{\gamma}$ is now 0.689 while it was 0.705 when no outliers were present. In general the bias of $\widehat{\gamma}$ due to outliers increases as ρ increases but it always remains very small.

If instead we take a look at the estimated standard errors of both $\widehat{\rho}$ and $\widehat{\gamma}$ (not shown in the Tables) and compare them to the corresponding standard errors obtained in the absence of outliers, we conclude that the implications are much stronger for the standard errors of $\widehat{\rho}$. In fact, while the standard errors of $\widehat{\gamma}$ show a small increment the standard errors of $\widehat{\rho}$ for higher values of ρ more than doubled. The implications for the standard deviations would naturally be reflected, for instance, in the properties of the interquartile range of each estimator. From column (5) we see (with the exception of the model with $\rho = 0.00$) that the interquartile range of the OLS estimator does not include the true ρ (nor the estimated ρ when no outliers are present). In contrast, the interquartile range for $\widehat{\gamma}$ (column (8)) always includes the true γ (or the estimated γ obtained in Table 3 when no outliers are present).

Results in Table 5 are of course specific to the particular way we generated the data, and less extreme outliers are expected to have less damaging consequences for the estimators. However, the exercise carried out shows that in general we can expect $\widehat{\gamma}$ to be more robust to the presence of additive outliers in the data than the OLS estimator of ρ.

4.3 Coverage ratio of empirical confidence intervals

In order to evaluate the consequences for inference stemming from the use of empirical estimates of the asymptotic variance of $\widehat{\gamma}$ in finite samples, in this sub-section we compute the coverage ratio of 95% confidence intervals for ρ and γ using the same AR(1) and AR(2) models, as before.

We restrict our simulations to the Bartlett kernel-based estimator as it is probably the most widely used kernel in so-called heteroskedasticity and autocorrelation consistent estimation and also the one that underlies some well-known nonparametric unit root and stationary tests as the ones suggested by Phillips and Perron (1988) and Kwiatkowski *et al.* (1992). Thus, to compute the long run variance of $\widehat{\gamma}$ we use the estimator

$$s^2_{\widehat{\gamma}} = T^{-1}\sum_{t=1}^{T}(x_t - \overline{x})^2 + 2T^{-1}\sum_{j=1}^{m_k}(1 - \frac{j}{m+1})\sum_{t=j+1}^{T}(x_t - \overline{x})(x_{t-j} - \overline{x})$$

for alternative number of lags, m_k, determined using three different procedures. The first approach, followed for instance in Schwert (1989), defines m_k as a function of the number of observations such that $m_k = int\left[k(T/100)^{1/4}\right]$, for alternative values of k[22]. The second approach uses the automatic data-based procedure suggested in Andrews (1991), using the AR(1) plug-in method. Finally, the third approach follows the methodology suggested in KV(2002), in which the bandwidth parameter is equal to the sample size. The two first approaches are examples of the conventional procedure that involves the use of a consistent estimator for the asymptotic variance of $\widehat{\gamma}$, while the KV(2002) approach uses an estimator for the variance of $\widehat{\gamma}$ which is not consistent.

The results of the Monte Carlo simulations for the AR(1) and AR(2) models with $T = 100$, $T = 250$ and $k = 8$, are in Tables 6 and 7. The confidence intervals for γ, in the first two approaches, and for ρ are constructed using the Normal distribution to approximate the true finite sample distribution of $\widehat{\gamma}$ and $\widehat{\rho}$. In the third approach the confidence intervals for γ are constructed using the critical values supplied by KV(2002)[23].

The first important point to note, regarding the use of γ, is that the first two conventional approaches deliver quite different results vis-à-vis the third approach. On the one hand, the exercise shows, for the type of models estimated, that the Bartlett estimator with a predetermined bandwidth parameter, usually underestimates the asymptotic variance of $\widehat{\gamma}$ so that the effective coverage ratio is below its nominal level[24]. Strangely

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[22] Below we present the results for $k = 8$. We carried out simulations also for $k = 4$ and $k = 12$, but the qualitative conclusions do not change.

[23] In the present case 4.771 is used as the critical value to construct the confidence intervals.

[24] This outcome accords with the available Monte Carlo evidence for stationarity tests that rely on kernel estimators (see, for instance, Kwiatkowski *et al.* 1992, Lee, 1996, Caner and Kilian, 2001).

enough the Andrews approach despite defining the bandwidth parameter as a function of the autoregressive structure of the data, does not significantly improve on the previous approach, notably so for higher values of ρ in the AR(1) case. Not surprisingly, given the simulations performed by the authors, the estimator suggested in KV(2002) performs significantly better than the other two, delivering confidence intervals for γ whose effective coverage ratio is closer to its nominal level. This of course also means that tests on γ based on the two conventional approaches tend to be oversized, especially so for larger values of ρ and γ, but tests on γ based on the KV(2002) approach have good size properties.

