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The Impact of Quick Response in Inventory-Based Competition

Felipe Caro
Anderson School of Management, University of California, Los Angeles, Los Angeles, California 90095, fcaro@anderson.ucla.edu

Victor Martínez-de-Albéniz
IESE Business School, University of Navarra, 08034 Barcelona, Spain, valbeniz@iese.edu

We propose an extension of the competitive newsvendor model to investigate the impact of quick response under competition. For this purpose, we consider two retailers that compete in terms of inventory: customers that face a stockout at their first-choice store will look for the product at the other store. Consequently, the total demand that each retailer faces depends on the competitor's inventory level. We allow for asymmetric reordering capabilities, and we are particularly interested in the case when one of the firms has a lower ordering cost but can only produce at the beginning of the selling season, whereas the second firm has higher costs but can replenish stock in a quick response manner, taking advantage of any incremental knowledge about demand (if it is available). We visualize this problem as the competition between a traditional make-to-stock retailer that builds up inventory before the season starts versus a retailer with a responsive supply chain that can react to early demand information. We provide conditions for this game to have a unique pure-strategy subgame-perfect equilibrium, which then allows us to perform numerical comparative statics. We confirm that quick response is more beneficial when demand uncertainty is higher or exhibits a higher correlation over time. We also find that the competitive advantage from quick response is larger when facing a slow response competitor, and interestingly, asymmetric competition can be desirable to both competitors.

Key words: operations strategy; supply chain management; inventory competition; game theory; fast fashion
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1. Introduction

In recent years, the apparel industry has seen the rise of what has been called *fast fashion* retailers. These are clothing companies that are able to respond quickly to market trends and introduce new products very frequently. Most of these products have a life cycle of no more than a few weeks, and by the time the cycle is over, they are promptly replaced by a more "fashionable" item.

In Europe, where the concept began, fast fashion has been denominated a *21st-century retailing phenomenon*, with representative companies such as H&M and the Inditex Group, owner of Zara (Davidson 2005). A crucial part of their success is due to their flexible supply chain and operational competencies (Ghemawat and Nueno 2003). In particular, fast fashion retailers can make in-season replenishments thanks to remarkably low lead times, in the order of weeks rather than months. The latter is achieved in most cases through local production or expediting, which obviously translates into higher unit costs.

In the case of North America, fast fashion remains a niche that represents no more than 2% of the apparel business (Foroohar 2006). Large clothing retailers like Gap, Inc., seem too big and might not have the incentives to restructure their entire supply chain to mimic their European competitors because their customers have been historically less fashion forward. They might, however, borrow a few elements of fast fashion. For example, they can strengthen the link with their suppliers, or they can move the production of trendier items to Mexico instead of Asia to shorten lead times. The extent to which traditional retailers should adopt or convert to fast fashion remains an open question and serves as part of the motivation for this paper.

Despite the incipient (but growing) success of fast fashion in North America, the concept itself builds upon *quick response* (QR), which was an apparel manufacturing initiative that started primarily in the United States during the mid-1980s (Hammond and Kelly 1990). The main objective of QR is to drastically reduce lead times and setup costs to allow the postponement of ordering decisions until right before (or during) the retail selling season, when better demand information might be available. A successful implementation of QR is typically based on the effective use of information technologies. Fast fashion has taken QR to a higher level and has leveraged on the minimal lead times by introducing new products on a regular basis, therefore enabling a dynamic assortment that basically fulfills the ideal of providing "fashion on demand."

The overall success of fast fashion is attributable to a combination of multiple factors. The interaction between all the elements involved is at a preliminary stage of being understood. There has been extensive qualitative work that describes the different cases or examples of fast fashion companies. However, the academic literature on this topic remains scarce. In this paper we aim at understanding the impact of one specific element of fast fashion. We focus on the QR component, which is arguably the basis for all the other elements that later come into play. Therefore, we disregard assortment, pricing, or market positioning decisions related to fast fashion, and we focus on the essential capability of having more flexibility in terms of inventory replenishment. We look at the problem in a competitive setting, because from its inception, QR advocates have claimed that it is the only viable strategy under the current conditions in the apparel market, similar to what just-in-time manufacturing has meant to the auto industry (Hammond and Kelly 1990).

We consider a model with two retailers selling a substitutable product over a finite horizon that is divided into two periods. The two retailers compete through their inventory levels. When a stockout occurs at one retailer, the unsatisfied customer walks into the second retailer, where she is served if stock is available. Thus, the inventory decision of a given retailer depends on the level of inventory at the competitor. Moreover, in the second period, the retailers can incorporate new demand information into their stocking decisions. The model allows for asymmetric retailers that may or may not be able to use the information updates, depending on their reordering capabilities. We then analyze the competitive strategies and the equilibrium inventory decisions. Our model can be seen as a two-period extension of the competitive newsvendor developed by Lippman and McCardle (1997). As in their case, we are interested in determining and characterizing the existence of a unique pure-strategy subgame-perfect equilibrium. This allows us to understand and compare the outcomes for each retailer, and in particular assess the potential benefits of implementing QR in a competitive setting.

Our paper makes contributions to the operations literature from both methodological and managerial standpoints. From a methodological perspective, we solve an asymmetric two-period inventory-based competition model, where the asymmetry is in terms of reordering and demand learning capabilities.[1] We are not aware of any other paper that studies horizontal inventory-based competition with demand correlation over time. Several authors have previously studied the infinite horizon case but under such conditions that it reduces to a myopic single-period problem. On the contrary, we formulate a two-period model, and we provide sufficient conditions that guarantee the existence of a unique equilibrium in pure strategies. The conditions for different cases are summarized in Table 1, and the meaning of each assumption is discussed later. Note that we focus on the two-period case because it is the most common approach that has been used in the literature to model QR (see, for instance, §10.4 in Cachon and Terwiesch 2005).[2]

From a managerial standpoint, this paper is a first attempt at understanding the competitive advantage of QR compared to more traditional retail operations. In particular, we provide a detailed numerical study where we compute the equilibrium profits achieved by two *slow response* (SR) firms engaged in

[1] Asymmetric net margins and demand-splitting functions are also allowed.

[2] The symmetric case allows for an arbitrary number of periods and is analyzed in §A of the online appendix available on the authors' website.

Table 1 Sufficient Conditions for a Unique Pure-Strategy Subgame-Perfect Equilibrium

	Case		
Assumption	Demand signal	Midseason replenishment	Symmetric retailers
Costs and prices	Different costs per period	Nondecreasing and nonincreasing respectively	—
Infinite support	—	Only in initial period	—
Log-concavity	—	Only in initial period	—
Demand			
Independence	—	$(D_2 - kD_1)$ independent of D_1 for some $k \geq 0$	—
Likelihood order	—	Nondecreasing	—
Linear split	—	Only in last period if $k > 0$	In all periods

Note. The symmetric case is studied in the online appendix.

inventory-based competition, and we compare it to those achieved when one or both retailers have QR capabilities. Our results indicate that both retailers are better off in the QR versus SR competition (Q-S) compared to the SR versus SR case (S-S). In other words, we show that part of the competitive advantage of a QR retailer comes from the asymmetry, i.e., from being faster than the competitor. These benefits are larger under higher demand uncertainty or higher correlation over time. We also observe that the QR firm might be willing to let demand leak to reduce inventory risk.

The remainder of the paper has the following structure. In §2 we review the existing literature, mostly on inventory-based competition models. Then, in §3 we develop our model, and in §4 we establish the existence and uniqueness of a pure-strategy subgame-perfect equilibrium for the demand signal and midseason replenishment cases. In §5 we study the equilibrium under different scenarios and focus on the role of cost asymmetries, demand variability, and correlation. Finally, in §6 we conclude and discuss future research directions. There is an electronic companion to this paper where we provide the essential proofs. In a separate online appendix (available on the authors' website) we provide supplemental results and the remaining analytical proofs not included in the electronic companion.

2. Literature Review

Fast fashion has been discussed extensively in the popular press; see, for instance, Foroohar (2006). In more academic terms, the literature is mostly descriptive with an emphasis on the qualitative aspects of the retailing strategy. Many cases have been written, in particular for the Spanish company Zara (e.g., Ghemawat and Nueno 2003, McAfee et al. 2004, Ferdows et al. 2004). From a quantitative perspective, during the 1990s significant progress was made in understanding the impact of QR in an isolated supply chain, mostly in a two-period setting (see Fisher and Raman 1996, Iyer and Bergen 1997, and references therein). More recently, Cachon and Swinney (2009) looked at QR in the presence of strategic customers. All this work is related to ours because we focus on the QR aspect of fast fashion. However, as mentioned before, fast fashion goes beyond QR, in particular by introducing a large number of new products during the retail selling season. In that respect, Caro and Gallien (2007) provide a closed-form policy for one of the distinctive operational challenges faced by fast fashion firms, namely, the dynamic assortment problem.

To the best of our knowledge, there has not been much analytical work that tries to identify the drivers of fast fashion's success in a competitive context. Clearly, the answer is not simple, because there are many interconnected factors that come into play. For that reason, as a first attempt to understand the (potential) competitive advantage, in this paper we focus exclusively on the QR capability, and we openly disregard other important elements of fast fashion. With this scope in mind, we are left with an inventory-based competition problem for substitutable products. Several models have been developed in the literature for this problem. In Table 2 we provide a noncomprehensive summary of preceding work. The key feature that all these papers have in common is that unmet demand at one firm is reallocated completely or partially among the competitors (i.e., unmet demand spills over to the competition). Our paper contributes to this stream of research by solving a two-period model that allows for asymmetric retailers in terms of reordering and demand learning capabilities.

