Online Appendix for "The Impact of Quick Response in Inventory-Based Competition"

General Remarks:

- The structure of the appendix is the following: we first present a general result which holds for an arbitrary number of periods when both firms are symmetric; then we report additional numerical computations using our model; and finally, we provide the proofs of the analytical results that were not included in the electronic companion.
- The numbering of the equations in the appendix continues the same sequence from the paper and the electronic companion.
- Throughout the proofs, any interchange of integration and differentiation in definite integrals is justified by Leibnitz' rule, which holds whenever the integrand is (almost everywhere) continuously differentiable.

A. Symmetric Retailers

When additional conditions are placed on the retailers and the demand structure, stronger results can be derived. Here we provide a general result when the retailers are symmetric, i.e., both retailers have identical costs and prices in every period. Notice that our definition of symmetry still allows the retailers to have different initial inventories and can receive unequal shares of demand. The result below is presented for only two periods, in line with the main results in the paper, but it can be extended to an arbitrary number of periods. The key assumption it requires, besides the symmetry, is that the demand splitting rule is linear and stationary. In contrast with Theorem 3, we make no assumptions on the demand distributions (see Table 1 in the paper).

Theorem 4 *Assume that costs and prices are identical for both firms, i.e.,* $c_t^i = c_t^j$ *and* $p_t^i = p_t^j$ *for* $t = 1, 2$*. In addition, assume that* $q_t^i(d) = \alpha^i d$ *for* $t = 1, 2$ *and* $i = 1, 2$*.*

Then, the two-period stocking game has a unique pure-strategy subgame-perfect equilibrium. This equilibrium is characterized by

$$e_1^i(x_1^i, x_1^j) = \max\{x_1^i, \alpha^i s_1\} \quad \text{and} \quad e_{2|I_2}^i(x_2^i, x_2^j) = \max\{x_2^i, \alpha^i s_{2|I_2}\} \tag{25}$$

where s_1 *and* $s_{2|I_2}$ *are the monopoly base-stock levels, i.e., the solution to the inventory problem for a single firm that faces* D_1 *and* $D_{2|I_2}$ *in periods 1 and 2 respectively.*

As stated in Equation (25), the equilibrium level of retailer i only depends on its own inventory level x_t^i and is independent of the competitor's inventory level x_t^j. In fact, we show that the best-response of each retailer is constant when the competitor carries inventory above a certain threshold. Moreover, we show that the constant level is greater or equal than the threshold. Hence, in equilibrium, by symmetry, both firms will play within that region. Interestingly, the portion where the best-response is constant involves the monopoly stocking quantities s_1 and $s_{2|I_2}$, defined through the following dynamic program. Letting $c_t = c_t^i = c_t^j$ and $p_t = p_t^i = p_t^j$ for $t = 1, 2$:

$$\begin{aligned} U_{2|I_2}(x) &= c_2 x + \max_{y \geq x} \left\{ -c_2 y + p_2 \mathbb{E} \min\{y, D_{2|I_2}\} \right\}, \\ U_1(x) &= c_1 x + \max_{y \geq x} \left\{ -c_1 y + p_1 \mathbb{E} \min\{y, D_1\} + \mathbb{E}\, U_{2|I_2}\left((y - D_1)^+ \right) \right\}. \end{aligned} \quad (26)$$

Since this is a standard inventory problem, it is easy to see that the optimal policy is to set the order-up-to levels equal to $\max\{x, s_1\}$ and $\max\{x, s_{2|I_2}\}$ in periods 1 and 2 respectively, where s_1 and $s_{2|I_2}$ are the base-stock levels of the respective unconstrained problem. These are the quantities used in Theorem 4. Figure 9 illustrates the shape of the best-response functions described by the theorem. This plot is for $c_t = 0.6$, $p_t = 1$, D_t uniform [0,1], for $t = 1, 2$, and $x_1^i = x_1^j = 0$ and $\alpha^i = 30\%$. We observe that, indeed, s_1^i is flat in y_1^j, when $\dfrac{y_1^j}{\alpha^j} \geq s_1$.

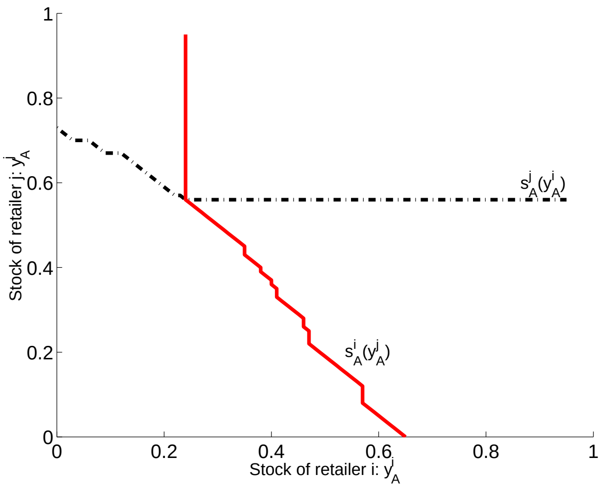

Figure 9: Shape of the best-response functions b_1^i in the case of symmetric firms.