If we consider the relative performance of $\widehat{\rho}$ and $\widehat{\gamma}$ we see that $\widehat{\rho}$ performs better than $\widehat{\gamma}$ when inference on $\widehat{\gamma}$ is made using the two conventional methods, but not when inference on $\widehat{\gamma}$ is made using the KV(2002) approach. In fact, in this latter case, if anything, the coverage ratio for γ emerges as slightly better than the coverage ratio for ρ[25]. Notice in addition that the performance of $\widehat{\rho}$ and $\widehat{\gamma}$ decreases not only as ρ and γ increase in the AR(1) case, as expected, but also as we move from model (1) to model (9) in the AR(2) case, despite ρ being held constant. Besides the increased biases in the estimators documented in sub-section 5.1, such an outcome must also be reflecting increased difficulties in the estimation of the variance of $\widehat{\rho}$ and $\widehat{\gamma}$. As expected, when T increases the difference in the performance of the different approaches is reduced.

An interesting final question regards the power of the tests involving $\widehat{\gamma}$. According to simulations in KV(2002) the Bartlett kernel estimator, among the common choices of kernels, produces the highest power function, when the bandwidth is equal to the sample size. However, the power of the tests based on the KV(2002) approach seems to be less than that which can be attained using conventional procedures involving consistent

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[25] Notice that the exercise performed here is very benevolent for ρ as it is conducted by estimating the correct model. In empirical applications the relative performance of $\widehat{\rho}$ in terms of its coverage ratio may be expected to be significantly weakened because the true model is unknown (and this would generally imply additional biases for $\widehat{\rho}$) and the data may contain some outliers (which, as we have seen, have a stronger negative impact on the performance of $\widehat{\rho}$).

estimators for the asymptotic variance of $\widehat{\gamma}$. In fact according to the simulations in Kiefer and Vogelsang (2005) for kernel and bandwidth choice there is a trade-off between size distortions and power. Smaller bandwidths lead to tests with higher power but greater size distortions while large bandwidths lead to tests with lower power but less size distortions.

5 Persistence in the United States and the Euro Area.

As noticed in the introduction of the paper, inflation persistence has been, over the last decade, one of the most intensely investigated topics in macroeconomics. Issues such as whether inflation is persistent or not, whether it has changed over time, whether it is structural or may vary according to the specific monetary policy regime, are examples of relevant questions that have been addressed in the literature. As far as the United States (U.S.) and the Euro Area (E.A.) are concerned, there is now a vast number of contributions aiming at quantifying the degree of inflation persistence and investigating whether it has changed over time. For the U.S., Burdekin and Siklos (1999), Bleaney (2001), Stock (2001), Willis (2003), Levin and Piger (2004), Pivetta and Reis (2007) and Cogley and Sargent (2001, 2007) are examples of important contributions. For the E.A. as whole the evidence is naturally more limited (see, however, Gadzinski and Orlandi, 2004).

In this section we contribute to this empirical literature by investigating the degree of inflation persistence in the United States (U.S.) and the Euro Area (E.A.) using γ and ρ as alternative measures of persistence for an updated sample that includes data until the end of 2006. The main purpose is, on the one hand, to illustrate the use of γ, as a measure of persistence and, on the other hand, to see if by using the new measure of persistence we are able to uncover some important features of persistence in these two countries that would have remained undetected if we stuck to ρ, as the single measure of persistence.

A major issue when evaluating the degree of inflation persistence regards the way the mean of inflation is dealt with. Assuming a constant or a time varying mean for inflation makes all the difference for the estimated persistence. The literature has addressed this issue by testing for breaks in the mean and computing persistence conditional on such breaks (see for instance, Burdekin and Siklos, 1999, Bleaney, 2001, Levin and Piger, 2004, Gadzinski and Orlandi, 2004), by dividing the samples according to historically different monetary policy regimes (Alogoskoufis and Smith, 1991, Burdekin and Siklos, 2001, Benati, 2007) or allowing for a pure time varying mean of inflation as a way of capturing shifts over time in the central bank inflation target (Smets and Wouters, 2003, Cogley and Sargent, 2001, 2007)[26]. Here, because our sample period covers only the last twenty three years of data we follow the first approach by conducting break tests in the mean of inflation and computing persistence conditional on the estimated means.

To test for a break in the mean we resort both to the Andrews (1993) and the Altissimo and Corradi (2003) tests that allow testing for changes in the mean at unknown points in the sample[27]. To run the Andrews test we impose 20% symmetric trimming to avoid detecting spurious breaks at the beginning or at the end of the sample. The Andrews test detects a break in 1991q3 for the U.S. and a break in 1993q2 for the E.A., while the Altissimo and Corradi test detects a break in 1991q1 for the U.S. and a break

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[26] Cogley and Sargent (2007) denote the difference between inflation and the time varying mean (which they define as a pure random walk component) as the "inflation gap". It is the persistence of this "inflation gap" that the authors investigate in their paper. In our view, however, there seems to be no reason to distinguish between inflation persistence and "inflation gap" persistence. By definition, measuring persistence means measuring the persistence of deviations from the mean. So what varies across the different approaches is just the way the mean is dealt with.