In the single-period case with N retailers, Lippman and McCardle (1997) prove the existence of a pure

Table 2 Inventory-Based Competition Models for Substitutable Products

Author/year	No. of firms	No. of periods	Myopic solution	Backlogging	Stochastic demand model	Demand learning	Type of asymmetric retailers allowed	Sufficient conditions for the existence of a unique Nash equilibrium
Parlar (1988)	2	1	Yes	Not allowed	Independent firm demands are aggregated to industry demand; deterministic spillovers	Not allowed	In terms of cost parameters and effective demand	Continuous and strictly increasing cumulative demand
Lippman and McCardle (1997)	N	1	Yes	Not allowed	Aggregate industry demand is allocated across firms; general splitting rules are considered	Not allowed	As in Parlar (1988)	Parlar (1988) + symmetric costs and revenues + strictly increasing deterministic demand split
Mahajan and van Ryzin (2001)	N	1	Yes	Not allowed	Based on utility maximization; customers can buy more than one item	Not allowed	As in Parlar (1988)	Symmetric retailers + incomplete nontrivial demand diversion + continuous demand
Avsar and Baykal-Gürsoy (2002)	2	∞	Yes	Not allowed	As in Parlar (1988)	Not allowed	As in Parlar (1988).	Continuous demand + stationary policies + low initial demand
Netessine and Rudi (2003)	N	1	Yes	Not allowed	As in Parlar (1988) but correlations are allowed	Not allowed	As in Parlar (1988)	Continuous demand + incomplete demand diversion
Netessine et al. (2006)	2	$1, \infty$	Yes	Allowed	Model I: Identical to Netessine and Rudi (2003). Models II–IV: Allow backorders	Not allowed	As in Parlar (1988)	Same as in Avsar and Baykal-Gürsoy (2002)
Li and Ha (2008)	2	1	Yes	Not allowed	As in Parlar (1988) but correlations are allowed	Not allowed	As in Parlar (1988)	Conditions on joint demand distribution
Nagarajan and Rajagopalan (2009)	2	T	No	Allowed	Total demand is deterministic or has compact support; allocation is random	Not allowed	As in Parlar (1988)	Newsvendor ratio large enough (then each retailer can ignore its competitor)
Present paper	2	2	No	Not allowed	As in Lippman and McCardle (1997) with a strictly increasing deterministic demand split	Allowed	As in Parlar (1988) + asymmetric reordering and demand learning capabilities	See Table 1

Nash equilibrium under the general assumption that the effective demand faced by a particular firm is stochastically decreasing in the inventory levels of the other firms, which comes naturally in the case of substitutable products (see Netessine and Zhang 2005). The existence of a *unique* Nash equilibrium requires additional assumptions as those listed in the last column of Table 2.

In the infinite horizon case, several authors have shown that, under suitable conditions, there exists a Nash equilibrium in which each retailer follows a stationary base-stock policy. All these results stem from the dynamic oligopoly model by Kirman and Sobel (1974). In practice, this is equivalent to solving a single-period problem. Note that even if the latter has a unique Nash equilibrium, that does not guarantee a unique subgame-perfect equilibrium in the multiperiod case.

Several other (retailing) competition models in which inventories play an important role have been studied in the literature. For instance, in an infinite horizon setting, Li (1992) looks at delivery-time competition and shows that when all retailers are identical they tend to make to stock. Anupindi and Bassok (1999) consider inventory-based competition à la Parlar (1988) and study the impact of "market search" (i.e., the spillover fraction) on the manufacturer's profit. Bernstein and Federgruen (2004) examine the case of retailers that compete on price and then set their inventory levels accordingly. Gaur and Park (2007) consider customers sensitive to negative experiences such as a stockout and study the competition of retailers on the basis of their service levels. As before, given the stationary model formulation in these papers, the solution is myopic in the sense of Sobel (1981), and the analysis reduces to a single-period problem.

Given the extremely short life cycle of fashionable clothing, finite-horizon models seem more appropriate. In that matter, the work by Hall and Porteus (2000) is conceptually close to ours because, despite the fact that they consider competition based on customer service instead of (nonperishable) inventory, and information updates are not allowed, the retailers can only take actions to prevent leakage of demand to the competitor rather than proactively attract demand to themselves. Under these conditions together with a multiplicative demand model, they are able to show the existence of a unique subgame-perfect equilibrium. Liu et al. (2007) extend the result to a more general demand model. Olsen and Parker (2008) provide an alternative extension in which a retailer can hold inventory over time and can advertise to attract dissatisfied customers from its competitor's market. The existence of a unique equilibrium is guaranteed by assuming a particular salvage value function and low initial inventory levels (as in Avsar and Baykal-Gürsoy 2002). Interestingly, in equilibrium, the game effectively becomes two parallel Markov decision processes where each firm can make its stocking decision independently of the other firm's choices. We obtain a similar result for the case of two symmetric retailers, although under a different set of assumptions.

3. A Two-Period Inventory Competition Model

In this section we formulate the inventory competition model that will be used later to study the benefits of QR. In §3.1 we present the basic features and assumptions. Then, in §3.2, we introduce the sequential game and the solution approach (i.e., subgame perfection).

3.1. Basic Features and Assumptions

In what follows, we present the assumptions that lead to our inventory competition model with an explanation or discussion whenever appropriate. Any related notation is introduced as well.

Assumption 1. *There are only two firms that sell substitute products, and each firm maximizes the total expected profits over a finite horizon divided into two periods.*

Let index $i = 1, 2$ denote the retailers. When necessary, we use the index j to denote the competitor, and throughout the paper it is understood that $i \neq j$. Because the retailers sell perfect substitutes, a customer that cannot find the product at her preferred retailer will check if it is available at the competitor. We consider a finite horizon to represent the short product life cycle in the fashion apparel industry. Following the QR literature, we focus on the two-period case, $T = 2$, which is rich enough to capture the key features that come into play. We use the index t to denote a period, with $t = 1$ and $t = 2$ representing the first/initial and second/final period, respectively.

Assumption 2. *The aggregate customer demand in period* $t = 1, 2$ (*denoted* D_t) *is continuous, stochastic, and may be correlated across periods.*

Let f_t and F_t be the probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of the demand in period t, respectively. Let $\bar{F}_t \equiv 1 - F_t$ with $\bar{F}_t^{-1}$ its inverse. When there is demand correlation across periods, we denote the p.d.f. and c.d.f. of the demand in period 2 as $f_{2|I_2}$ and $F_{2|I_2}$, respectively, where I_2 is the demand information available at the beginning of the second period, which is assumed to be common knowledge. Typically, the information would be the previous demand realization D_1 and/or any data or demand signal that has become available. An important remark is that we do not allow the information I_2 to be a function of past decisions. In other words, information is *uncensored*, just as in most QR models.

Assumption 3. *In period* $t = 1, 2$, *the effective demand faced by retailer* i *is composed of two parts*: (i) *the original demand and* (ii) *the spillover demand*. *The original demand is expressed as* $q_t^i(D_t)$, *where* q_t^i *is the demand allocation function* (*also referred to as the demand-splitting function*), *which is assumed to be strictly increasing, and we have that* $D_t = q_t^i(D_t) + q_t^j(D_t)$.[3]

The original demand is made of customers that naturally choose retailer i over the competitor j, and the spillover demand is made of those customers that originally choose j but end up buying at i because j runs out of stock. This spillover demand is equal to

$$\max\{0, q_t^j(D_t) - y_t^j\}, \tag{1}$$

where y_t^j is firm j's inventory level (after replenishment) in period t. Then the effective (realized) demand faced by retailer i is given by $R_t^i(y_t^j) \equiv q_t^i(D_t) + \max\{0, q_t^j(D_t) - y_t^j\}$. Because this is a key assumption in our model, several important observations follow:

- The effective demand $R_t^i(y_t^j)$ depends only on the competitor's inventory level. Therefore, competition is based on the inventory levels, but retailer i can only limit the customers it loses rather than influence those it gains. In an authentic fast fashion setting, a retailer would typically attract more demand by sustaining a high assortment rotation. Our model does not consider such a feature because we focus on understanding the impact of the QR capability for one particular product. We also note that, except for Netessine et al. (2006), all of the papers mentioned in Table 2 consider competitive models in which the retailer can only prevent leakage rather than attract additional demand. The same happens in Hall and Porteus (2000) and Liu et al. (2007).
- In Parlar (1988) and other similar papers, independent firm demands are aggregated into industry demand. On the contrary, in Lippman and McCardle (1997) and follow-up work, aggregate industry demand is allocated across firms. If the allocation is deterministic, then in each period there is only one source of uncertainty, namely, the total demand D_t. We have followed the latter, and therefore our approach is most appropriate when the main source of uncertainty is the size of the market (i.e., how well a product will sell) rather than the initial allocation across retailers. It is worth noting that our results can be extended to the case when the original allocation in the second period depends on the demand information I_2.
- As in the proof of uniqueness by Lippman and McCardle (1997), we require the demand-splitting function q_t^i to be strictly increasing in D_t. It must be strictly monotone because we need the inverse $(q_t^i)^{-1}$ to be well defined, and it must be increasing because our analysis requires that once the stocking decisions have been made, the retailer that runs out of stock first is the same one under all possible demand scenarios (in a given period). These conditions implicitly impose a positive correlation between the original demands of both firms. Again, this is reasonable when the main source of uncertainty is the market size. Note that the correlation can be anywhere between 0 to 1. It is perfect (equal to one) for the linear demand-splitting case, but can be close to zero as well.[4] Some models in the literature, e.g., Nagarajan and Rajagopalan (2009), assume a negative correlation between the original

[3] In the demand signal case studied in §4.2.1, because there is no demand realization in the first period, we let $q_1^i = 0$ for $i = 1, 2$.

[4] Consider the following example: $q_t^i(D_t) = 0$ for $0 \le D_t \le a$ and $q_t^j(D_t) = a$ when $a \le D_t \le 1$, and D_t is uniform in $[0, 1]$. Then $\mathrm{Cov}(q_t^i(D_t), q_t^j(D_t)) = a^2(1-a)^2/4$, $\mathrm{Var}(q_t^i(D_t)) = (1-a)^3(1+3a)/12$, and $\mathrm{Var}(q_t^j(D_t)) = a^3(4-3a)/12$. Hence, $\mathrm{Corr}(q_t^i(D_t), q_t^j(D_t)) = 3\sqrt{a}\sqrt{1-a}/(\sqrt{1+3a}\sqrt{1+3(1-a)})$, which is close to 0 for $a \approx 0$ or $a \approx 1$.

demands of the two firms, which is an appropriate assumption when the retailers are competing for a fixed pool of customers. Another class of models assume that the effective demand allocation is proportional to the individual stocking levels (see Cachon 2003). In that case, the retailers face perfectly correlated demands, and a retailer that stocks more will get a larger share of demand. This goes back to the previous discussion about the retailer being able to influence its effective demand but also has the inconvenience of making the multiperiod analysis intractable.

• Our model can be directly extended to the case of imperfect substitution, that is, when only a fraction δ of customers choose to substitute when they face a stockout. It suffices to multiply Equation (1) by δ, and all the equilibrium results follow through.

Assumption 4. *At the beginning of period $t = 1, 2$ both retailers decide simultaneously the order-up-to levels (y_t^i, y_t^j) based on the initial stock levels (x_t^i, x_t^j). When there is demand correlation, the decision in the second period also depends on the demand information I_2.*

The status of both retailers at the beginning of period $t = 1, 2$ is described by the initial stock levels (x_t^i, x_t^j), and when there is demand correlation, the status of the market in the second period is described by the demand information I_2. In the first period, the state of the system (i.e., the retailers and the market) is given by the vector (x_1^i, x_1^j). Based on that state, both retailers decide the order-up-to levels (y_1^i, y_1^j). Demand D_1 is then realized and depletes the inventory level of the system down to (x_2^i, x_2^j). Finally, given the information update I_2, the retailers may place a second order to bring the inventory levels up to (y_2^i, y_2^j). Therefore, the retailers decide their actions, i.e., the order-up-to levels (y_t^i, y_t^j), contingent on the respective state of the system. In other words, the retailers play Markovian strategies (see Fudenberg and Tirole 1991). As seen later in the discussion after Theorem 2, the analysis of our model can be restricted to Markovian strategies without any loss of generality.