We thus completely characterize the two-period equilibrium in the case of symmetric firms. Note that the firms are allowed to start with *any initial inventory*. This contrasts with the work by Avsar and Baykal-Gürsoy (2002), who require that the initial inventory levels of retailers are below their equilibrium stationary base-stock levels. Also note that it is still possible for spill-overs between retailers to occur, even though the equilibrium quantity of each retailer is independent of the competitor's inventory level, see Equation (25). However, if both retailers start with zero inventory, there will be no spill-overs: in other words, the outcome of the game is the same as not having inventory-based competition.

Finally, when both firms start with zero inventory at $t = 1$, the aggregate industry inventory is equal to $\alpha^i s_1 + \alpha^j s_1 = s_1$, which is also the industry inventory target level under a monopoly, i.e., when firms i and j merge. The same proof can be extended to $T \geq 2$, thus our result extends Theorem 3 of Lippman and McCardle (1997) to a multi-period setting with demand learning. Unfortunately, when costs are asymmetric, it turns out that the best response is never independent of y_t^j, which prevents the use of some of the arguments of the proof. The same sort of property is used in Nagarajan and Rajagopalan (2008) by requiring that the newsvendor ratio is larger than 0.5.

B. Supplemental Numerical Study

B.1 Best-responses in the Mid-Season Replenishment Case

The following numerical examples complement Figure 4. The purpose is to confirm that the best-response of the QR firm in the mid-season replenishment case is likely to have a dip that goes below the constant level (i.e., the level of inventory that is ordered when there are no spill-overs). The two examples described below have discrete demand, which also shows that the numerical observation goes beyond the cases that satisfy Proposition 2.[21]

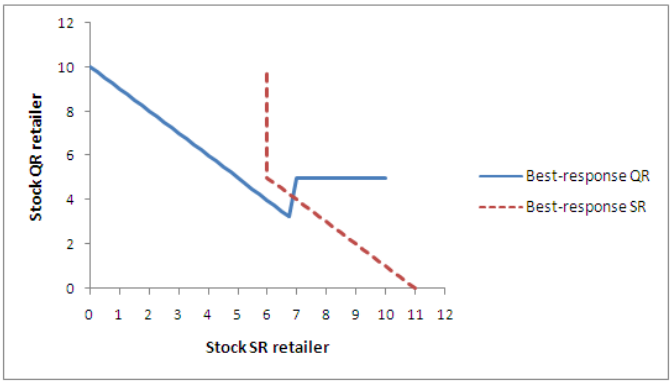

Figure 10: First period best-response functions under Q-S competition with $\rho = 0$, and $c^{\mathrm{QR}} = c^{\mathrm{SR}} = 0.4$.

We begin by considering a case in which the demand across periods is independent. In

[21]In general, with discrete demand the unconstrained expected profit r_1^i is not unimodal (quasi-concave) and the equilibrium is not necessarily unique.

fact, let the first period demand, D_1, take only the values 0 and 10 with equal probability and let the second period demand, D_2, be uniformly distributed over $[0, 10]$. We consider a fixed unit price, an ordering cost equal to 0.4, and an equitable linear split. The first period best-response functions for both firms under Q-S competition are depicted in Figure 10.

For simplicity, let i represent the QR firm. Figure 10 shows that the best-response of the QR retailer has a dip before it becomes constant and equal to 5. If fact, for $y_1^j = 6.75$, the best-response of the QR firm is to order 3.25 units in the first period (see Table 4 below).

y_1^j	y_1^i	D_1	$sales_1^i$	y_2^i	$\mathbb{E}[sales_2^i]$	Expected Values			
						Costs	Sales	Profits	
6.75	3.25	0 10	0.00 3.25	3.25 6.00	2.19 4.20	2.50	4.82	2.32	← best-response
	5.00	0 10	0.00 5.00	5.00 4.25	2.50 2.76	2.85	5.13	2.28	
7.25	3.25	0 10	0.00 3.25	3.25 5.50	2.19 3.73	2.40	4.58	2.18	
	5.00	0 10	0.00 5.00	5.00 3.75	2.50 2.46	2.75	4.98	2.23	← best-response

Table 4: Expected profits for the QR firm under Q-S competition when the competitor's stock is $y_1^j = 6.75$ or 7.25 ($c^{\text{QR}} = c^{\text{SR}} = 0.4$ and firm i orders optimally in the second period).

When the QR firms orders less in the first period, there are two effects. On the one hand, it reduces the amount of unsold inventory when the demand in the first period is low ($D_1 = 0$). This is the positive effect. On the other hand, it allows demand to leak so it reduces sales in the first period, especially when demand is high ($D_1 = 10$). This is the negative effect, but remarkably, it is diminished if the SR firm does not have too much inventory. Indeed, the demand that leaks in the first period has a side-effect: it depletes the competitor's stock and that allows the QR firm to get more spill-over back in the second period. This is shown in Table 4 for $y_1^j = 6.75$: if the QR retailer orders 3.25 it sells less in the first period, but this is mitigated by the additional spill-over it gets in the second period, and overall, the decrease in total expected costs pays off. Put differently, the spill-over in the second period allows the QR firm to reduce inventory risk in the first period. The argument fails if the SR firm has a larger amount of initial stock (e.g., $y_1^j = 7.25$) since then the QR firm does not get enough reward for letting its demand leak in the first period.