[27] We thank Fillipo Altissimo and Valentina Corradi for sharing with us the code of the Altissimo and Corradi test.

in 1993q2 for the E.A..[28] Given that the analysis does not change in any significant way, in what follows we stick to the break dates identified by the Andrews tests.

Graph No.2 displays inflation for the U.S. and the E.A. together with their (constant) means, where inflation is measured by the first difference of logged GDP deflator for the period 1984q1-2006q4[29]. The upper panels of Graphs No.3 and No.4 display inflation for the U.S. and the E.A. allowing for a break in the mean, at the above identified dates, while the lower panels of these two Graphs depict the corresponding deviations from the mean.

Table 8 displays the main results for the two series[30]. Column (1) displays the full sample estimates of γ and ρ under the assumption of a constant mean, while column (2) reports the corresponding estimates under the assumption of a break in the mean. Below $\widehat{\gamma}$ and $\widehat{\alpha}_2$ are the KV(2002) standard-errors obtained with the Bartlett window. Conditional on a constant mean we get point estimates for persistence that are somewhat higher for the E.A. than the U.S. ($\widehat{\gamma}$=0.813 and $\widehat{\rho}$=0.879 for the E. A. and $\widehat{\gamma}$=0.703 and $\widehat{\rho}$=0.752 for the U.S.), even though the null of equal persistence in the two countries may not be rejected for a 5% test.

If we look at persistence conditional on a break in the mean for the whole sample period, in column (2), we find that estimated persistence is now significantly lower as expected ($\widehat{\gamma}$=0.725 and $\widehat{\rho}$=0.559 for the E.A. and $\widehat{\gamma}$=0.615 and $\widehat{\rho}$=0.615 for the U.S.) and that once again the null of equal persistence in the two countries may not be rejected

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[28] These two series (for a smaller sample) have been studied in Gadzinski and Orlandi (2004) who found the breaks in the mean to occur in 1991q2 for the U.S. and 1993q2 for the E.A.. Notice that 1991q2 is also the date of the break identified in Levin and Piger (2004) for the U.S..The change of one quarter in the date of the break for the U.S., found in our case, is probably due to the fact that we are using a larger sample.

[29] The data on GDP deflator for the U.S. were downloaded from the Bureau of Economic Analysis website while that for the E.A. were constructed by udating the data in Fagan et al. (2001) with the ECB official series from 1995q1 onwards. Both series are seasonally adjusted.

[30] Estimates of γ were obtained by using equation (8) in Section 3, while estimates of ρ were obtained by estimating an autoregressive model for the series of the deviations from the mean. For the autoregressive model a general-to-specific approach was followed which delivered a model with 5 lags for the U.S. and with 3 lags for the E.A..

for a 5% test[31]. These results suggest that, irrespective of the measure used, persistence of inflation in the E.A. and the U.S. is not high and does not differ significantly between the two countries. Of course, we must be aware that these conclusions are conditional on an estimated mean, which as we have seen in section 5 is likely to introduce some downward bias into the estimators of our measures of persistence.

Column (3) displays the results of a Chow-type test for a change in persistence, occurring at the dates identified for the break in the mean: 1991q3 for the U.S. and 1993q2 for the E.A.. The idea is to investigate whether persistence differs for high and low levels of inflation. From Table 8 we see that the null of unchanged persistence for the two sub-periods cannot be rejected both for the U.S. and the E.A., irrespective of whether we use γ or ρ to measure persistence (the t-ratios of $\widehat{\lambda}$ are clearly below the 5% critical value of the normal distribution and the t-ratios of $\widehat{\alpha}_2$ are well below the 5% critical value reported in Table 1 of KV(2002))[32]. Thus, from this exercise we conclude that, for this sample period, there is no significant evidence that persistence has changed over time with the average level of inflation.

This exercise, however, does not preclude the possibility of changes in persistence having occurred in a different part of the sample, not directly related to changes in the average level of inflation. To investigate this possibility we tested for changes in persistence occurring at an unknown point in the sample using again the Andrews and the Altissimo and Corradi tests. When we assume a constant mean, neither test detects

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[31] Notice that the estimates for ρ compare very closely with the ones obtained in Gadzinski and Orlandi (2004). For the the period 1984q1-2003q3, these authors, using median unbiased estimators, obtained $\widehat{\rho}$=0.60 for the E.A. and $\widehat{\rho}$=0.52 for the U.S.