Assumption 5. *The unit cost and price for retailer i in period t are constant parameters denoted c_t^i and p_t^i, respectively. The retailers are said to be symmetric if they face the same cost and price in all periods.*

We exclude pricing decisions from the model. This allows us to focus on the use of inventory as a competitive lever. This assumption is consistent with the fact that fast fashion retailers rely less on markdowns (see Ghemawat and Nueno 2003). Note that our definition of symmetry is only on costs and prices. The firms can differ on any other parameter. Clearly, if $c_t^j \geq p_t^j$, then retailer j will not order in period t. Therefore, by choosing the appropriate cost and price parameters we can model the asymmetric case of two retailers that have different reordering and demand learning capabilities. This represents the situation in which firm j has a lower ordering cost but can only produce before the selling seasons starts, whereas retailer i has higher costs but can replenish stock every period, taking advantage of any incremental knowledge about demand if it is available.

Assumption 6. *We ignore holding and lost sales penalty costs, and there is no minimum ordering quantity.*

Overall, we aim at formulating a parsimonious model. We omit inventory holding costs, as they are less relevant for short-life-cycle products. However, these costs can also be incorporated in the model.[5] For lost sales penalty costs, the revenue function needs to be redefined, but a similar analysis would go through.[6] On the contrary, the QR problem with minimum ordering quantities lies beyond the scope of this paper.

Assumption 7. *Leftover inventory can be carried over from the first to the second period and is lost at the end of the season. If both retailers stockout in a given period, the unsatisfied demand is lost as well.*

The leftover inventory in period t is equal to $(y_t^i - R_t^i(y_t^j))^+$. A salvage value at the end of the season could be easily incorporated in the model. Similarly, a straightforward extension allows for backlogged demand to be shared between the firms in a deterministic way.

[5] To incorporate holding costs, it suffices to redefine p_1^i, c_1^i, p_2^i, and c_2^i, as $p_1^i - h_1^i - h_2^i$, $c_1^i - h_1^i - h_2^i$, $p_2^i - h_2^i$, and $c_2^i - h_2^i$, respectively, where h_t^i is the holding cost paid at the end of period t.

[6] If one wished to incorporate lost sales penalty costs, one would subtract the term $v_t^i \mathbb{E}\{R_t^i - y_t^i\}^+ = v_t^i \mathbb{E}\{R_t^i\} - v_t^i \mathbb{E}\min\{y_t^i, R_t^i\}$ from the revenue.

3.2. Subgame-Perfect Strategies

Because the spillover demand depends on the inventory level of the competitor (see Equation (1)), we must use game-theoretical tools to analyze the replenishment decision, and we proceed by backward induction (see Fudenberg and Tirole 1991). To be precise, we begin with the terminal period $t = 2$ and then use the latter to establish the outcome in the first period. We are interested in pure strategies, which in our two-period game are given by two functions, one for each period, that dictate the action a firm plays in any possible state of the system.[7] For expositional purposes, we initially consider the case when the demand information I_2 is void. Throughout this paper, when it is clear from the context, we omit the arguments of a given function.

First, let r_2^i be firm i's *unconstrained* expected profit, which can be expressed as

$$r_2^i(y_2^i, y_2^j) = \mathbb{E}\{-c_2^i y_2^i + p_2^i \min\{y_2^i, R_2^i(y_2^j)\}\}. \tag{2}$$

Second, if firm i knew firm j's order-up-to level y_2^j, then firm i's best response would come from maximizing expected profits, taking into account its initial stock x_2^i and the competitor's action. In other words, retailer i would solve

$$\max_{y_2^i \geq x_2^i} c_2^i x_2^i + r_2^i(y_2^i, y_2^j). \tag{3}$$

Because the term $c_2^i x_2^i$ in Equation (3) is constant, firm i's best response actually comes from maximizing $r_2^i(y_2^i, y_2^j)$ subject to $y_2^i \geq x_2^i$. If the solution to this optimization problem is unique, then we can define the best-response function $b_2^i(x_2^i, y_2^j) \equiv \arg\max_{y_2^i \geq x_2^i}\{r_2^i(y_2^i, y_2^j)\}$, which represents the optimal stocking level in period 2 for firm i, in response to a level y_2^j from firm j, starting with a position of x_2^i.

Per Assumption (4), both firms decide the inventory levels simultaneously. Therefore, firm i does not know in advance what level y_2^j firm j will select (as it usually occurs in practice). Thus, to analyze the competition we use the notion of Nash equilibrium. In our setting, a Nash equilibrium, if it *exists*, is given by two functions e_2^i and e_2^j that might depend on the initial stock levels (x_2^i, x_2^j) and are such that $b_2^i(x_2^i, e_2^j) = e_2^i$ and $b_2^j(x_2^j, e_2^i) = e_2^j$. Put differently, no player is better off by unilaterally deviating from the equilibrium. It is important to note that if the equilibrium is *unique* for any initial conditions (x_2^i, x_2^j), then e_2^i and e_2^j are real single-valued functions. Otherwise, e_2^i and e_2^j are only correspondences unless one of the (multiple) equilibriums is somehow specified. In Theorem 1 we show that the equilibrium in period 2 is indeed unique, and therefore we can define the *equilibrium* expected profit π_2^i by replacing the equilibrium actions in the objective function of Equation (3) to obtain

$$\pi_2^i(x_2^i, x_2^j) = c_2^i x_2^i + r_2^i(e_2^i(x_2^i, x_2^j), e_2^j(x_2^j, x_2^i)). \tag{4}$$

If we now consider two periods, the notion of Nash equilibrium as a solution concept can be extended to this setting through a refinement known as subgame perfection. An equilibrium is subgame perfect if it induces a Nash equilibrium in each subgame of the original game (see Fudenberg and Tirole 1991). In our context, a subgame corresponds to a game that is similar to the original one but with one less period to go. In particular, the last period is a subgame of the two-period game. Therefore, in the first period we can construct the best-response functions just as we did for the last period, but the only caveat is that now the expected profit is the sum of the immediate profit plus the future profit to go, and the latter must be the equilibrium profit of the single-period subgame. Formally, the unconstrained expected profit in period 1 is given by

$$r_1^i(y_1^i, y_1^j) = \mathbb{E}\{-c_1^i y_1^i + p_1^i \min\{y_1^i, R_1^i(y_1^j)\} + \pi_2^i((y_1^i - R_1^i(y_1^j))^+, (y_1^j - R_1^j(y_1^i))^+)\}, \tag{5}$$

where π_2^i is the equilibrium expected profit of the last period subgame defined in Equation (4). Note that the latter is evaluated in $x_2^i = (y_1^i - R_1^i(y_1^j))^+$ and $x_2^j = (y_1^j - R_1^j(y_1^i))^+$, and therefore the order-up-to level decisions in period 1 affect the initial conditions (and equilibrium) in period 2.

The best-response functions in period 1 are obtained from maximizing $r_1^i(y_1^i, y_1^j)$ now subject to the constraint $y_1^i \geq x_1^i$, i.e.,

$$b_1^i(x_1^i, y_1^j) = \arg\max_{y_1^i \geq x_1^i}\{r_1^i(y_1^i, y_1^j)\}, \tag{6}$$

[7] By definition, a mixed strategy is a probability distribution over pure strategies (see Fudenberg and Tirole 1991).

and a Nash equilibrium in period 1, if it exists, is given by two functions e_1^i and e_1^j that might depend on the initial stock levels (x_1^i, x_1^j) and are such that $b_1^i(x_1^i, e_1^j) = e_1^i$ and $b_1^j(x_1^j, e_1^i) = e_1^j$. As before, we assume (and later prove in Theorems 2 and 3) that equilibrium in period 1 is unique for any given initial conditions (x_1^i, x_1^j), and therefore the expected equilibrium profit in period 1 is given by

$$\pi_1^i(x_1^i, x_1^j) = c_1^i x_1^i + r_1^i(e_1^i(x_1^i, x_1^j), e_1^j(x_1^j, x_1^i)). \tag{7}$$

The previous definitions were given for the case when the demand information in the second period I_2 is void. The latter would be valid if the demand across periods was independent. However, if the retailers can use demand information from the first period to predict demand in the second period, then their actions, and consequently the competitive equilibrium, will be contingent on the information that is actually available. Therefore, in that case, we write $r^i_{2|I_2}$, $b^i_{2|I_2}$, $e^i_{2|I_2}$, and $\pi^i_{2|I_2}$ instead of r_2^i, b_2^i, e_2^i, and π_2^i, respectively, in the equations above, and all the expectations are conditional on I_2.

4. Existence and Uniqueness of Equilibrium

We now present the structural results that we use later to compare different competitive settings. Our goal in this section is to prove the existence of a unique pure-strategy subgame-perfect equilibrium for the three cases mentioned in Table 1. These results have theoretical value but also allow us to understand the benefits of QR by computing comparative statics of the unique equilibrium, which is what we do in §5.

We start by considering the single-period problem in §4.1. Then, in §4.2 we consider the two-period case, and we study the two most cited QR models in the literature, namely, the *demand signal* and the *midseason replenishment* models.[8] It is worth pointing out that, when both retailers are symmetric, the results can be extended to an arbitrary number of periods under a linear demand splitting, as shown in the online appendix.

4.1. Single-Period Case

The single-period subgame that takes place in period 2 is an essential building block in our model. To simplify the exposition, we omit the dependence on the demand information vector I_2, but all the discussion throughout this section remains valid if we replace the subindex $t = 2$ with $2 \mid I_2$, and all the expectations are conditional on I_2.

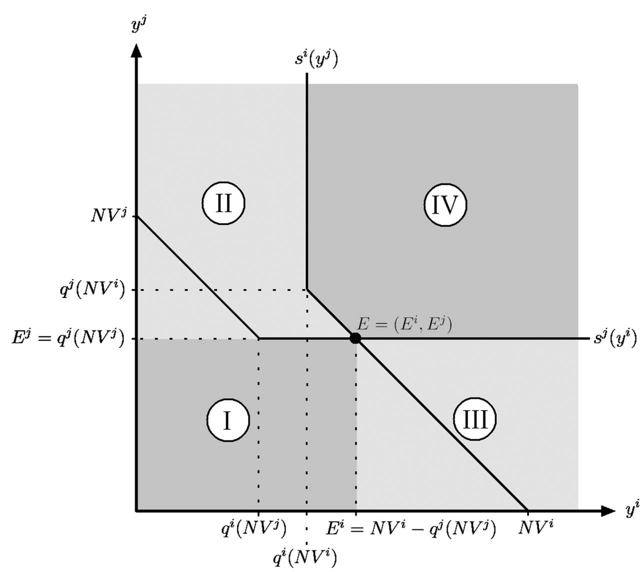
We first consider the *unconstrained* game, i.e., the game when the initial inventory levels are equal to zero. From Equation (2), it is clear that r_2^i is concave in y_2^i, for all y_2^j. Thus, the optimal inventory policy is a base-stock policy with target level $s_2^i(y_2^j)$, which can be obtained from the first-order conditions $\Pr(R_2^i(y_2^j) \geq s_2^i(y_2^j)) = c_2^i/p_2^i$. Solving the latter yields

$$s_2^i(y_2^j) = \begin{cases} NV^i - y_2^j & \text{when } y_2^j \leq q_2^j(NV^i), \\ q_2^i(NV^i) & \text{otherwise,} \end{cases} \tag{8}$$

where $NV^i \equiv \bar{F}_2^{-1}(c_2^i/p_2^i)$ corresponds to the newsvendor stocking quantity when firm i faces all demand, i.e., when $y_2^j = 0$ (recall that $1 - c_2^i/p_2^i$ is the newsvendor critical ratio).