Now consider the same case as before but with perfect (positive) correlation between the demand in the first and second period. Hence, D_2 is no longer uniformly distributed but rather takes the value 0 or 10 depending on the realization of D_1. This case is depicted in Figure 11. Here the dip in the best-response function of the QR retailer is even more

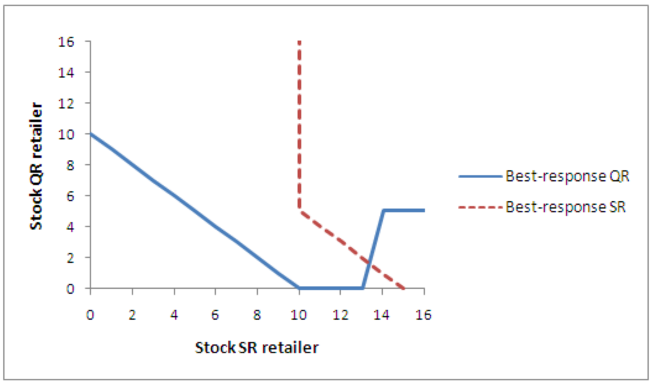

Figure 11: First period best-response functions under Q-S competition with perfect correlation ($\rho = 1$), $c^{\text{QR}} = c^{\text{SR}} = 0.4$, and D_1 equals 10 or 0.

noticeable because now it can also learn about demand at the competitor's expense. In fact, a priori there is a 50% chance that there will be no demand at all, and if the SR firm has a moderate amount of initial stock, then the QR retailer is better off waiting until uncertainty is resolved. For instance, suppose that the SR firm starts with $y_1^j = 10$. In that case, the best-response of the QR retailer is not to order in the first period since then it can sell 10 units for sure in the second period if $D_1 = 10$ and loses nothing if $D_1 = 0$. By the contrary, if the QR firm starts with $y_1^i = 5$, which is the optimal constant level when there are no spill-overs, then it also sells 10 units when demand is high, but loses 5 unsold units otherwise. Note that our observation relies on uncensored demand, see assumption (A2). Demand censoring reduces the advantage of QR when there is correlation over time.

B.2 The Demand Signal Case

In what follows, we present the same figures reported in §5 but now for the demand signal case, and we point out any differences that occur. To be consistent, we use the same demand model as in the mid-season replenishment case, namely, $D_{2|I_2} = kD_1 + \epsilon$, but D_1 represents the additional market information gained in the initial period instead of the actual demand in $t = 1$. Note that since in our simulations we assume $c_1^i = c_2^i$ for $i = 1, 2$, the equilibrium in the demand signal case might not be unique (see the discussion after Theorem 2). In those situations, we consider the equilibrium in which a QR retailer does not order anything in the first period.

Figure 12 is analogous to Figure 4 in the main paper and shows the first period best-response functions under S-S and Q-S competition. Note that Proposition 3 is again verified. The main difference with respect to the mid-season replenishment case is that the best-response of the QR retailer under Q-S competition is not to order.[22] Therefore, we do not observe the decreasing, then increasing, and finally constant, behavior described for the mid-season replenishment case (cf. §5.2).

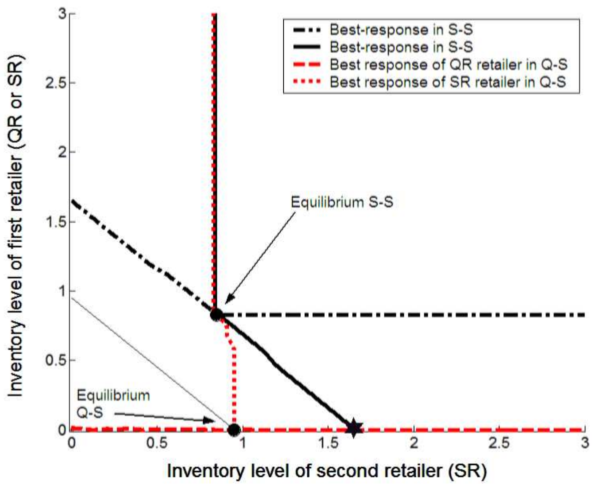

Figure 12: First period best-response functions for the demand signal case.