[32] Tests on γ were performed as explained in section 3, by estimating equation (14) and computing the variance of $\widehat{\alpha}_2$ using the KV(2002) estimator with the Bartlett window. Tests of a change in ρ were performed by estimating autoregressive models that allow for the possibility of a break in the persistence parameter. In particular, the estimates $\widehat{\lambda}$ in Table 8 were obtained from the model

$$z_t = \sum_{j=1}^{p-1} \delta_j \Delta z_{t-j} + \rho_1 z_{t-1} + \lambda d_t . z_{t-1} + \varepsilon_t$$

where z_t is the series of inflation (deviations from the mean) and d_t is a dummy variable which is zero until the date of the break ($t \leq s$) and equals 1 thereafter ($t > s$). More general models were also estimated (for example allowing for the possibility of changes in the δ_j parameters) but the conclusions do not change.

any break in persistence for the U.S. or the E.A., irrespective of whether we look at γ or ρ as alternative measures of persistence. Under the assumption of a break in the mean, the two tests still do not find any evidence of a change in persistence for the U.S.. However, the results for the E.A. are quite different according to whether one looks at ρ or γ as the relevant measure of persistence. In fact, if we stick to ρ as the single measure of persistence we are unable to detect any change in persistence, but if we look at γ instead, both the Andrews as well as the Altissimo and Corradi tests suggest that a change in persistence has occurred in 2001q4 (according to the Andrews test) or in 2000q1(according to the Altissimo and Corradi test).

We can get some intuition for this result by looking at Chart 4. It is immediate to recognise that mean reversion is clearly stronger for the period 2001/2002-2006 than it is in the period before (1984-2000) which is a clear sign that persistence has changed around 2000/2001. In fact, for the period 1984q1-2001q4 we get $\widehat{\gamma} = 0.803$ (sd. 0.022) while for the period 2002q1-2006q4 we get $\widehat{\gamma} = 0.450$ (sd. 0.033), suggesting that persistence in the E.A. has completely vanished during the last 5 years or so, as γ is now not significantly different from 0.50.[33]

Thus, even though with some uncertainty about the specific quarter for the date of the break, the relevant point to be stressed is that when γ is used to measure persistence, both tests suggest strong evidence of a break in persistence coinciding with the launching of the euro as the common currency, and the implementation of single monetary policy by the European Central Bank for the twelve euro area countries. This, we think, is a very important result that to our knowledge had so far remained undetected in the literature[34].

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[33] If we take 2000q1 as the date of the break we get $\widehat{\gamma}$=0.828 (sd. 0.032) for the period 1984q1-2000q1, while for the period 2000q2-2006q4 we get $\widehat{\gamma}$=0.481 (sd. 0.027)

[34] Of course whether this strongly reduction in persistence in the E.A. is a direct consequence of the implementation of a common monetary policy by the ECB, remains an open issue. In fact, the possibility of such a change stemming from a change in the type of shocks that hit the E.A. during the last five years or so, may not be ruled out on a priori grounds. But such an investigation is outside the scope of the present paper.

Overall, conditional on a break in the mean, the U.S. and the E. A. do not differ significantly as far as inflation persistence for the period 1984-2006 is concerned. Both countries exhibit low levels of persistence and there is no significant evidence that the degree of persistence has changed over time with the average level of inflation. This evidence is basically in line with the results in Gadzisnki and Orlandi (2004) for the U.S. and E.A. and in Levin and Piger (2004) for the U.S.. However, when we look for changes in persistence not related to changes on average inflation, the new measure of persistence suggested in this paper allows us to detect a significant reduction in persistence for the E.A. occurring in 2000/2001, following the creation of the euro area and the implementation of a common monetary policy by the European Central Bank.

6 Conclusions

This paper suggests a new scalar measure of persistence, denoted by γ, together with a companion estimator, $\widehat{\gamma}$, which explores the relationship between persistence and mean reversion. The new measure is defined as the unconditional probability of a stationary stochastic process not crossing its mean in period t, and has the property of being broader in scope than other measures used in the literature and, in particular, the widely used ρ, the sum of the coefficients in a pure autoregressive process.

The process of obtaining the theoretical value of γ for a stationary ARMA process under the assumption of normal innovations is derived and the properties of the non-parametric estimator $\widehat{\gamma}$ are investigated. It is shown that $\widehat{\gamma}$ is an unbiased estimator of γ when the mean of the process is known and a consistent estimator of γ when the mean is unknown. Moreover, $\widehat{\gamma}$ is asymptotically normal distributed.

Inference on γ may be conducted resorting either to the conventional approach in which a consistent kernel estimator for the asymptotic variance of $\widehat{\gamma}$, $\sigma^2_{\widehat{\gamma}}$, is used, or to the approach suggested in Kiefer and Vogelsang (2002), in which a non-consistent kernel estimator for $\sigma^2_{\widehat{\gamma}}$ is used.