In Figure 1 we plot $s_2^i(y_2^j)$ and $s_2^j(y_2^i)$. For simplicity, we omit the subindex $t = 2$, and firm i is such that $c^i/p^i < c^j/p^j$. The base-stock functions $s^i(y^j)$ and $s^j(y^i)$ intersect only once, which means that in the unconstrained competitive game there exists a unique

Figure 1 **Unconstrained Single-Period Base-Stock Functions**



[8] In Cachon and Terwiesch (2005), these two models are defined in terms of the reactive capacity and are referred to as *limited* and *unlimited but expensive*, respectively.

Nash equilibrium, which we denote $E = (E^i, E^j)$. The shaded regions (I)–(IV) are used in the proofs provided in the electronic companion.

If now we allow the initial inventory levels to be nonzero (i.e., we consider the *constrained* game), given the concavity of r_2^i, it follows that firm i's best response is $b_2^i(x_2^i, y_2^j) = \max\{x_2^i, s_2^i(y_2^j)\}$. Therefore, in the constrained competitive game, a graph of the best responses would look just as Figure 1, except that the vertical and horizonal stretches would move right and down, respectively. Hence, we obtain our first result.

Theorem 1. *For all* (x_2^i, x_2^j), *there exists a unique Nash equilibrium* $(e_2^i(x_2^i, x_2^j), e_2^j(x_2^j, x_2^i))$ *of the stocking game. In addition, we can characterize* (e_2^i, e_2^j) *as follows. Without loss of generality, assume that* $c_2^i/p_2^i \le c_2^j/p_2^j$ (*firm i has a* higher *newsvendor critical ratio*). *Then*

$$e_2^j(x_2^j, x_2^i) = \max\{x_2^j, q_2^j(NV^j)\} \quad and$$
$$e_2^i(x_2^i, x_2^j) = \max\{x_2^i, q_2^i(NV^i), NV^i - e_2^j(x_2^j, x_2^i)\}.$$

We can see from Theorem 1 that the equilibrium strategy of the firm with the lower critical ratio is independent of the competitor's inventory level. This is intuitive because, for that firm, leftover inventory impacts profits more. The reverse is not true: the equilibrium strategy of the higher critical ratio firm may depend on how much inventory is available at the competitor.

Theorem 3 of Lippman and McCardle (1997) proves that the unconstrained competitive game with symmetric retailers has a unique Nash equilibrium. The result requires the demand allocation functions to be deterministic and strictly increasing, just as in our setting. Theorem 1 in this paper extends the result by Lippman and McCardle (1997) in the sense that we consider the constrained game (i.e., the initial inventory levels can be nonzero), and we allow for asymmetric retailers (i.e., they can face different costs and prices).

From a technical standpoint, the unique equilibrium in the single-period problem follows from the fact that the unconstrained expected profit r_2^i is concave in firm i's action y_2^i. To prove the existence of a unique pure-strategy subgame-perfect equilibrium in the two-period game, we will need to show that r_1^i is (strictly) quasiconcave in y_1^i. For that, we first need to show that π_2^i, the equilibrium expected profit in the single-period problem, is concave in firm i's initial stock level x_2^i. Note that we have to consider the equilibrium expected profit because, by the definition of subgame perfection, the retailers assume that in the last period a Nash equilibrium will be played, given any initial state (x_2^i, x_2^j, I_2) (we refer the reader back to the discussion at the end of §3.2). The following proposition provides the theoretical result that we need to analyze the two-period game.

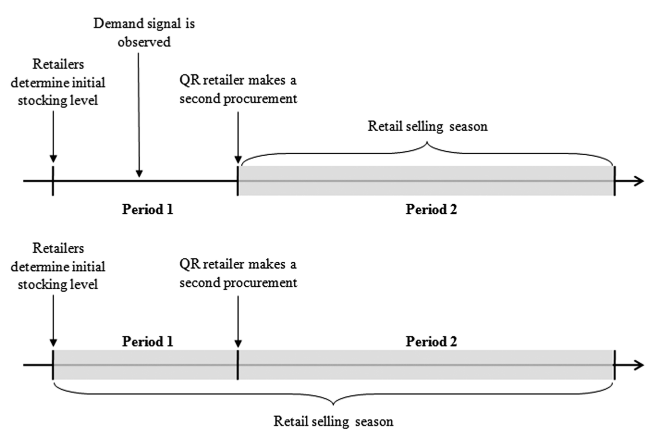
Proposition 1. *For* $i = 1, 2$ *and for any* (*uncensored*) *market information* I_2, *the expected equilibrium profit* $\pi_{2|I_2}^i(x_2^i, x_2^j)$ *is concave in* x_2^i, *for all* x_2^j.

4.2. Two-Period Case

We now examine the two-period case. We consider two models that seem to concentrate most of the attention in the literature (see, for instance, Cachon and Terwiesch 2005). Figure 2 shows a schematic description of both models.

4.2.1. Demand Signal. The first QR model we consider is based on the one studied (for a single firm) in Iyer and Bergen (1997). A similar sequence of events has been used in several other papers (see, for instance, Cachon and Swinney 2009). The planning horizon is divided into two periods. The last one represents the retail selling season, whereas the first one represents a period during which a demand signal is revealed. The latter could simply represent data that is collected right before the season starts (for example, in fashion shows, mock stores, and focus groups, or by consulting experts). We assume

Figure 2 **QR Models: Demand Signal (Top) and Midseason Replenishment (Bottom)**



that the demand signal is informative, meaning that it is correlated with the actual demand during the season. Otherwise, this model reduces to the single-period problem studied in the previous section. A QR retailer can place orders before and after observing the demand signal, whereas a traditional SR retailer, because of longer lead times, can only place a single order before the additional demand information becomes available. The sequence of events for a QR retailer is depicted in Figure 2 (top timeline).

Given that there is no demand realization in the initial period ($R_1^i = R_1^j = 0$), the unconstrained expected profit for either an SR or QR retailer reduces to

$$r_1^i(y_1^i, y_1^j) = -c_1^i y_1^i + \mathbb{E}\{\pi_{2|I_2}^i(y_1^i, y_1^j)\}, \tag{9}$$

where the expectation is with respect to the (a priori) distribution of I_2. Note that $r_1^i(y_1^i, y_1^j)$ is concave in y_1^i, for any y_1^j (from Proposition 1). Therefore, the existence of a pure-strategy (subgame-perfect) Nash equilibrium in the unconstrained competitive game is guaranteed by Theorem 1.2 in Fudenberg and Tirole (1991).[9] Moreover, also because of the concavity of r_1^i, firm i's best response is a base-stock policy $s_1^i(y_1^j)$. Therefore, just as in the single-period model, the best-response function in the constrained game is equal to $b_1^i(x_1^i, y_1^j) = \arg\max_{y_1^i \geq x_1^i}\{r_1^i(y_1^i, y_1^j)\} = \max\{x_1^i, s_1^i(y_1^j)\}$. The following theorem provides conditions under which the equilibrium is unique.[10]

Theorem 2. *If $c_1^i \neq c_2^i$ for $i = 1, 2$, then the stocking game with a demand signal has a unique subgame-perfect equilibrium. In that equilibrium, both retailers play pure strategies.*

Three technical observations about Theorem 2 are worth noting. First, when $c_1^i = c_2^i$, the equilibrium exists; in fact, not ordering in the initial period would be an equilibrium, but in general it might not be unique (though the profits achieved are the same under any equilibrium).[11] Second, the theorem rules out the existence of an equilibrium in which the retailers randomize over pure strategies, because otherwise it would contradict the fact that there is a unique subgame-perfect equilibrium. Third, per Assumption (4), the strategies played by the retailers must be Markovian. Hence, what we actually prove is that there exists a unique Markov-perfect equilibrium (see §13 in Fudenberg and Tirole 1991). However, because the state (x_2^i, x_2^j, I_2) contains all the payoff-relevant information in period 2 (from Theorem 1), any strategy that induces a subgame-perfect equilibrium must be Markovian, or at least equivalent to a Markovian strategy. To be precise, all histories that lead to the same state (x_2^i, x_2^j, I_2) result in the same best responses and same equilibrium. Hence, the histories can be partitioned according to the state (x_2^i, x_2^j, I_2) they lead to, and thus any subgame-perfect strategy can be mapped into a Markovian strategy. This observation justifies the claim that there is a unique subgame-perfect equilibrium. A similar justification is given in Hall and Porteus (2000).

Example 1 (Iyer and Bergen 1997). The two-period demand signal case allows us to model the competition between a traditional retailer (i.e., one with very long lead times) and a QR retailer that is modeled as in Iyer and Bergen (1997). In that paper, demand during the retail selling season is assumed to be normally distributed; i.e., $D_2 \sim N(\theta, \sigma^2)$. The variance σ^2 is assumed to be known, whereas the average size of the market θ is uncertain. Information about θ in the initial period is modeled as a normal distribution with mean μ and variance τ^2. Thus, at time $t = 1$, the prediction of season demand is normally distributed with mean μ and variance $\sigma^2 + \tau^2$. Then the demand signal d is realized, and the QR retailer performs a Bayesian update of its belief regarding θ. In other words, we have that $I_2 = \{d\}$, and $D_{2|I_2} \sim N(\mu(d), \sigma^2 + 1/\rho)$, where

$$\mu(d) = \frac{\sigma^2\mu + \tau^2 d}{\sigma^2 + \tau^2} \quad \text{and} \quad \rho = \frac{1}{\sigma^2} + \frac{1}{\tau^2}.$$

Note that $1/\rho < \tau^2$. Therefore, after the realization of the demand signal d is observed, the QR retailer has a more accurate prediction of season demand. The traditional retailer cannot make use of the demand signal because of long lead times. In other words, it cannot place a second order after the demand signal

[9] The theorem actually requires that the strategy space is compact. Because the retailers would never order an infinite stock, it is always possible to restrict their actions to a compact set.

[10] When the equilibrium is not unique, $s_1^i(y_1^j)$ and $b_1^i(x_1^i, y_1^j)$ are actually correspondences rather than functions.