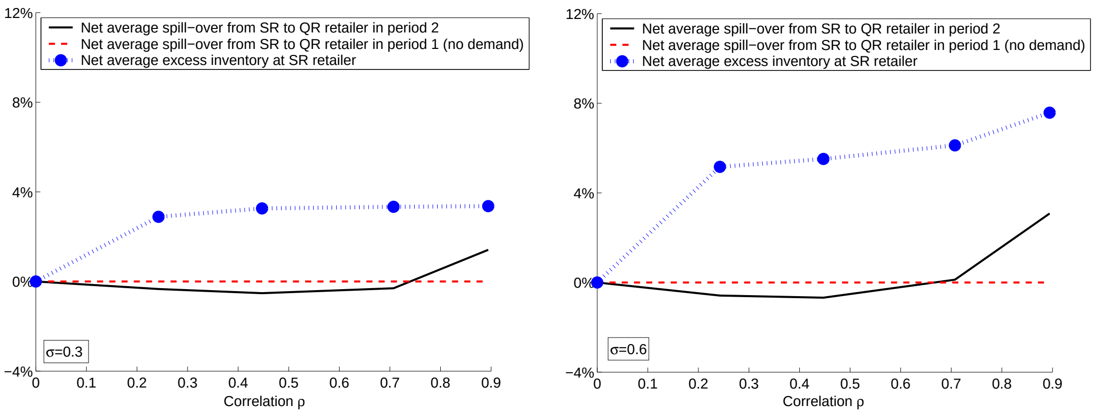

Figure 13: Net average spill-over from the SR to the QR firm and net average excess inventory at the SR firm as a function of ρ (values normalized by $\alpha\mathbb{E}\{D_1 + D_2\}$), for the demand signal case.

Figure 13 is equivalent to Figure 5 and plots the spill-overs as a function of demand correlation (ρ). In the demand signal case, spill-overs can only occur in the last period. Note that, without correlation, the net spill-over is zero. This is true because, even though retailers order in different periods, they both place the same level of inventory as in S-S competition, where no spill-overs occur. Interestingly, for lower values of demand correlation

[22]Again, when the best-response of the QR retailer is not unique, we deliberately assume that it will not place an order in the first period.

($\rho \leq 0.7$), the average spill-over, though small, goes from the QR to the SR firm. Indeed, in all the scenarios in which the signal indicates that demand will be low in the second period, the QR firm decides to order less and sometimes will face a stockout (because despite the signal, demand in the second period is still random). In those cases, the SR firm, which is oblivious to the demand signal, will have stock available to meet the spill-over demand. However, note that the QR firm is still better off since, on average, it carries less excess inventory at the end of the season.

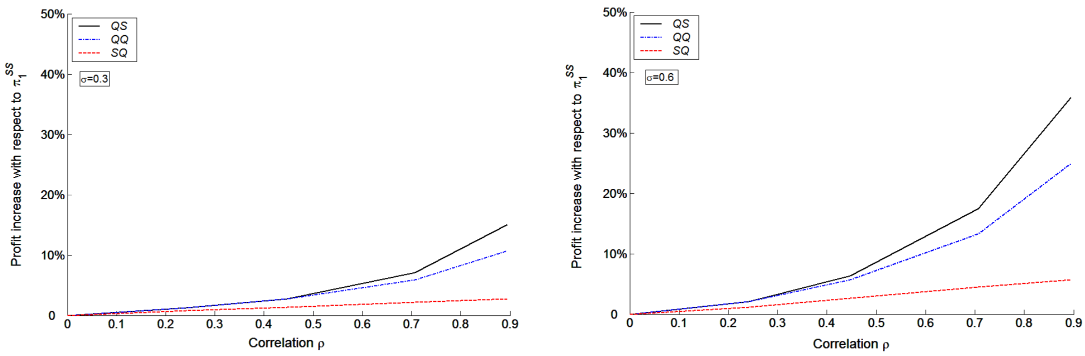

Figure 14: Percentage increase in the equilibrium expected profits with respect π_1^{SS} as a function of ρ, for the demand signal case.

Figure 14 is equivalent to Figure 6. Compared to the mid-season replenishment model, in the demand signal case, the increase with respect to the base case (S-S) is lower, especially for low levels of demand correlation. This is what should be expected since there is significantly less demand spill-over (cf. Figure 13). However, the fact that the QS curve continues to do dominate the QQ shows that, even when correlation is low and the SR firm receives most of the spill-over demand, a QR retailer still prefers an asymmetric scenario where spill-overs occur. In fact, the spill-overs allow the QR firm to reduce inventory risk (in particular, over-stock costs) which at the end brings higher profits.

Finally, Figure 15 is analogous to Figure 7. As expected, the break-even threshold is lower for the demand signal case. Similarly, in Figure 16 which is equivalent to Figure 8), the range of the cost differential (Δ) in which Q-S is preferred is smaller than in the mid-season replenishment case.

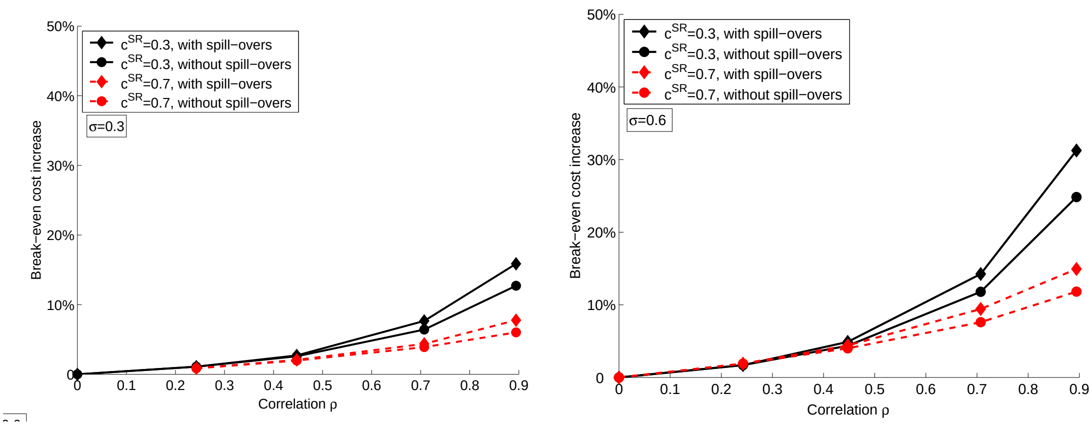

Figure 15: Break-even cost that makes a retailer indifferent between being QR or SR, with and without spill-overs, as a function of ρ, for the demand signal case.