Using Monte Carlo simulations the finite sample properties of $\widehat{\gamma}$ are compared to those of $\widehat{\rho}$, the OLS estimator of ρ. We find that $\widehat{\gamma}$, which by construction is immune to potential model misspecifications, is also robust to the presence of outliers in the data and that the coverage ratio of their empirical confidence intervals compares favourably to that of $\widehat{\rho}$, when inference on $\widehat{\gamma}$ is made using the approach suggested in Kiefer and Vogelsang (2002). However, in empirical applications the use of this approach, as a valid alternative to conventional methods, must be weighted against the implied loss in the power of the tests.

The relation between γ and some alternative measures of persistence available in the literature, namely ρ, is also analysed. It is shown that there is a monotonic relationship between ρ and γ when the data is generated by an AR(1) process, but for higher order processes the different scalar measures of persistence may deliver conflicting views on the degree of persistence. For that reason the use of γ and ρ as companion measures of persistence is suggested, as a strategy that may allow uncovering important features of the underlying process, that would otherwise remain unidentified.

Finally, the use of the new measure of persistence is illustrated by evaluating inflation persistence in the United States and the Euro Area for the period 1984q1-2006q4. In line with other empirical studies for a similar period, we find that, conditional on a break in the mean of inflation, both countries exhibit low levels of persistence and there is no evidence that persistence has changed with the average level of inflation. However, when we look for changes in persistence not related to changes in the average level of inflation, the use of γ allows us to detect a significant reduction in persistence in the Euro Area, occurring in 2000/2001, coinciding with the launch of the euro and the implementation of a common monetary policy by the European Central Bank. This reduction, which would have remained undetected if we stuck to ρ as the single measure of persistence, is such that persistence in the Euro Area becomes virtually nil for the period 2000-2006.

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Table 1

Comparing γ with other measures of persistence

AR(1) case

ρ	γ	h	m_{50}	m_{95}	m_{99}
(1)	(2)	(3)	(4)	(5)	(6)
0.00	0.500	1	1	1	1
0.20	0.564	1	1	1	2
0.40	0.631	1	1	3	5
0.60	0.705	1	1	5	9
0.70	0.747	1	1	6	10
0.80	0.795	3	3	13	20
0.90	0.856	6	6	28	43
0.95	0.899	13	13	58	89

Table 2

Comparing γ with other measures of persistence

Different AR(2) models with $\rho = \rho_1 + \rho_2 = 0.80$

Model		γ	h	lar	m_{50}	m_{95}	m_{99}
(1)		(2)	(3)	(4)	(5)	(6)	(7)
(1)	$\rho_1 = 1.7, \rho_2 = -0.9$	0.853	5	–	1	3	3
(2)	$\rho_1 = 1.5, \rho_2 = -0.7$	0.844	4	–	1	3	3
(3)	$\rho_1 = 1.2, \rho_2 = -0.4$	0.828	4	–	2	5	6
(4)	$\rho_1 = 1.0, \rho_2 = -0.2$	0.814	3	0.724	2	9	14
(5)	$\rho_1 = 0.8, \rho_2 = 0.0$	0.795	3	0.800	3	13	20
(6)	$\rho_1 = 0.6, \rho_2 = 0.2$	0.770	2	0.839	3	16	25
(7)	$\rho_1 = 0.4, \rho_2 = 0.4$	0.732	1	0.863	4	20	31
(8)	$\rho_1 = 0.2, \rho_2 = 0.6$	0.667	1	0.881	4	23	35
(9)	$\rho_1 = 0.0, \rho_2 = 0.8$	0.500	1	0.894	6	26	40

Table 3

Monte Carlo simulations – AR(1) model (T=100)

True ρ	True γ	Model with no intercept			Model with an intercept			
		$\overline{\rho}$	Mean bias of $\widehat{\rho}$ (%)	$\overline{\gamma}$	$\overline{\rho}$	Mean bias of $\widehat{\rho}$ (%)	$\overline{\gamma}$	Mean bias of $\widehat{\gamma}$ (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.00	0.500	0.001	—	0.500	-0.010	—	0.497	-0.60
0.20	0.564	0.197	-1.50	0.564	0.184	-7.78	0.560	-0.71
0.40	0.631	0.393	-1.75	0.631	0.378	-5.41	0.626	-0.80
0.60	0.705	0.589	-1.83	0.705	0.572	-4.66	0.698	-1.00
0.70	0.747	0.687	-1.86	0.747	0.669	-4.47	0.739	-1.07
0.80	0.795	0.785	-1.88	0.796	0.765	-4.36	0.786	-1.13
0.90	0.856	0.883	-1.89	0.857	0.861	-4.37	0.842	-1.64
0.95	0.899	0.933	-1.79	0.899	0.907	-4.52	0.878	-2.34

Table 4

Different AR(2) models with $\rho = \rho_1 + \rho_2 = 0.80$ (T=100)