[11] Notice that if $c_1^i > c_2^i$, then the equilibrium is to not order anything in the initial period. On the contrary, if $c_1^i < c_2^i$, then a positive amount is ordered in $t = 1$.

is realized. This is incorporated in our model by letting $c_2^i = p_2^i$ for the traditional retailer. As long as it is cheaper to order in the initial period (for both retailers), the conditions of Theorem 2 hold, and the competition between the QR firm and the traditional retailer has a unique subgame-perfect equilibrium.

4.2.2. Midseason Replenishment. In the previous QR model, the demand signal does not deplete any stock. That is, the acquisition of additional demand information in the initial period does not affect the inventory that is carried over to the final period. This simplifies the analysis and allows us to prove the existence of a unique equilibrium under fairly general conditions (see Table 1). In the current section, we consider a second QR model that differs from the previous one in few subtle but fundamental aspects. Specifically, the selling season comprises both periods, the actual sales that occur in the initial period play the role that the demand signal previously had, and the procurement in the final period comes to replenish the inventory that has been depleted (hence the name for this case). The sequence of events is depicted at the bottom of Figure 2. An example that would fit this QR model is the Sport Obermeyer case (Fisher and Raman 1996), where 20% of initial sales provides an excellent estimate of the remaining 80%.

The unconstrained expected profit $r_1^i(y_1^i, y_1^j)$ in the initial period ($t = 1$) is given by Equation (5). This differs from the expression we considered in the demand signal case because now sales can occur in period 1 (see Equation (9)). Moreover, the profit-to-go $\pi_{2|I_2}^i$ is evaluated in the remaining inventory, and r_1^i is no longer guaranteed to be concave. Fortunately, we are able to show that it is quasiconcave under certain conditions to be introduced next, and hence, the optimal policy is still a base-stock policy dependent on y_1^j.

Proposition 2. *Assume the following for $i = 1, 2$:*

(i) $p_1^i \geq c_2^i \geq c_1^i$;

(ii) *D_1 has infinite support and a log-concave p.d.f., i.e., $\log(f_1(d))$ is concave in d;*

(iii) *$I_2 = \{D_1\}$ and $D_{2|I_2} = kD_1 + \epsilon$, where $k \geq 0$ and ϵ is a random variable independent of D_1 with p.d.f. g such that for all x*

$$\max\left\{0, \frac{f_1'}{f_1}((q_1^i)^{-1}(x))\right\} \leq \max\left\{0, \frac{g'}{g}((q_2^i)^{-1}(x))\right\}; \quad (10)$$

(iv) *if $k > 0$, then $q_2^i(d) = \alpha_2^i d$.*

Then, $r_1^i(y_1^i, y_1^j)$ is quasiconcave for all y_1^j, and the constrained best response is $b_1^i(x_1^i, y_1^j) = \max\{x_1^i, s_1^i(y_1^j)\}$, where $s_1^i(y_1^j)$ is the (unconstrained) base-stock level, which is unique.

Condition (i) in Proposition 2 requires that the cost does not decrease over time and that the initial price is not smaller than the midseason replenishment cost. This would be the case if the price is fixed throughout the planning horizon and the midseason replenishment is more expensive than the initial procurement. Condition (ii) requires demand D_1 to be log-concave with an infinite support.[12] Corollary (2) below provides examples of common distributions that meet this requirement. Condition (iii) specifies the dependency between D_1 and D_2 that is allowed. Notice that Equation (10) is satisfied if f_1 has a decreasing p.d.f. or if it is not larger than g in the likelihood ratio order. For example, this is the case when the initial period represents a small fraction of the total season and demand is Normal (see Example 2). Finally, condition (iv) requires a linear splitting rule in the final period whenever D_1 and D_2 are not independent.

The central idea in the proof of Proposition 2 is to show that the following inequality holds:

$$\frac{\partial^2 r_1^i}{(\partial y_1^i)^2} < \phi_1((q_1^i)^{-1}(y_1^i))\left(\frac{\partial r_1^i}{\partial y_1^i}\right),$$
$$\text{where } \phi_1(y) = \max\left\{0, \frac{f_1'}{f_1}(y)\right\}. \quad (11)$$

Note that quasiconcavity and uniqueness follow directly from (11). In fact, for a given $y_1^j \geq 0$, consider a critical point s^* such that $(\partial r_1^i/\partial y_1^i)(s^*, y_1^j) = 0$.[13] From inequality (11), s^* is necessarily a strict maximum; i.e., $(\partial^2 r_1^i/(\partial y_1^i)^2)(s^*, y_1^j) < 0$. Furthermore, this maximizer must be unique because otherwise there would have to be a minimum in between any two maxima that would contradict (11). This shows that $r_1^i(y_1^i, y_1^j)$ as a function of y_1^i (and for a given y_1^j) is first increasing and then decreasing; i.e., it is quasiconcave.

Proving Equation (11) is not straightforward, and we rely on conditions (i)–(iv) in Proposition 2 to

[12] To be precise, the support must be either the real line or an interval of the type $[a, +\infty)$.

[13] Note that a critical point must exist because for y_1^i very large, r_1^i is eventually decreasing.

bound the second derivative of r_1^i by its first derivative. Condition (i) is needed to rule out local maxima and nonsensical solutions (think of a newsvendor in which the salvage value is greater than the selling price). The log-concavity in condition (ii) is needed to generate bounds in the proof and the unbounded support is used to guarantee that the inequality (11) is strict. The last two conditions ((iii) and (iv)) provide a correlation structure that is amenable to analysis. Note that when there is correlation, the second-period profit $\pi^i_{2|I_2}(x_2^i, x_2^j)$ depends on the first-period demand D_1 in two ways: through the inventory that is carried over, (x_2^i, x_2^j), and through the market information, I_2. Thus, computing expectations with respect to D_1 becomes quite involved. We can overcome this obstacle by imposing condition (iv), which allows us to make a linear change of variables that reduces the double dependency on D_1 to a single dimension.[14] We do this for tractability reasons, but it does not affect the essence of the game in the second period (per Theorem 1, the inventory level of the firm with the higher critical ratio still depends on the competitor's action).

We can now state the main result of this section. Note that this result applies both to the cases of QR versus QR competition and QR versus SR competition by appropriately setting the price/cost parameters if necessary (the same holds for Theorem 2).

Theorem 3. *If conditions* (i)–(iv) *of Proposition* 2 *are satisfied and* $p_1^i \geq p_2^i$ *for* $i = 1, 2$, *then the stocking game with midseason replenishment has a unique pure-strategy subgame-perfect equilibrium.*

As in Theorem 2 for the demand signal case, Theorem 3 shows existence and uniqueness of a pure-strategy subgame-perfect equilibrium. However, there are a some differences. First, Theorem 3 requires a few more conditions than Theorem 2 (see Table 1 for a comparison). In particular, the condition $p_1^i \geq p_2^i$ is used in the proof of uniqueness to show that $ds_1^i/dy_1^j \geq -1$; i.e., if firm j increases (decreases) its inventory level by one unit, then firm i does not decrease (increase) its stock by more than one. Second, because in the midseason replenishment case r_1^i is quasiconcave rather than concave, we cannot rule out the existence of an equilibrium in which firms play randomized strategies. Another consequence is that the equilibrium profit π_1^i defined in Equation (7) is not concave either, and therefore we are not able to extend Theorem 3 to a larger number of periods.

[14] See Equation (20) in the proof, provided in the electronic companion.

Despite the additional conditions required in Theorem 3, there are several interesting cases for which they hold. Two of them are given in the next corollaries. Corollary 1 shows the simplest application of Theorem 3 by assuming independent and identically distributed (i.i.d.) demand. On the contrary, Corollary 2 shows an application with demand that is correlated across periods. We then use the latter in an example that resembles the QR model in the Sport Obermeyer case (Fisher and Raman 1996).

Corollary 1 (Independent Demands). *Assume that* $c_1^i \leq c_2^i$, $p_1^i \geq p_2^i$, *and* $q_1^i = q_2^i$ *for* $i = 1, 2$. *If the demands* D_1, D_2 *are i.i.d. and* D_1 *is log-concave with infinite support, then the stocking game with midseason replenishment has a unique pure-strategy subgame-perfect equilibrium.*

Corollary 2 (Correlated Demands). *Assume that* $c_1^i \leq c_2^i$, $p_1^i \geq p_2^i$, *and* $q_1^i(d) = q_2^i(d) = \alpha^i d$ *for* $i = 1, 2$. *Let* ϵ_1, ϵ_2 *be two independent random variables such that* $D_1 = \epsilon_1$ *and* $D_{2|I_2} = kD_1 + \epsilon_2$, *with* $k > 0$ (*thus*, $\rho \equiv \mathrm{Corr}(D_1, D_2) = k\sqrt{\mathrm{Var}(D_1)/\mathrm{Var}(D_2)} > 0$). *Furthermore, let*

- ϵ_1, ϵ_2 *follow normal distributions with parameters* (μ_1, σ_1) *and* (μ_2, σ_2), *respectively, and* $\mu_1 \leq \mu_2$ *and* $\sigma_1 \geq \sigma_2$; *or*
- ϵ_1, ϵ_2 *follow truncated normal distributions with parameters* (μ_1, σ_1) *and* (μ_2, σ_2), *respectively, and* $\mu_1 \leq \mu_2$ *and* $\sigma_1/\mu_1 \geq \sigma_2/\mu_2$; *or*
- ϵ_1, ϵ_2 *follow gamma distributions with parameters* (a_1, θ_1) *and* (a_2, θ_2), *respectively, and* $\theta_1 \leq \theta_2$ *and* $1 \leq a_1 \leq a_2$; *or*
- ϵ_1, ϵ_2 *follow exponential distributions.*

In the four cases above, the stocking game with midseason replenishment has a unique pure-strategy subgame-perfect equilibrium.

Example 2 (Fisher and Raman 1996). Consider the case when the demand vector (D_1, D_2) follows a multivariate normal with marginal distributions $D_t \sim N(\mu_t, \sigma_t)$ and covariance $\mathrm{Cov}(D_1, D_2) = \rho\sigma_1\sigma_2$, with $\rho \geq 0$. Then, we have that the demand in the last period conditional on the demand realization in the initial period is given by $D_{2|I_2} = kD_1 + \epsilon_2$ with $k = \rho\sigma_2/\sigma_1$, and ϵ_2 is normally distributed with parameters $(\mu_2 - k\mu_1, \sigma_2\sqrt{1-\rho^2})$ and is independent of D_1 $(=\epsilon_1)$. Notice that if there is positive correlation $(\rho > 0)$, then $\mathrm{Var}(D_{2|I_2}) = \sigma_2^2(1-\rho^2) < \sigma_2^2 = \mathrm{Var}(D_2)$. In other words, as in Example 1, the updated forecast $D_{2|I_2}$ is more accurate than the unconditional prediction of D_2.[15] It can be shown that the condition on the normal distributions in Corollary 2, in this case, reduces to

$$\mu_1 < \mu_2 \quad \text{and} \quad \sigma_1 \geq \sigma_2 \max\left\{\sqrt{1-\rho^2}, \rho\frac{\mu_1}{\mu_2 - \mu_1}\right\}. \quad (12)$$

Hence, if the latter holds, then Theorem 3 applies. In particular, this would be the case if a QR retailer can place an order after observing a small fraction (e.g., 20%) of the total season demand (so that $\mu_1 \ll \mu_2$), and these early sales provide relevant information about what should be expected in the remainder of the season (i.e., the correlation ρ is high).

