Optimizing long term planning for underground copper mines

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ABSTRACT

Mining production can be modeled as a group of operations that begin with geologic ore deposit estimations, and finish with refined mineral and sub-products sale. Intermediate stages are mainly rock size reduction, transportation and mineral concentration, eliminating waste material. This paper presents a mathematical programming model that optimizes long term plans and large investments in underground copper mines. We present a multi-period capacitated network design/flow model, which considers different mineral demands, technical mining exploitation constraints and environmental limitations, maximizing net profit over a 25-year horizon. Based on mixed integer programming, the model analyses several mines and processing facilities as a whole, performing a simultaneous evaluation that allows interaction among these elements and the investment options, assigning ore production for each mine over the horizon and providing a global optimal solution. The resulting combinatorial model is large in size and theoretically hard to solve. This paper presents the results of its implementation in El Teniente, the largest underground copper mine in the world.

INTRODUCTION

Long term mine operation planning is a complex problem. Many interrelated decisions must be made through a long evaluation period where the profitability of tomorrow decisions depends on choices made today. A mine, like El Teniente, usually can be divided into sectors. The mining process in one sector can last decades and essentially involves four main activities:

a) Geologic tasks: definition of the characteristics of a mineralized body.
b) Mining tasks: mineral accessibility and extraction operations.
c) Concentration tasks: elimination of the waste material.
d) Benefit: elaboration of final products.

This paper focuses on the mining and concentration tasks. It is assumed that during the geologic tasks the mineralized body has been identified and set into a collection of regular blocks with known properties. The benefit (refinement) activity is also left out and can be considered separately because it is naturally seen as an independent stage. Also, the model to be presented is specialized for underground mining with *block caving*. Other mining methods could be considered, and in particular an extension to open pit mining has been done but will be reported in a future work.

The main problem covered in this paper is how to maximize the long term net present value of an underground mine. We define a methodology for finding long term production plans that are technically feasible guided by a profit maximizing criteria. Moreover, if there is a set of possible investments that can be done during the planning horizon (for example a capacity increase of a crusher), we propose a tool that can help to find the best combination.

Depending on the size of the mine and the planning horizon, the number of variables involved in defining long term plans becomes rapidly overwhelming and reaching global optimality is extremely hard, if not impossible. Then obtaining a near optimal solution is the best chance. There are mainly two approaches. One is to break down the problem into several smaller problems and then try to solve each one of these to optimality. This would be the case when the definition of the economic mineral reserve, the production plan and the possible investments are treated separately and/or when each sector of a mine is considered independently. Then a particular local optimal solution is obtained for each individual problem and a set of rules must be defined in order to have a long term plan. These rules are necessary because solving separate problems oversees the interactions among them. The second approach is to directly try to solve the global long term optimization problem. Again in most cases only near optimality can be achieved, but now the interactions between different level of decisions and/or several sectors of the mine are considered explicitly. Based on the case study here reported we claim that this unified approach, compared to the break down approach, provides space for profit improvement even when the global problem is solved suboptimally. We propose a comprehensive model that considers the whole mine and different level of decisions simultaneously but without loosing the long run perspective. In other words, we are careful in choosing the appropriate level of aggregation so that interactions are captured without getting confused with the details, and solutions can be obtained in reasonable time.

In the literature most of the models that can be found address separately only one of the components involved in long term planning. This make them more suitable for the break down approach mentioned above. Also most of the literature focuses in open pit mining. Therefore this work is a two-fold contribution, it enriches the knowledge of the underground mining community, and it proposes a unified approach that has not received much attention until now. The approach and model here described have been successfully implemented at El Teniente, the largest underground copper mine in the world.

The rest of the paper is organized as follows. The next section has a brief literature review. Then in the third section we describe the underground mining operation. In section four we explain the traditional planning scheme previously used at El Teniente. In section five the unified model is presented and in section six the solution procedure is explained and the results of a case study at El Teniente are shown. Conclusions are drawn in section seven.

LITERATURE REVIEW

As mentioned before, in the literature most models have been developed for open pit mines and solve only a partial version of the long term planning problem. The problem that has received most attention is the one of determining the economic mineral reserve in an open pit mine. This is known as the ultimate pit problem. It is simple enough so theoretic analysis is tractable and specialized algorithms can be developed. Several Operations Research techniques have been used to solve it, starting with the classic “moving cone” heuristic, see for example Kim (1), which can be sub-optimal but is intuitively appealing. Among the algorithms with proven optimality, historically the most important are Lerchs and Grossmann (2) and Picard (3). The first one is based on graph theory but it can be shown that its structure is very similar to the dual simplex algorithm (see Underwood and Tolwinski (4)). The second one proves the equivalence of the ultimate pit problem with finding a maximum closure in a graph, so then it reduces to a maximum flow problem that can be solved, for example, with a push-relabel procedure. A good explanation and comparison of both procedures together with a complete literature review of the ultimate pit problem can be found in Hochbaum and Chen (5). Even though these efficient algorithms exist, the problem itself does not consider what happens with the mineral beyond the extraction stage and, for instance, does not provide a production schedule that considers all the downstream capacities.