Model		Model with no intercept		Model with an intercept		True	Zero mean	Estimated mean	
		$\overline{\rho}$	Bias of $\widehat{\rho}$ (%)	$\overline{\rho}$	Bias of $\widehat{\rho}$ (%)	γ	$\overline{\gamma}$	$\overline{\gamma}$	Bias of $\overline{\gamma}$ (%)
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	$\rho_1 = 1.7, \rho_2 = -0.9$	0.800	0.00	0.797	-0.38	0.853	0.853	0.852	-0.12
(2)	$\rho_1 = 1.5, \rho_2 = -0.7$	0.796	-0.50	0.789	-1.38	0.844	0.844	0.842	-0.24
(3)	$\rho_1 = 1.2, \rho_2 = -0.4$	0.790	-1.25	0.777	-2.88	0.828	0.828	0.823	-0.60
(4)	$\rho_1 = 1.0, \rho_2 = -0.2$	0.786	-1.75	0.769	-3.88	0.814	0.814	0.806	-0.98
(5)	$\rho_1 = 0.8, \rho_2 = 0.0$	0.783	-2.13	0.760	-5.00	0.795	0.796	0.786	-1.13
(6)	$\rho_1 = 0.6, \rho_2 = 0.2$	0.779	-2.63	0.751	-6.13	0.770	0.771	0.758	-1.56
(7)	$\rho_1 = 0.4, \rho_2 = 0.4$	0.775	-3.13	0.743	-7.13	0.732	0.733	0.716	-2.19
(8)	$\rho_1 = 0.2, \rho_2 = 0.6$	0.772	-3.50	0.734	-8.25	0.667	0.667	0.645	-3.30
(9)	$\rho_1 = 0.0, \rho_2 = 0.8$	0.768	-4.00	0.725	-9.38	0.500	0.499	0.471	-5.80

Table 5

Robustness to additive outliers

AR(1) process with no intercept (T=100)

True ρ	True γ	$\overline{\rho}$	Bias of $\widehat{\rho}$ (%)	Inter Quartile Range of $\widehat{\rho}$	$\overline{\gamma}$	Bias of $\widehat{\gamma}$ (%)	Inter Quartile Range of $\widehat{\gamma}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0.00	0.500	0.002	—	-0.059-0.063	0.500	0.14	0.47-0.53
0.20	0.564	0.105	-46.59	0.036-0.172	0.559	-0.87	0.53-0.59
0.40	0.631	0.219	-44.34	0.134-0.302	0.621	-1.60	0.59-0.66
0.60	0.705	0.359	-39.07	0.257-0.463	0.689	-2.30	0.65-0.72
0.70	0.747	0.448	-34.76	0.342-0.561	0.729	-2.50	0.69-0.76
0.80	0.795	0.560	-28.62	0.459-0.677	0.774	-2.77	0.74-0.81
0.90	0.856	0.713	-19.33	0.628-0.818	0.834	-2.71	0.80-0.87
0.95	0.899	0.815	-12.61	0.755-0.903	0.877	-2.46	0.84-0.91

Table 6

Coverage ratio of 95% confidence intervals for ρ and γ

AR(1) model with an intercept

True	True	Coverage ratio for ρ		Coverage ratio for γ					
				K=8		Andrews		KV	
ρ	γ	T=100	T=250	T=100	T=250	T=100	T=250	T=100	T=250
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.00	0.500	0.951	0.951	0.923	0.939	0.946	0.952	0.950	0.952
0.20	0.564	0.949	0.951	0.922	0.937	0.943	0.949	0.950	0.950
0.40	0.631	0.946	0.950	0.917	0.932	0.934	0.940	0.946	0.950
0.60	0.705	0.942	0.948	0.910	0.929	0.924	0.932	0.946	0.949
0.70	0.747	0.938	0.946	0.907	0.931	0.920	0.929	0.941	0.947
0.80	0.795	0.935	0.943	0.899	0.923	0.902	0.914	0.938	0.948
0.90	0.856	0.922	0.937	0.867	0.900	0.855	0.877	0.918	0.939
0.95	0.899	0.885	0.921	0.825	0.869	0.800	0.834	0.894	0.924

Table 7

Coverage ratio of 95% confidence intervals for ρ and γ

AR(2) models with an intercept and $\rho_1 + \rho_2 = 0.80$

AR(2) Models [(a)]	True γ	Coverage ratio for ρ		Coverage ratio for γ					
				K=8		Andrews		KV	
		T=100	T=250	T=100	T=250	T=100	T=250	T=100	T=250
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	0.853	0.947	0.952	0.983	0.985	0.998	0.999	0.986	0.973
(2)	0.844	0.945	0.950	0.946	0.953	0.987	0.983	0.961	0.955
(3)	0.828	0.937	0.946	0.918	0.936	0.949	0.949	0.943	0.947
(4)	0.814	0.933	0.944	0.910	0.930	0.923	0.931	0.944	0.947
(5)	0.795	0.928	0.943	0.895	0.923	0.901	0.914	0.937	0.948
(6)	0.770	0.923	0.943	0.887	0.917	0.881	0.903	0.929	0.942
(7)	0.732	0.921	0.940	0.875	0.907	0.866	0.896	0.926	0.941
(8)	0.667	0.917	0.939	0.856	0.896	0.849	0.888	0.918	0.937
(9)	0.500	0.913	0.938	0.815	0.867	0.811	0.866	0.907	0.931