5. Quick vs. Slow Response Competition

In this section we provide an extensive study on the two-period $(T = 2)$ inventory game. We do this analytically when tractable; otherwise, we proceed numerically. Theorems 2 and 3 provide the theoretical grounds that guarantee the existence and uniqueness of the equilibrium. We now perform comparative statics; i.e., we study how the equilibrium depends on the key parameters of the model. Because the equilibrium is unique, we can indeed build one-to-one mappings with respect to the input parameters.

In §5.1 we first define the scenarios used in our study. Then, in §§5.2 and 5.3 we compare inventory levels and profits, respectively. For expositional purposes, we focus on the midseason replenishment case. The results presented below hold for the demand signal case as well, except for a few numerical observations. These minor differences are pointed out when they occur, and we describe them more extensively in the online appendix.

[15] Under the conditions of Proposition 2, $\mathrm{Var}(D_2) = k^2\mathrm{Var}(D_1) + \mathrm{Var}(\epsilon) \geq \mathrm{Var}(\epsilon) = \mathrm{Var}(D_{2|I_2})$.

5.1. Definition of Three Market Configurations

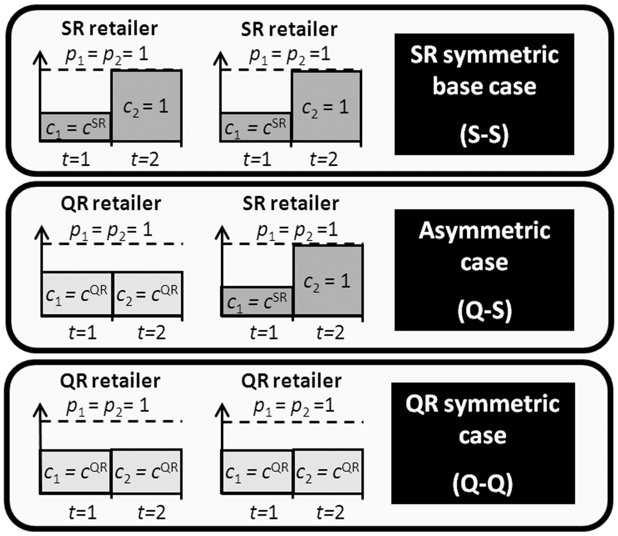
We are interested in quantifying the competitive advantage of QR over SR (slow response). In particular, we compare inventory levels and profits vis-à-vis the "traditional" competition between two SR firms. For this purpose, we consider three main situations, which we will denote by S-S, Q-S, and Q-Q, all falling within the generic model presented above (see §3). The first scenario, S-S, which is the base case, considers two SR retailers that can only place orders before the start of the selling season at a cost c^{SR} (i.e., they are not able to replenish stock at time $t = 2$). Here, the competition corresponds to the single-period equilibrium studied in Lippman and McCardle (1997). The second case, Q-S, which is the focus of the paper, considers the asymmetric competition between a QR and an SR retailer. The SR retailer has the same replenishment capabilities as in the base case (S-S), but the QR firm is characterized by the ability to place orders in both periods at a cost c^{QR}. As in reality, we allow the QR retailer to have the same, or possibly higher, ordering cost than the SR firm (i.e., $c^{\mathrm{QR}} \geq c^{\mathrm{SR}}$). Finally, the third case, Q-Q, considers two symmetric QR retailers (i.e., both firms can replenish in the second period at a cost c^{QR}).

In terms of notation, when we need to specify the type of a firm *and* the scenario, we use two italic letters of the form T_1T_2, which indicates a firm type T_1 facing a competitor type T_2. For instance, $s_1^{i,SQ}(y_1^j)$ denotes the unconstrained base-stock level of firm i, which is an SR retailer facing a QR competitor (abbreviated SQ).

In our simulations, we assume a fixed price equal to one in both periods and for both firms ($p_1^i = p_2^i = p_1^j = p_2^j = 1$). Therefore, if firm i is a SR retailer, then $c_1^i = c^{\mathrm{SR}}$ and $c_2^i = p_2^i = 1$, whereas if firm i is a QR retailer, then $c_1^i = c_2^i = c^{\mathrm{QR}} < 1$. Figure 3 summarizes the three cases that we analyze throughout this section.

Our demand model follows the structure presented in Proposition 2, namely, $D_{2|I_2} = kD_1 + \epsilon$. This allows retailers to learn from the realization of D_1 and improve the forecast of the last period demand D_2. The parameter k determines the correlation between D_1 and D_2. Specifically, $\rho = \mathrm{Corr}(D_1, D_2) = k/\sqrt{k^2+1}$. In Figures 4–8, we plot our results directly as a function of the demand correlation instead of k. For simplicity, and to avoid negative demand, we assume

Figure 3 Three Market Configurations: SR Symmetric Competition (S-S, Top), Asymmetric Competition (Q-S, Middle), and QR Symmetric Competition (Q-Q, Bottom)



that D_1 and ϵ are identically distributed, and follow a gamma distribution with mean $\mu = 1$ and standard deviation σ. In terms of the demand allocation function, we use a linear splitting rule $q_1^i(d) = q_2^i(d) = \alpha^i d$ with a 50% market share for each retailer (i.e., $\alpha^i = 0.5$, for $i = 1, 2$). In the online appendix we provide a numerical study for the case when the demand split in the first period is nonlinear. Finally, we assume that the initial stock is equal to zero ($x_1^i = 0$, for $i = 1, 2$).

Note that under the last two assumptions, linear splitting and zero initial inventory, it can be shown that when the retailers are symmetric (i.e., they share the same price and cost structure so they are both SR or QR), the equilibrium outcome is the same as when there are no spillovers.[16] In other words, in the S-S (respectively, Q-Q) scenario, an SR (respectively, QR) firm makes the same inventory decisions and achieves the same expected profit as when there are no spillovers. This allows us to compare our results with those available in the literature for the noncompetitive single-firm case.

5.2. Equilibrium Inventory and Spillovers

We start by comparing the (first period) equilibrium inventory levels between the base scenario (S-S) and the case when the retailers have asymmetric ordering capabilities (Q-S). We first present an analytical result, which we then complement with our simulations. The following proposition shows that when the retailers have different replenishment capabilities, the base-stock levels, and consequently, the best-response functions, are shifted downward compared to the base case. This holds for any deterministic splitting (i.e, not necessarily linear) that satisfies Assumption (3).

Proposition 3. *In the first period, the unconstrained base-stock level is greater in* S-S *than in* Q-S *competition, for both firms. In other words,*

$$s_1^{i,SS}(y_1^j) \geq s_1^{i,QS}(y_1^j), \quad \forall y_1^j \geq 0 \quad \text{and} \tag{13}$$

$$s_1^{j,SS}(y_1^i) \geq s_1^{j,SQ}(y_1^i), \quad \forall y_1^i \geq 0. \tag{14}$$

Moreover, in equilibrium, the first-period industry inventory level is greater in S-S *than in* Q-S *competition. Formally,*

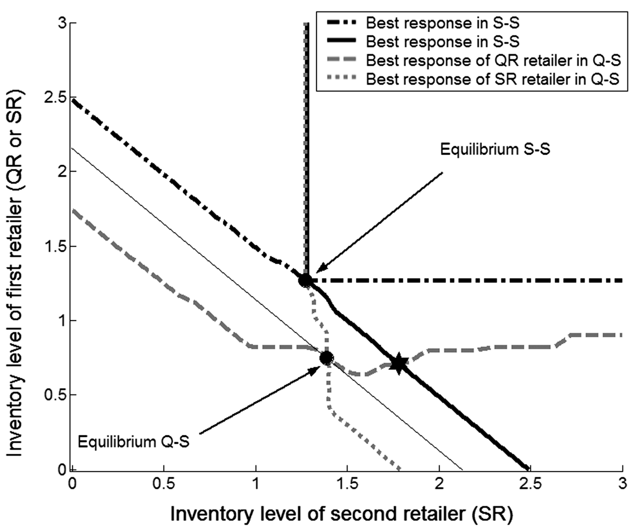
$$e_1^{i,SS}(x_1^i, x_1^j) + e_1^{j,SS}(x_1^j, x_1^i) \geq e_1^{i,SQ}(x_1^i, x_1^j) + e_1^{j,QS}(x_1^j, x_1^i), \quad \forall x_1^i, x_1^j \geq 0. \tag{15}$$

The first part of Proposition 3 is a pointwise comparison of the unconstrained base-stock levels for each firm, which are shifted downward when one of them has QR capabilities. This can be seen in Figure 4. The horizonal axis represents an SR firm, and the vertical axis represents either an SR firm (under S-S) or a QR firm (under Q-S). Note that the best-response functions under asymmetric competition (Q-S) are no greater than the respective functions in the base case (S-S), which graphically confirms Equations (13) and (14).

In Figure 4, the two solid dots represent the equilibrium under S-S and Q-S, respectively. Note that, compared to the base case S-S, the equilibrium under Q-S is shifted down and to the right. The shift down should be expected, because a firm that has QR capabilities will use its flexibility to order less in the first period. The shift of the equilibrium to the right shows that the reaction of the SR firm is to stock more. This outcome is not obvious and is the net result of two counteracting effects. On the one hand, the downward shift of the best-response function of the QR firm (see Equation (13)) induces the SR firm to stock more, because there are more opportunities for demand

[16] We refer the reader to §A in the online appendix for a detailed discussion of symmetric competition.

Figure 4 First-Period Best-Response Functions



Notes. The two solid dots represent the equilibriums (S-S and Q-S), and the star has an expositional purpose (see the discussion in §5.2). Here, we use $\rho = 0.7$, $\sigma = 0.6$, and $c^{QR} = c^{SR} = 0.6$.

spillover to occur. This first shift is represented by the star in Figure 4. On the other hand, the best response of the SR firm is also shifted downward under Q-S competition (see Equation (14)), which makes it stock less. Numerically, we have observed that the former effect (to stock more) dominates the latter (to stock less), and overall, the equilibrium shifts to the right. Finally, despite the fact that the equilibrium inventory levels of both firms move in opposite directions (i.e., the QR firm stocks less and the SR stocks more), the total inventory level of the industry decreases, as stated in Equation (15) of Proposition 3. This can be seen in Figure 4 by comparing the isoinventory lines with slope −1.