Several models can be found for the production-scheduling problem in the mining industry, again most of them specialized for the open pit case. For example Tolwinski and Underwood (6) propose a dynamic programming approach while Gershon (7) uses mixed integer programming. See also Caccetta and Hill (8) and all the references within there. In the case of underground mines, two important works are Kuchta et al. (9) and Newman et al. (10). Both cover the same topic, the implementation of a production scheduling mixed integer program at LKAB's Kiruna Mine. The first paper presents the model and the preliminary results and the second one explains additional pre-processing routines that significantly reduce the size of the problem. These papers are related to ours even though their model only copes with the scheduling problem and the objective is to minimize the deviation from pre-specified demands per period. The sizes of the instances are not comparable either but there are some similarities in the modeling and pre-processing techniques.

We suggest the reader to see Caccetta and Giannini (11) for other applications of Operations Research methods to specific (open pit) mining problems, as for example the determination of optimal blends or the determination of equipment maintenance and replacement policies. Very few papers can be found regarding models that simultaneously try to solve the entire mine planning problem. One attempt would be Dagdelen and Johnson (12) but the instances are still small compared to real-life cases. Another example is Barbaro and Ramani (13) where discrete optimization is used to solve a facility location model that also decides if a sector of a mine should produce or not. It assumes that the economic mineral reserve has somehow already been defined and takes it as an input. To our knowledge the most closely related paper to ours is Carlyle and Eaves (14). Their mixed integer model was developed for an underground platinum and palladium mine and considers several mine planning decisions, including development, drilling, preparation, and production of each mine segment for an horizon of 10 quarters. The model is relatively small and near optimal integer solutions are obtained with a commercial software package.

Finally, a completely different approach for determining long term mining plans is the optimal cut-off grade methodology made popular through the work of Lane (15). It can be applied to either open pit or underground mines. The concept of optimal cut-off grades is simple but powerful. Moreover it has important operational advantages and is embedded in the background of all the mining practitioners. The drawback is that finding the optimal cut-off grade is not trivial. This methodology was adapted to the reality of El Teniente by De la Huerta (16) and has been extensively used to generate long term plans, as for example, the annual *Base Case Report* (17), but the analysis of one particular scenario can take several weeks. This procedure will be the benchmark for the model presented in this paper.

UNDERGROUND MINING AT EL TENIENTE

We here present a brief description of the mining operations at El Teniente. The first step is the selection of reserves. This consists in defining the boundaries of the orebody, delimiting the material to be removed. This is done according to economic criterions that determine what is profitable based on the grade distribution. In addition suitable technologies of operation are considered according to the type of mineral.

For planning and operational purposes, the geological model of the whole deposit is expressed through a *block model*. Each block is uniquely identified together with its geologic characteristic, in particular the ore grades. These values are estimated using geostatistic procedures (kriging). A column (or extraction point) is the vertical aggregation of blocks, and a mine or sector corresponds to a set of neighboring columns. Then, defining the orebody consists in determining which blocks of each sector are profitable.