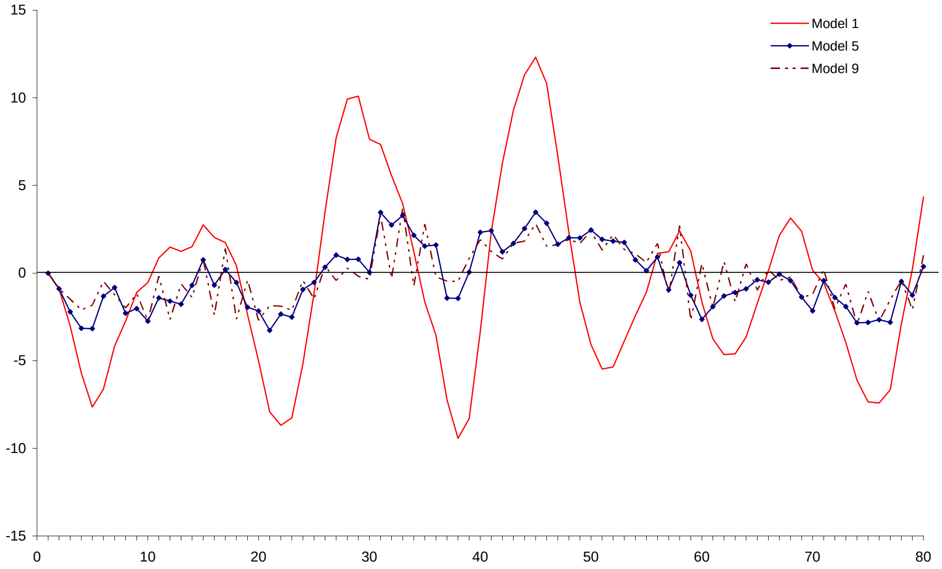
[(a)] The AR(2) models are the same as in Table 2;

Table 8

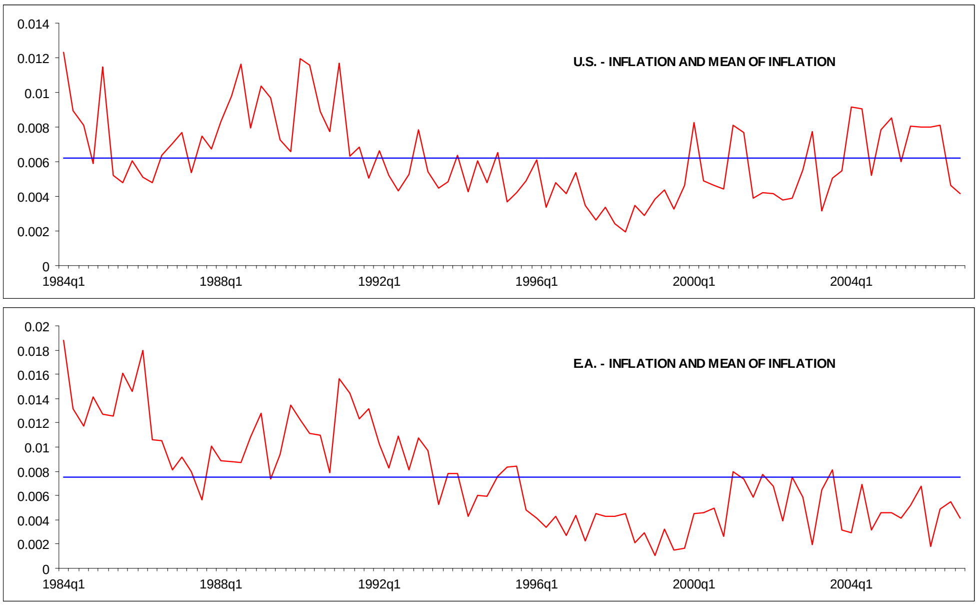
Inflation persistence in the U.S. and the Euro Area

U.S.		
No break in the mean	Break in the mean	
full sample estimates[(*)]	full sample estimates[(*)]	change in persistence[(*)]
(1)	(2)	(3)
$\widehat{\gamma} = 0.703$ (0.025)	$\widehat{\gamma} = 0.615$ (0.030)	$\widehat{\alpha}_2 = 0.027$ (0.057)
$\widehat{\rho} = 0.752$ (0.101)	$\widehat{\rho} = 0.615$ (0.126)	$\widehat{\lambda} = 0.132$ (0.176)
Euro Area		
$\widehat{\gamma} = 0.813$ (0.025)	$\widehat{\gamma} = 0.725$ (0.059)	$\widehat{\alpha}_2 = 0.144$ (0.060)
$\widehat{\rho} = 0.879$ (0.065)	$\widehat{\rho} = 0.559$ (0.126)	$\widehat{\lambda} = -0.083$ (0.155)