An interesting observation from Figure 4 is that the best-response function of the QR firm is decreasing, then increasing, and eventually becomes constant. We found this behavior in most of the simulations performed for the midseason replenishment case and the rationale is as follows.[17] On the one hand, the decreasing segment exists because the QR firm will receive less spillover as the SR firm stocks more. On the other hand, if the SR firm has an extremely large quantity of inventory, then the best response of the QR firm is to ignore the competitor because it will not receive any spillover demand. At that point, the QR firm is better off choosing the inventory level that it would order if spillovers did not occur at all. This explains the constant segment.

For intermediate inventory levels, the best-response function increases. Thus, there is a range in which the best response dips below the constant level. In fact, in that range the QR firm can reduce overall costs by ordering less than the constant level in the first period. Of course, by cutting the initial procurement, some of its demand will leak to the SR competitor, but that can pay off because it depletes inventory that would otherwise be carried over to the second period. In fact, if the SR retailer is left with little stock, then the QR firm can recoup some of the demand it lost because it will receive more spillover in the last period. Hence, by letting demand leak in the first period, the QR firm reduces inventory risk. This becomes even more pronounced with demand correlation because the QR firm can also learn at the competitor's expense.[18] To the best of our knowledge, this numerical observation has not been reported in the literature. We refer the reader to §B.1 in the online appendix for more examples.

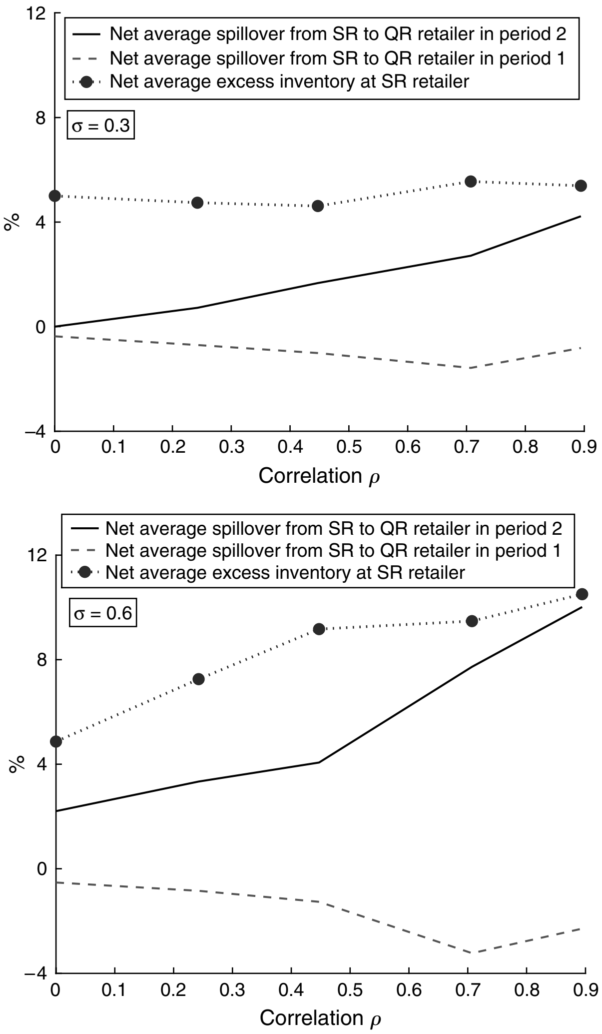
To conclude this section, we investigate the magnitude of the spillovers under Q-S competition. Figure 5 shows the net average spillover from the SR to the QR firm in each period. A positive (respectively, negative) value for period $t = 1, 2$ means that, on average, the SR (respectively, QR) retailer is leaking demand to the competitor. The figure also shows what we call the net average excess inventory, i.e., the average difference ("SR minus QR") in leftover stock. For this curve, a positive value means that, on average, the SR retailer has more unsold inventory at the end of the second period. Note that the values in Figure 5 have been normalized by $\alpha\mathbb{E}\{D_1 + D_2\}$, which is the total expected demand for a single firm without inventory-based competition.

The first observation from Figure 5 is that the average spillover in the first period goes from the QR to

[17] This numerical observation does not hold for the demand signal case because demand only occurs in the last period; see §B.2 in the online appendix for details.

[18] Recall that we do no allow demand to be censored; see Assumption (2). Any benefits from learning would be reduced if the QR firm could not forecast sales effectively because of low inventory levels and demand censoring.

Figure 5 Net Average Spillover from the SR to the QR Firm, and Net Average Excess Inventory at the SR Firm as a Function of ρ (Values Normalized by $\alpha\mathbb{E}\{D_1 + D_2\}$), with $c^{QR} = c^{SR} = 0.5$, $\sigma = 0.3$ (Top), and $\sigma = 0.6$ (Bottom)



the SR firm. As mentioned above, in equilibrium the SR firm stocks more (mostly because the QR competitor stocks less), and therefore it benefits from some spillover in the first period. However, this is reversed in the second period, and the spillover goes in larger quantities from the SR to the QR retailer. Figure 5 confirms that, overall, the QR firm gets a larger share of spillover demand. Remarkably, despite the eventual stockout that generates spillover, on average, the SR retailer finishes the season with more unsold stock, which confirms that the QR firm faces lower inventory risk.[19] This holds even without demand correlation ($\rho = 0$) and becomes more significant as it increases ($\rho \nearrow 1$). For instance, for high demand variability ($\sigma = 0.6$) and high demand correlation ($\rho \geq 0.9$), the average spillover received by the QR firm in the second period may be as large as 10% of the total expected demand without competition (recall that we use the latter as the normalizing constant), whereas in the same setting the SR firm receives less than 3% in the first period, and it finishes the season with much higher inventory.[20]

5.3. Profit Comparison

After analyzing the inventory levels in equilibrium, we move on to study the retailers' profits. As in the previous section, we first present an analytical result regarding the equilibrium profits, which we then complement with numerical evidence. The following proposition compares the profits obtained by a firm in the different competitive scenarios.

Proposition 4. *Under linear demand splitting, if $c^{QR} = c^{SR}$ and the initial stock levels are zero, then the profits a firm* (SR *or* QR) *can achieve in the different competitive scenarios can be ordered as follows*:

$$\pi_1^{QS} \geq \pi_1^{QQ} \geq \pi_1^{SQ} \geq \pi_1^{SS}, \tag{16}$$

where $\pi_1^{T_1T_2}$ represents the first period expected equilibrium profit of a firm type T_1 facing a competitor type T_2.

A few observations are worth pointing out from Equation (16). (i) The rightmost inequality shows that the least preferred situation is to be an SR firm facing another SR competitor. This is because when both retailers are symmetric, the equilibrium resembles the outcome of two firms that do not compete with each other, and therefore, spillovers do not occur (see the

[19] The reader should keep in mind that Figure 5 reports averages. Clearly, for a given demand realization, the SR retailer either stocks out and generates spillover to the QR firm, or it finishes with unsold inventory.

[20] In the demand signal case, spillovers can only occur in the second period and can favor the SR retailer when demand correlation is low. Similarly, if the demand split is nonlinear, then the firms face different coefficients of variation, and the total spillover across both periods might favor the SR retailer. However, in either case, the QR firm is still better off. See §§B.2 and B.3 in the online appendix for more details.

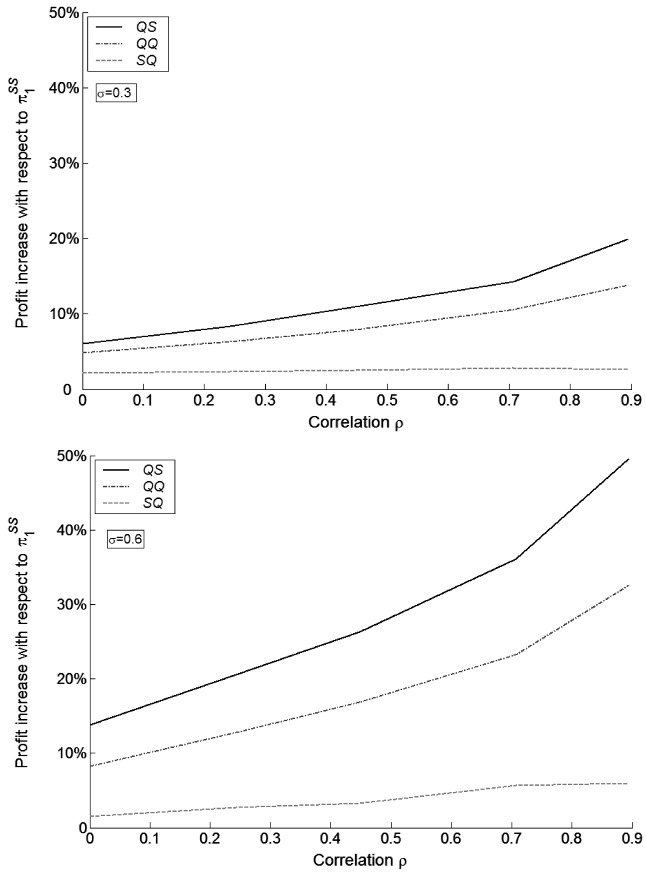
discussion at the end of §5.1). In contrast, under asymmetric competition, the SR retailer can receive some spillover (see Figure 5), and overall it can achieve higher expected profits. Hence, an SR retailer would prefer a QR competitor. (ii) The middle inequality shows that an SR retailer competing against a QR opponent would rather be a QR retailer itself. This confirms that flexibility pays off, and that a retailer benefits from being able to replenish in the second period (even if the competitor has QR capabilities). (iii) The leftmost inequality shows that the best scenario for a QR retailer is to have an SR competitor. As mentioned in §5.1, in the symmetric Q-Q scenario there are no spillovers. Hence, the QR firm prefers an SR competitor so it can benefit from the spillovers, either by selling more or by reducing inventory risk (see the discussion following Figure 5). Thus, part of the competitive advantage of QR comes from the replenishment agility asymmetry.

Proposition 4 shows that, for both types of firms, asymmetric competition is preferred over a noncompetitive scenario in which spillovers do not occur. To be precise, an SR firm prefers Q-S over S-S, and a QR firm prefers Q-S over Q-Q. Of course, this result relies on the assumptions that lead to Proposition 4 and is driven by the structure of our model. In particular, recall that we consider a competitive setting in which firms can prevent demand from leaking, but they cannot attract (or deviate) demand to themselves. From that perspective, the competition in our model is less aggressive, and the asymmetry is beneficial because both retailers can eventually receive spillover demand.