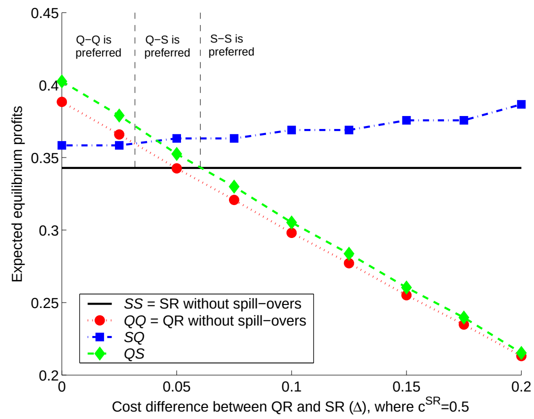

Figure 16: Expected equilibrium profits as a function of $\Delta \equiv c^{\mathrm{QR}} - c^{\mathrm{SR}}$, for the demand signal case

B.3 Non-Linear Demand Split

In all the previous numerical simulations we assumed a linear demand allocation function. However, except for Proposition 4, our theoretical results are more general since they also hold when the demand split in the first period is non-linear. In this section we provide a numerical study for the latter case. In particular, we consider the demand allocation function $\ln(1 + 5D_1)/5$ and its complement $D_1 - \ln(1 + 5D_1)/5$. Note that the former is a strictly concave function and represents the case when observable demand is a deterrent, whereas the latter is strictly convex and represents the case when demand is an attractive force (see Lippman and McCardle 1997). Therefore, the firm that is allocated the convex share will receive more customers when the demand realization is larger.

As in our previous simulations, we assume that the demand in the first period is gamma distributed with a mean equal to one, and both firms have zero initial inventory. We focus on the mid-season replenishment case with $c^{\mathrm{QR}} = c^{\mathrm{SR}} = 0.5$, and for simplicity, we consider $\sigma = 0.3$. Then, the first period expected demand is 0.35 and 0.65 for the firm that is allocated the concave and convex share respectively. Note that with a non-linear split the retailers can face different levels of demand uncertainty. Indeed, the coefficient of variation can differ significantly across firms. In our example, it is equal to 14% and 38% for the firm with the concave and convex allocation respectively. This contrasts with the linear splitting rule where both firms face the same coefficient of variation.

In Figure 17 we first show the best-response functions in the S-S and Q-S scenarios. In the left graph, firm 1 is allocated the concave share, $q_1^1(D_1) = \ln(1+5D_1)/5$, whereas in the right graph it receives the convex complement, $q_1^1(D_1) = D_1 - \ln(1+5D_1)/5$. As expected, in both cases all the observations made under the linear splitting rule (cf. Figure 4) continue to hold. In particular, the Q-S equilibrium is shifted downwards and to the right with respect to the S-S equilibrium, and under Q-S competition the best-response function of the QR firm has an increasing segment.

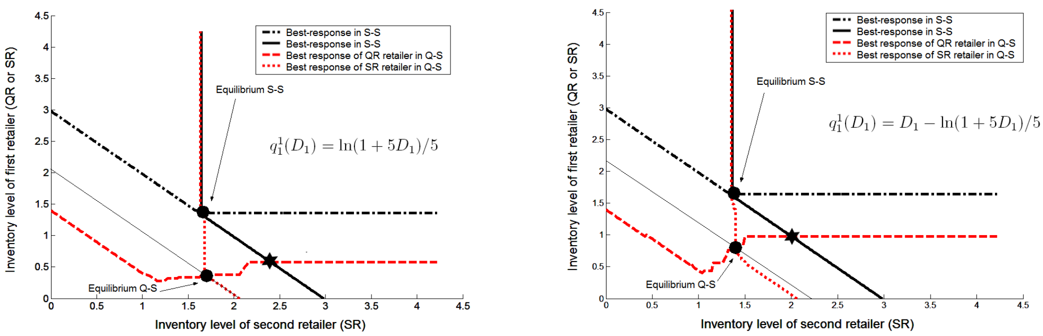

Figure 17: Best-response functions when the demand split in the first period is non-linear, with $\rho = 0.7$, $\sigma = 0.3$, $c^{\mathrm{QR}} = c^{\mathrm{SR}} = 0.5$, and $q_1^1(D_1)$ convex (left) and concave (right).

We next look at the spill-overs under Q-S competition. Figure 18 shows the net average spill-over from the SR to the QR firm and the net average excess inventory at the SR firm. In this figure, without loss of generality we assume that firm 1 is QR, and again in the left and right graphs $q_1^1(D_1)$ is concave and convex respectively. Note that we used the same normalization constant as in Figure 5, namely $\mathbb{E}[D_1 + D_2]/2$. However, given the non-linear split, this constant no longer represents the total expected demand for a single firm without inventory-based competition.