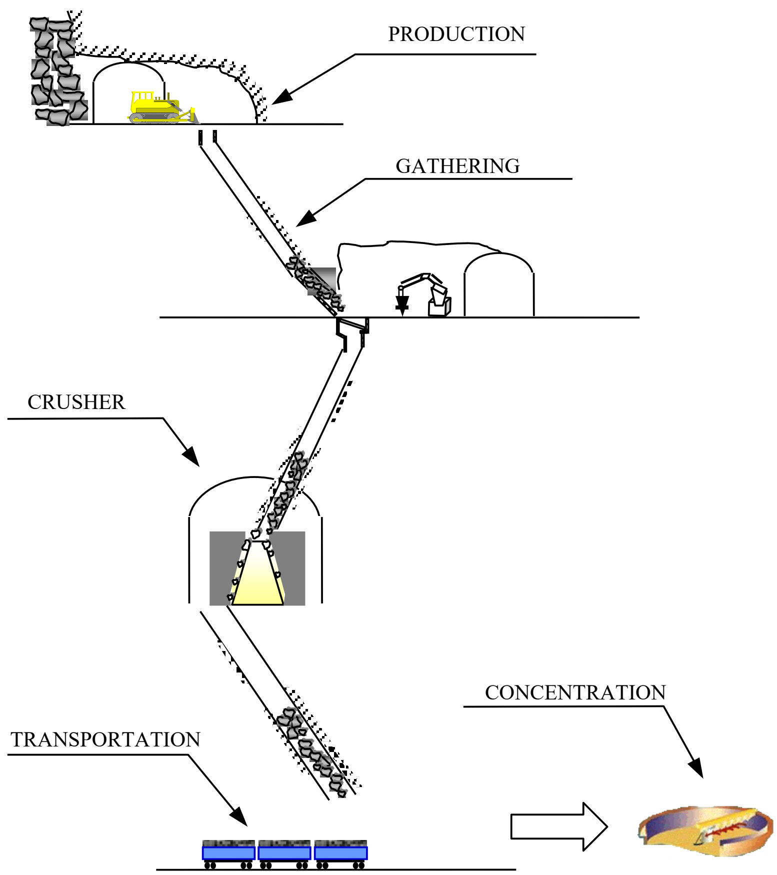

Figure 1 – Undergound Mining Stages at El Teniente

Once the reserves are identified, the following step is to program the ore extraction on a time scale taking into account technical mining restrictions and the downstream capacities. This involves the configuration of a network of processes through which the extracted ore must pass before becoming a final product. These intermediate processes are basically rock size reduction (i.e. crushers and mills), transportation, and concentration of the mineral, eliminating the waste material. Figure 1 shows the typical flow from the extraction points to the concentration plants. Each one of these intermediate processes can be characterized through technical coefficients. In general this means a capacity (usually measured in tons that can be processed per period), a variable operational cost, and a fixed installation cost. In addition, some processes may have alternative operation modes where there is a trade-off between the capacity and the size or concentration of the output material.

PREVIOUS PLANNING SCHEME

Currently El Teniente has migrated from a traditional planning scheme based exclusively on cut-off grades to a methodology based on the model presented in this paper. In order to compare the results we outline the former method.

a) The available resources are determined considering the current geologic characteristics of the deposit and the mine’s history.
b) A selection of resources is performed. This is done considering Macro Options (i.e. alternative technologies, economic indicators, management directives, etc) in order to delimit the floor, perimeter and ceiling of the resources to extract during the planning horizon. The outcome is known as the Model of Mining Reserve that contains the resources with positive benefit taking into account the processing costs and the projected recovery.
c) A Resource Consumption Strategy is defined. This specifies an extraction plan of Prepared Reserves (i.e. those that can be removed in the short term) that complies with the Planning Parameters (mostly technical mining constraints like the length of the planning horizon, the preparation rates, extraction sequences, extraction rates, or in situ mineral dilution), plus some additional external restrictions for example limitations on the amount of contaminant (arsenic) produced.
d) The Consumption Strategy defines a Mining Plan, which contains for every period: tons of mineral to extract, productive sectors involved, ore grades and level of uncertainty based on the drillings and samples.
e) The Mining Plan is economically evaluated, providing Profit Indicators, and measures of Technical Risk and Vulnerability. If the result is not satisfactory, the Planning Parameters are modified and the process is repeated from point c).
f) If an external change occurs affecting the Macro Options, these are incorporated into the model in point b), and the process is repeated from that stage.
g) The final result is a plan that provides the relevant information for operating each sector, including the starting period, operation rates and termination period.

This scheme is complete enough and allows evaluating different options by means of alternative plans, but it is not a unified (global) approach since each sector is treated separately and the selection of reserves is done ignoring the consumption strategy. In addition, due to the high complexity of the procedure and the involved calculations, several weeks (if not months) are spent in obtaining one particular plan, preventing the planner from evaluating more options.

MODEL DESCRIPTION

The following are the key features of the unified model we propose:

- We suggest an mathematical programming approach with two main parts: one representing the mining resource (reserves); and one representing the network of processes where the mineral flows and in which alternative technologies of operation and/or investments can be considered.
- The basic modeling unit in the mining resource part is a block (the same ones from the *block model*). A binary variable is associated to each block in each period indicating when it will be extracted.
- The planning horizon is set to 25 years, with shorter periods at the beginning (when more detail is required and deviations in the production have a major impact in the results), and longer periods towards the end.
- The objective function is to maximize the net present value, i.e. the benefits (obtained from the sales of the final products) minus the variable and fixed costs.
- Three different ore products are considered: copper, molybdenum and arsenic. The first two are commercial products while the last one is a contaminant on which additional constraints may be imposed.
- The equipment, machinery and facilities that are "consumed" during the planning horizon are prorated and included as variable costs.
- All the processes posterior to the ore extraction are modeled as a capacitated network design/flow problem. Then future investments translate into alterations of the existing capacities and/or additional nodes and arcs in the network.
- The main stages (nodes) of the network are the crosscuts, the ore passes, the crushers, the mills, and the concentration plants. The arcs represent the existing means of transportation between the nodes.