To further illustrate Proposition 4, Figure 6 shows the increase in the equilibrium expected profits with respect to the base case π_1^{SS} when $c^{\mathrm{QR}} = c^{\mathrm{SR}}$. The top graph has $\sigma = 0.3$, and the bottom graph has $\sigma = 0.6$. In each graph, three curves appear: the top curve represents the profit increase for a QR retailer under Q-S competition, namely, $\pi_1^{QS}/\pi_1^{SS} - 1$, which for simplicity is denoted by QS in the graph; the middle curve shows the profit increase for a QR retailer in the symmetric scenario Q-Q ($\pi_1^{QQ}/\pi_1^{SS} - 1$, denoted by QQ); and the bottom curve represents the profit increase for an SR retailer under Q-S competition ($\pi_1^{SQ}/\pi_1^{SS} - 1$, denoted by SQ).

The three curves in Figure 6 are positive and have a clear order, as expected by Equation (16). Moreover, the curves are increasing with respect to ρ, the

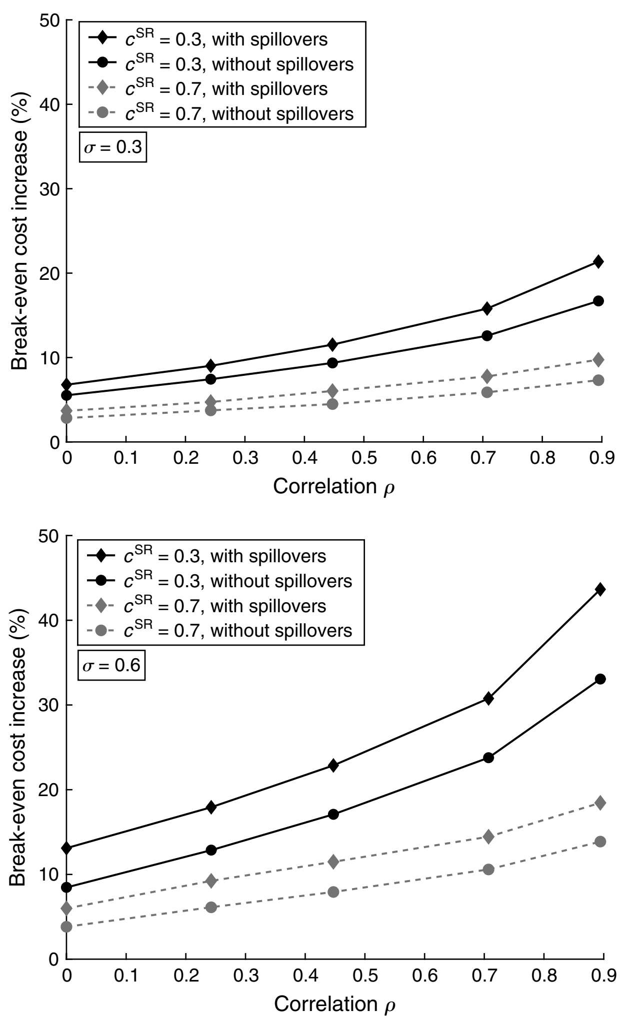
Figure 6 **Percentage Increase in the Equilibrium Expected Profits with Respect to the Base Case π_1^{SS} as a Function of ρ, with $c^{\mathrm{QR}} = c^{\mathrm{SR}} = 0.5$, $\sigma = 0.3$ (Top), and $\sigma = 0.6$ (Bottom)**



demand correlation across periods, and are higher when there is more demand variability ($\sigma = 0.6$). The fact that the QQ curve is positive and increasing confirms that, even in the absence of spillovers, QR is preferred over SR and is more profitable with higher demand correlation. This is a restatement of what is already known in the single-firm literature. What is new from our model is the SQ curve, which shows the value of the spillovers for an SR firm under asymmetric competition. Similarly, the difference between the QS and the QQ curves confirms that spillovers are also valuable to a QR firm. In the online appendix, we show through numerical examples that the order given by Equation (16) remains valid even in cases where the demand split in the first period is nonlinear. However, asymmetric competition becomes relatively less rewarding for the QR firm if, because of the nonlinear splitting rule, it faces more uncertainty than its SR counterpart. This is quite intuitive: if the SR retailer faces relatively stable demand, then the QR firm receives less spillovers, which makes asymmetric competition less attractive.

Figure 6 is based on the assumption $c^{QR} = c^{SR}$. However, as c^{QR} increases, the profit of a QR retailer decreases. We can thus compute the *break-even cost* at which a retailer would be indifferent between being a QR or SR firm when it faces an SR competitor. This cost differential, expressed as a percentage of c^{SR}, represents the threshold below which it is advantageous to be a QR firm. The threshold is depicted in Figure 7 as a function of demand correlation, for values of c^{SR} equal to 0.3 and 0.7. In the same figure, we also show the cost differential that makes a retailer indifferent between being a QR or SR firm in a setting without spillovers. Therefore, we identify the additional cost increase that a QR firm can handle under asymmetric competition.

Figure 7 **Break-Even Cost That Makes a Retailer Indifferent Between Being QR or SR, With and Without Spillovers, as a Function of ρ**

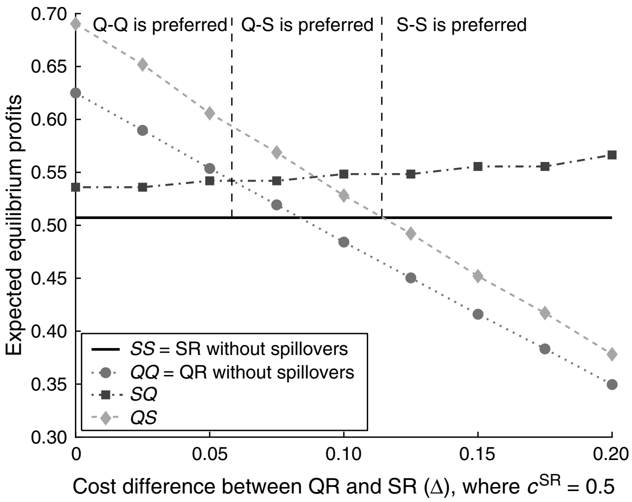


Note. Here, $\sigma = 0.3$ (top) and $\sigma = 0.6$ (bottom).

Whereas the SR retailer always gains when its competitor moves from SR to QR, this is not always true for the QR retailer: it depends on the cost increase associated with implementing QR. For fast fashion retailers, the literature estimates the cost increase to be 15%–20% compared to traditional SR firms producing in Asia (see Ghemawat and Nueno 2003, p. 11). As Figure 7 shows, this is insufficient to justify QR for small demand variability and/or small demand correlation. For instance, the break-even cost is below 15% for $\rho < 0.5$ and $\sigma = 0.3$. Thus, our model shows that for low demand variability and low correlation, a retailer would prefer less flexibility and lower cost; i.e., it would prefer being SR. This is the case for "basic" items, e.g., white T-shirts. On the other hand, for high demand variability and high correlation, a retailer is better off having a higher production cost but a faster response. This applies to "fashion" goods, as those typically found in a Zara store. In other words, in a competitive setting, our results confirm the fundamental rule that the supply chain (in particular, its costs and flexibility) should match the type of product. Functional products, such as standard garments, should have an efficient (i.e., low cost, and usually less flexible) supply chain, whereas innovative products, such as trendy items, should have a supply chain that is responsive, which typically requires excess buffer capacity, and therefore implies higher operational costs (see Fisher 1997).

Finally, in the analysis throughout this paper, we have taken the retailers' replenishment capabilities as given. We have shown that asymmetries are beneficial for both players, compared to the symmetric scenario, due to spillovers. However, if the retailers could choose whether to be QR or SR, one could think that these asymmetries might disappear. Figure 8 shows that this is not the case: *There are situations where asymmetric competition is desirable for both players, even when retailers can endogenously choose to be QR or SR*. In other words, if both retailers had

Figure 8 **Expected Equilibrium Profits as a Function of $\Delta \equiv c^{QR} - c^{SR}$, with $c^{SR} = 0.5$, $\sigma = 0.6$, and $\rho = 0.7$**



to choose simultaneously their replenishment capabilities (QR or SR), there are instances in which the cost differential makes the Q-S scenario able prefer to both players. Figure 8 depicts the expected equilibrium profits as a function of the cost differential $\Delta \equiv c^{QR} - c^{SR} > 0$, where we use *SS* as a shorthand notation for π_1^{SS} (the same for *QQ*, *SQ*, and *QS*). Note that when the *QQ* curve is above the *SQ* curve, both retailers would choose to be QR (i.e., Q-Q is preferred); this corresponds to a low cost differential Δ. When the *QQ* curve is below the *SQ* curve and the *QS* curve is above the *SS* curve, one retailer would choose to be QR and the other would choose to be SR (i.e., Q-S is preferred); this happens for intermediate values of Δ. Moreover, when the *QS* curve is below the *SS* curve, both retailers would choose to be SR (i.e., S-S is preferred); this corresponds to a high Δ.

6. Conclusions and Future Research

In this paper, we formulated a two-period inventory competition model for two retailers selling substitutable items. The model can be used to analyze the impact of (asymmetric) production costs and ordering flexibility on the competitive outcome, and specifically on retailer inventory levels and profits. This is the case when one of the firms has a lower production (ordering) cost but can only produce at the beginning of the selling season, whereas the second firm has higher costs but can replenish stock during the planning horizon, taking advantage of any additional demand information that might become available. We visualize the problem as the competition between a traditional SR retailer that makes to stock before the season starts versus a QR firm that has a flexible supply chain and can place orders more than once.

For asymmetric retailers, we provided conditions that guarantee the existence of a unique pure-strategy subgame-perfect equilibrium for the demand signal and the midseason replenishment cases. In addition, we performed an extensive numerical study to understand the impact of cost asymmetries, demand variability, and correlation across periods on the equilibrium inventory levels and the corresponding profits. We made several important observations. For instance, that a QR firm might be willing to let demand leak to reduce inventory risk. We also confirmed that all the benefits from QR identified in models without competition remain when retailers compete. However, these benefits are larger when the competitor is SR. Thus, part of the competitive advantage of QR in inventory-based competition comes from the ordering flexibility asymmetries. Furthermore, we found that an SR retailer would also prefer Q-S competition, compared to S-S. In other words, Q-S competition is preferred by both retailers, over S-S. Finally, depending on the cost differential between QR and SR, the preference for Q-S remains even when retailers can endogenously decide their ordering capabilities (QR or SR).

Several extensions of this work are possible. First, in terms of inventory competition models, an ideal extension would be to prove existence and uniqueness of equilibrium for the asymmetric case with an arbitrary number of periods, and possibly more general demand allocation rules and a larger number of retailers. However, the analysis is presumably not straightforward. Second, in terms of understanding the fast fashion phenomenon, there is still plenty to be done. In fact, in this paper we have ignored other distinctive aspects such as the endogenous effect of higher fill rates on market share, similar to Gaur and Park (2007). Incorporating these elements into our model is a challenging strand of future research.

Electronic Companion

An electronic companion to this paper is available on the *Manufacturing & Service Operations Management* website (http://msom.pubs.informs.org/ecompanion.html).

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