The main observations made under the linear splitting rule (cf. Figure 5) continue to hold in Figure 18. In particular, the SR and QR firm receives net average spill-over in the first and second period respectively, and on average the SR firm ends the season with more leftover stock. The only difference is that with a linear splitting rule the QR firm receives more spill-over in the second period than the SR competitor in the first period. This is still the case when $q_1^1(D_1)$ is concave, but might not hold when it is convex as shown in the right graph of Figure 18. The reason is that in the latter case the SR firm has very stable demand (recall, a coefficient of variation of 14%), whereas the QR firm is facing most of the uncertainty, and therefore there is a higher chance that the spill-over will go from the QR to the SR firm. In any case, the QR firm is still better off, as shown in the next figure.

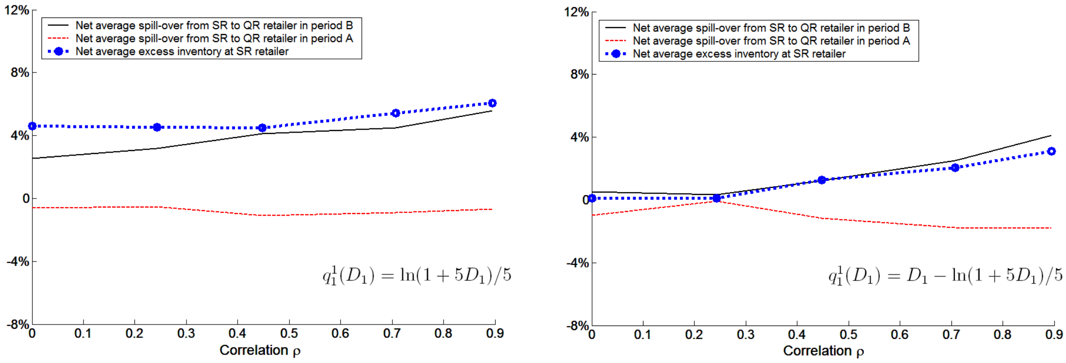

Figure 18: Net average spill-over from the SR to the QR firm and net average excess inventory at the SR firm when the demand split in the first period is non-linear, with $\sigma = 0.3$, $c^{\text{QR}} = c^{\text{SR}} = 0.5$, and $q_1^1(D_1)$ convex (left) and concave (right).

In Figure 19 we finally compare the profit increase with respect to those achieved in the S-S base scenario. We do so for firm i and as before consider the two cases when $q_1^i(D_1)$ is concave and convex. The first observation from Figure 19 is that in both graphs the order of the curves is the same as in Figure 6, which implies that the results of Proposition 4 still apply to cases with non-linear splitting rules. In particular, the graphs confirm that both retailers are better off under asymmetric Q-S competition than in the S-S scenario. Between the two firms, the QR retailer obtains a larger profit increase, independent of whether it is allocated the concave or convex demand share. The latter follows from the fact that the QS curve in one graph dominates the SQ curve in the other graph.

Interestingly, in our simulations of the Q-Q and S-S scenarios with a non-linear demand allocation, we did not observe any spill-overs, just as in symmetric case with a linear splitting rule (see §A). Therefore, the QQ curve in Figure 19 represents the benefit of QR over SR for a single firm in the absence of spill-overs. Clearly, this benefit is larger in the right graph where $q_1^i(D_1)$ is convex, which should be expected since in that case the firm faces more demand uncertainty, and that makes the QR capability more profitable. Finally, recall that the difference between the QQ and the QS curves represents the additional value due to spill-overs under asymmetric Q-S competition. The difference is significant in the left graph, but is very small in the right graph. In other words, when $q_1^i(D_1)$ is convex the main benefit comes from the QR capability alone and the spill-overs that take place under asymmetric competition provide little additional value.

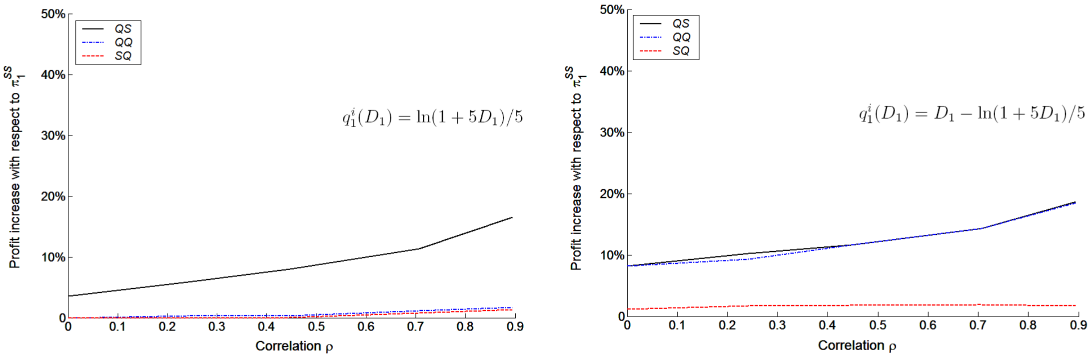

Figure 19: Percentage increase in the equilibrium expected profits with respect to π_1^{SS} as a function of ρ, with $c^{\text{QR}} = c^{\text{SR}} = 0.5$, $\sigma = 0.3$, and $q_1^i(D_1)$ convex (left) and concave (right).