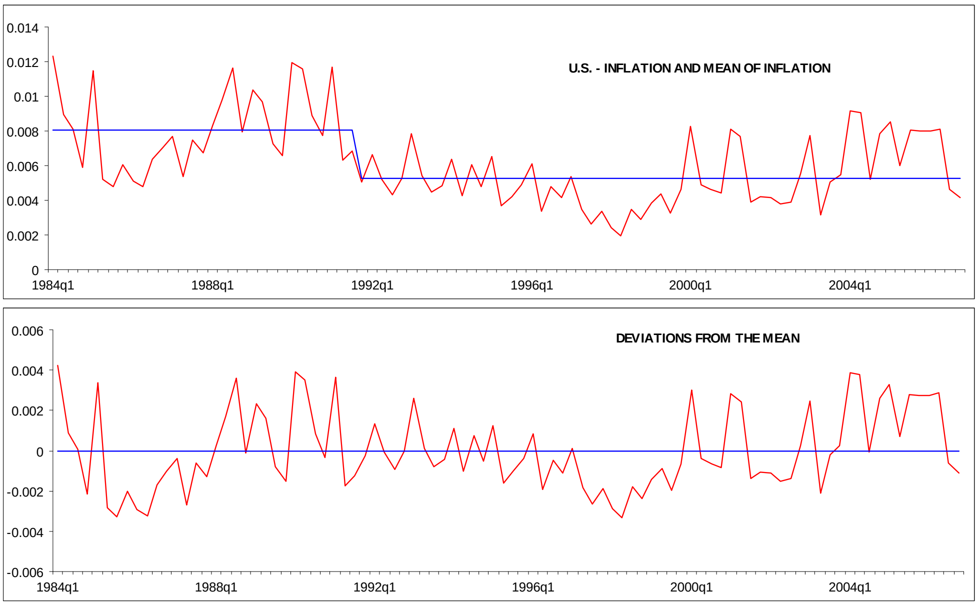
[(*)] For $\widehat{\gamma}$ and $\widehat{\alpha}_2$ KV (2002) standard-errors are given in parentheses; $\widehat{\alpha}_2 = \widehat{\gamma}_1 - \widehat{\gamma}_2$ and $\widehat{\lambda} = \widehat{\rho}_2 - \widehat{\rho}_1$ where $\widehat{\gamma}_1$ and $\widehat{\rho}_1$ refer to the first, and $\widehat{\gamma}_2$ and $\widehat{\rho}_2$ to the second sub-period, respectively.



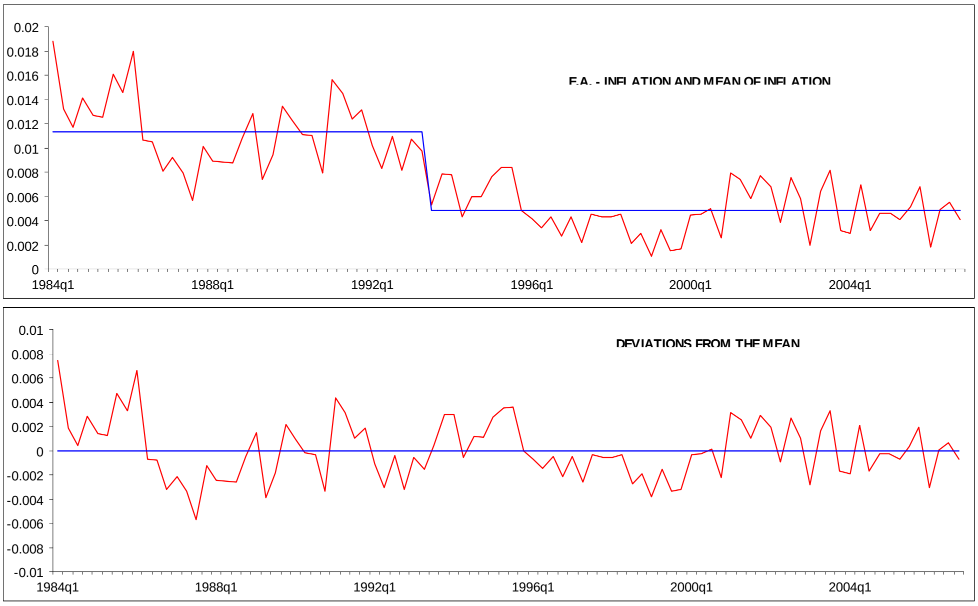
Graph No.1 - Realization of models 1, 5 and 9.



Graph No.2 - Quarter-on-quarter U.S. and E.A. inflation.



Graph No.3 - Quarter-on-quarter U.S. inflation.



Graph No.4 - Quarter-on-quarter E.A. inflation.