Figure 2 provides a schematic representation of the mathematical model just described. In order to obtain a technically feasible solution several mining constraints must be added, these are:

- Extraction regularity: the extraction must be even and smooth to allow a correct breaking of the rock.

- Extraction sequence: the opening of the extraction points must comply with a pre-specified sequence along with geo-mechanic considerations and possible interactions among different sectors.
- Minimum extraction: depending on the ore's properties there exists a minimum level of extraction to avoid the formation of solid rock pillars.
- Rate of extraction: upper and/or lower bounds may be set depending the characteristics and state of each sector.

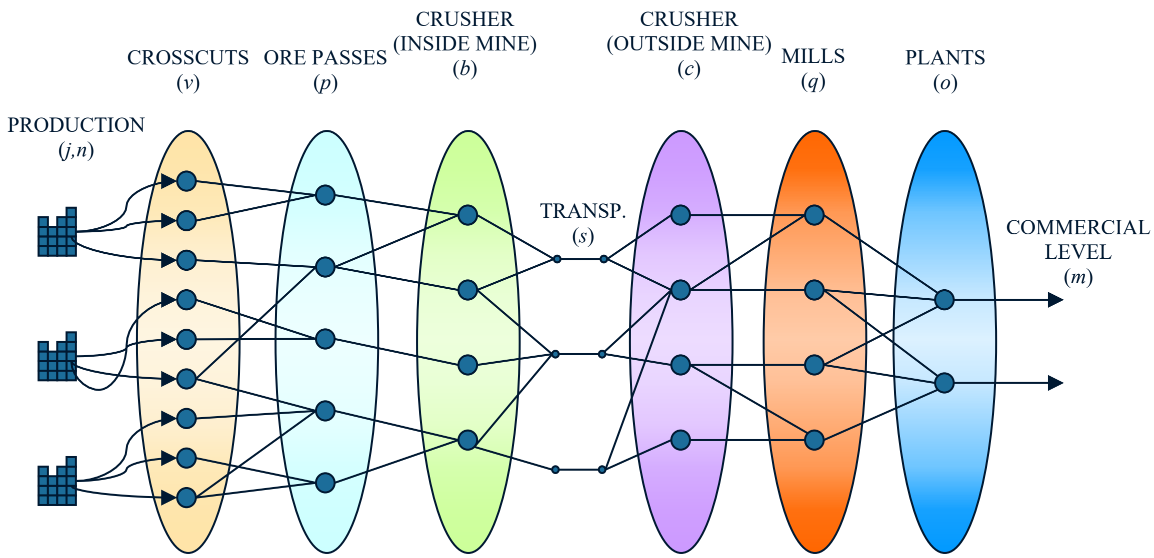

Figure 2 – The Production Stage and the Network of Intermediate Processes

SOLVING THE MODEL

Description of the Case Study

The case study analyzed in this work corresponds to all of the sectors currently in operation at El Teniente. For evaluation purposes two different price levels were considered (high and low) and the discount rate was set at 10%. The planning horizon was first divided in 3 periods of 1, 4 and 20 years, respectively, but then the second period was divided into 4 annual sub-periods. The current state of the mine was set as the initial condition. All sunk costs were excluded from the evaluation. The size of the mathematical model depends directly on the number of blocks, columns and periods. The number of equations and variables are reported in Table I.

All the data was extracted directly from the respective databases at El Teniente and some pre-processing routines were performed in order to reduce the number of binary variables. The model was implemented using GAMS version 19.0 and was solved using the parallel barrier interior point algorithm of CPLEX version 6.6. A computer running Windows 2000 with two Pentium III processors of 550 MHz and 1024 Mb of RAM was used.