C. Remaining Proofs

C.1 Proof of Theorem 4

Proof. While this proof focuses on the two-period problem, it is possible to extend it to any number of periods, by recursion. We first define

$$U_{2|I_2}(x) = c_2 x + \max_{y \geq x} \left\{ -c_2 y + p_2 \mathbb{E} \min\{y, D_{2|I_2}\} \right\},$$
$$U_1(x) = c_1 x + \max_{y \geq x} \left\{ -c_1 y + p_1 \mathbb{E} \min\{y, D_1\} + \mathbb{E}\, U_{2|I_2}\left(\left(y - D_1\right)^+ \right) \right\}.$$

These are standard inventory problems. For any demand distribution, for any correlation structure, $U_{2|I_2}$ and U_1 is concave. It follows that the optimal policy in each period is a base-stock policy with levels $s_{2|I_2}$ and s_1 respectively. where

$$c_2 = p_2 \mathbb{P}\{s_{2|I_2} \leq D_2\} \text{ and } c_1 = \mathbb{E}\left\{ p_1 1_{s_1 \leq D_1} + \frac{dU_{2|I_2}}{dx_2}(s_1 - D_1) 1_{s_1 \geq D_1} \right\}. \tag{27}$$

To show the statement for $t = 2$, we apply Theorem 1:

$$e^i_{2|I_2} = \max\left\{ x^i_{2|I_2}, \alpha^i \overline{F}^{-1}_{2|I_2}\left(\frac{c_2}{p_2}\right) \right\} = \max\left\{ x^i_{2|I_2}, \alpha^i s_{2|I_2} \right\}.$$

As a result, when $\frac{x_2^i}{\alpha^i} \leq \frac{x_2^j}{\alpha^j}$, then $\frac{e_{2|I_2}^i}{\alpha^i} \leq \frac{e_{2|I_2}^j}{\alpha^j}$. Hence, there is no spill-over from j to i in equilibrium, and thus

$$\pi_{2|I_2}^i\left(x_2^i, x_2^j\right) = \alpha^i U_{2|I_2}\left(\frac{x_2^i}{\alpha^i}\right). \tag{28}$$

At $t = 1$, we can thus show that, for any y_1^j, $r_1^i(\cdot, y_1^j)$ is concave and $\frac{\partial^2 r_1^i}{(\partial y_1^i)^2} \leq \frac{\partial^2 r_1^i}{\partial y_1^i \partial y_1^j} \leq 0$. Indeed,

$$\frac{\partial r_1^i}{\partial y_1^i} = -c_1 + p_1 \mathbb{P}\{y_1^i \leq R_1^i(y_1^j)\} + \mathbb{E}\frac{\partial}{\partial y_1^i}\pi_{2|I_2}^i\left(\left(y_1^i - R_1^i(y_1^j)\right)^+, \left(y_1^j - R_1^j(y_1^i)\right)^+\right).$$

Since

$$\begin{aligned}
&\pi_{2|I_2}^i\left(\left(y_1^i - R_1^i(y_1^j)\right)^+, \left(y_1^j - R_1^j(y_1^i)\right)^+\right) \\
&= \begin{cases} \text{either} & \pi_{2|I_2}^i(y_1^i + y_1^j - D_1, 0) \\ \text{or} & \pi_{2|I_2}^i(0, y_1^i + y_1^j - D_1) = \alpha^i U_{2|I_2}(0) \\ \text{or} & \pi_{2|I_2}^i(y_1^i - \alpha^i D_1, y_1^j - \alpha^j D_1) \\ \text{or} & \pi_{2|I_2}^i(0,0) = \alpha^i U_{2|I_2}(0) \end{cases}
\end{aligned}$$

and using Claim 1, we have that $\frac{\partial^2 r_1^i}{(\partial y_1^i)^2} \leq \frac{\partial^2 r_1^i}{\partial y_1^i \partial y_1^j} \leq 0$.

This implies that the optimal policy is a base-stock policy with base-stock level $s_1^i(y_1^j)$, and from the implicit function theorem, $s_1^i(y_1^j)$ is non-increasing. In addition, if $\frac{y_1^i}{\alpha^i} \leq \frac{y_1^j}{\alpha^j}$, then (i) there is no spill-over from j to i in period 1, and hence $\min\{y_1^i, R_1^i(y_1^j)\} = \min\{y_1^i, \alpha^i D_1\}$; and (ii), since $\frac{\left(y_1^i - R_1^i(y_1^j)\right)^+}{\alpha^i} \leq \frac{\left(y_1^j - R_1^j(y_1^i)\right)^+}{\alpha^j}$, there is no spill-over from j to i in period 2, and, from Equation (28),

$$\pi_{2|I_2}^i\left(\left(y_1^i - R_1^i(y_1^j)\right)^+, \left(y_1^j - R_1^j(y_1^i)\right)^+\right) = \alpha^i U_{2|I_2}\left(\frac{(y_1^i - \alpha^i D_1)^+}{\alpha^i}\right).$$