Table I – Size of the Model

Instance	Nº Equations	Nº Variables
3 Periods Model	688,516	498,796
4 Periods Model	1,002,000	664,451
Total	1,690,516	1,163,247

Rounding Heuristic

Given the large amount of binary variables, especially those representing the extraction stage, solving the model using a mixed integer programming routine is nonviable. Therefore the integrality condition is relaxed and a continuous version is solved. The obtained solutions do not present many fractional values, which can be explained primarily by two reasons: i) the model formulation itself is very tight; ii) the copper grades in each column are decreasing with height, then greater benefits are obtained when the inferior blocks are extracted first. To obtain a final integer solution (for all the blocks except the last one) a rounding heuristic is applied. The heuristic proceeds iteratively from the inferior blocks to the higher ones fixing those that already have an integer value. During this process the tons of ore extracted form each column is fixed to the value obtained in the relaxed solution. Figure 3 shows the passages of the heuristic for one particular column.

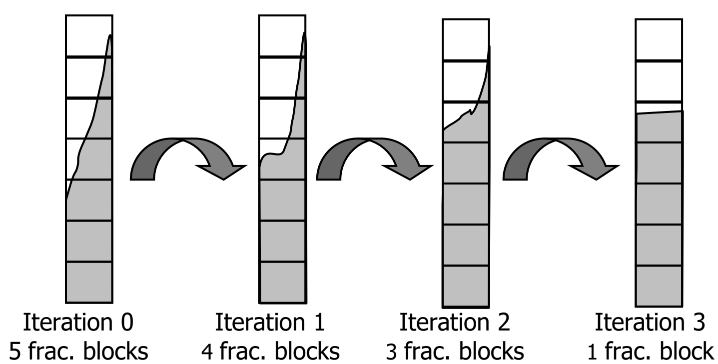

Figure 3 – The Rounding Heuristc

General Results

In this section we provide a qualitative description of the results. More detailed information can be found in Araneda et al. (18).

A direct comparison of our solutions with the plans obtained using the conventional methodology explained in section four, shows that both provide production schedules that are very similar in terms of total extracted tons per period and operation profiles for each sector, but the unified model yields higher grades of copper and molybdenum, which means a higher income and therefore a superior solution. This improvement mainly comes from considering the mine as a whole and deciding when and how much each sector must operate. A local approach may decided that certain blocks are not profitable when comparing them with its neighbors, but this can change when the blocks of the rest of the deposit are considered simultaneously. Then more alternatives are available and as an outcome a better blend is obtained.

We note that the average grade of copper obtained for the whole mine is decreasing along the planning periods, reflecting that the model decides to extract first those blocks with better grades, which is consistent with the fact that a discount factor is being used. The same behavior can be observed in each individual sector, but this is not the case for the molybdenum. Its average grade (either for the entire mine or per sector) does not follow any specific pattern. This is because the molybdenum represents a low fraction of the total income and therefore can be considered as a byproduct with no major strategic impact.

Regarding the global capacity of the intermediate processes, it is reached during the first periods, showing that the models provides an appropriate ore blend from all sectors so that the concentration plant receives a constant supply. This conclusion cannot be extended to the final period since the results correspond to the average of 20 years. In order have more detail this period should be at least decomposed into four 5-years sub-periods.

The model was solved with an explicit constraint that limits the total amount of arsenic produced (as a byproduct) each period. The conventional methodology has had difficulties in dealing with this restriction but with our model the mining plan is guaranteed to satisfy it.

When considering the two different price levels (high and low), the main difference is that the total tons extracted are lower for the second case but the "closing" grade of each column is higher. This is a direct consequence of the fact that the mineral extracted in the optimistic scenario is not profitable in the other.

The running times (CPU time) for solving the model are shown in Table II. These values do not consider the time spent in generating the input files from the database, they only reflect the time used by the algorithm to find an optimal solution of the relaxed problem. As the scenarios were solved in two steps, first with 3 periods and then disaggregating the second period, the total time presented is the sum of both phases.

Table II – Running Times

Instance	Time [sec]
Scenario 1 (high price)	
1st phase	4,505.2
2nd phase	5,559.9
Total	10,065.1
Scenario 2 (low price)	
1st phase	4,218.8
2nd phase	6,526.6
Total	10,745.4

CONCLUSIONS

We have presented a comprehensive model that considers the complete mining operation as a production stage plus a network of intermediate processes. The model was implemented and has been used at El Teniente during the last two years. The robustness of the provided solutions has been tested in practice and the validation phase is over. The model is now widely used to support the decisions of the long term planners.

Compared to the conventional methodology previously used at El Teniente, the results of the model are successful since the generated mining plans are feasible and report better economic indicators. Furthermore, once the data is loaded in system, obtaining a solution for a particular scenario only requires a few hours. This has been crucial allowing the planners to use the model as a tool for answering "what if" questions or to perform a sensitivity analysis of the current plan.

From a theoretic perspective it will be interesting to study further enhancements of the model formulation or better solution procedures so that the planning horizon can be decomposed into more sub-periods.

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