This is true because the splitting ratio α^i is stationary over time. Thus, one can observe that, when $\frac{s_1^i\left(y_1^j\right)}{\alpha^i} \leq \frac{y_1^j}{\alpha^j}$, the optimality equation is independent of y_1^j and corresponds to Equation (27): $s_1^i\left(y_1^j\right) = \alpha^i s_1$. With such best-response functions s_1^i, we have existence and uniqueness of equilibrium and $e_1^i(x_1^i, x_1^j) = \max\{x_1^i, \alpha^i s_1\}$. This completes the proof. ■

C.2 Proof of Proposition 3

Proof. We start by proving Equations (13) and (14). Consider the decision at $t = 1$ of a slow retailer: maximizing, under $y_1^i \geq x_1^i$,

$$r_1^i(y_1^i, y_1^j) = -c_1^i y_1^i + p_1^i \mathbb{E}\left\{y_1^i, R_1^i(y_1^j)\right\} + \mathbb{E}\left\{\pi_{2|I_2}^i\left(\left(y_1^i - R_1^i(y_1^j)\right)^+, \left(y_1^j - R_1^j(y_1^i)\right)^+\right)\right\}.$$

Since we are focusing on a slow retailer, with no replenishment capabilities in the final period, $\pi^i_{2|I_2}$ above is non-zero only when $x^i_2 = \left(y^i_1 - R^i_1(y^j_1)\right)^+ > 0$. As a result,

$$\begin{aligned}\frac{\partial r^i_1}{\partial y^i_1} &= -c^i_1 + p^i_1 \mathbb{P}\left\{y^i_1 \le R^i_1(y^j_1)\right\} \\ &\quad + \mathbb{E}\left\{\frac{\partial \pi^i_{2|I_2}}{\partial x^i_2}\left(\left(y^i_1 - R^i_1(y^j_1)\right)^+, \left(y^j_1 - R^j_1(y^i_1)\right)^+\right) 1_{y^i_1 - R^i_1(y^j_1) > 0}\right\} \\ &\quad - \mathbb{E}\left\{\frac{\partial \pi^i_{2|I_2}}{\partial x^j_2}\left(\left(y^i_1 - R^i_1(y^j_1)\right)^+, \left(y^j_1 - R^j_1(y^i_1)\right)^+\right) \frac{\partial R^j_1}{\partial y^i_1} 1_{y^j_1 - R^j_1(y^i_1) > 0}\right\} \\ &= -c^i_1 + p^i_1 \mathbb{P}\left\{y^i_1 \le R^i_1(y^j_1)\right\} + \mathbb{E}\left\{\frac{\partial \pi^i_{2|I_2}}{\partial x^i_2}\left(y^i_1 - R^i_1(y^j_1), \left(y^j_1 - R^j_1(y^i_1)\right)^+\right) 1_{y^i_1 - R^i_1(y^j_1) > 0}\right\}.\end{aligned}$$

In the expression above, the term with $\frac{\partial \pi^i_{2|I_2}}{\partial x^j_2}$ is zero when $\frac{\partial R^j_1}{\partial y^i_1} > 0$ (since in that case, $x^i_2 = 0$). Using Theorem 1, $\frac{\partial \pi^i_{2|I_2}}{\partial x^i_2} = p^i_2 \mathbb{P}\Big\{x^i_2 \le R^i_{2|I_2}(\max\{x^j_2, s^j_{2|I_2}(x^i_2)\})\Big\}$, which is non-decreasing in c^j_2, because $\frac{ds^j_{2|I_2}}{dc^j_2} \le 0$. As a result, $\frac{\partial r^i_1}{\partial y^i_1}$ is non-decreasing in c^j_2. Thus, a slow retailer places more inventory against a slow retailer than against a quick one: $s^{i,SS}_1(y^j_1) \ge s^{i,SQ}_1(y^j_1)$.

Consider now the decision of a quick or slow retailer, against a slow one, who cannot place any order in the final period. We have that

$$\begin{aligned}\frac{\partial r^i_1}{\partial y^i_1} &= -c^i_1 + p^i_1 \mathbb{P}\left\{y^i_1 \le R^i_1(y^j_1)\right\} \\ &\quad + \mathbb{E}\left\{\frac{\partial \pi^i_{2|I_2}}{\partial x^i_2}\left(\left(y^i_1 - R^i_1(y^j_1)\right)^+, \left(y^j_1 - R^j_1(y^i_1)\right)^+\right) 1_{y^i_1 - R^i_1(y^j_1) > 0}\right\} \\ &\quad - \mathbb{E}\left\{\frac{\partial \pi^i_{2|I_2}}{\partial x^j_2}\left(\left(y^i_1 - R^i_1(y^j_1)\right)^+, \left(y^j_1 - R^j_1(y^i_1)\right)^+\right) \frac{\partial R^j_1}{\partial y^i_1} 1_{y^j_1 - R^j_1(y^i_1) > 0}\right\}.\end{aligned}$$

Similarly as before, from Theorem 1 we can show that $\frac{\partial \pi^i_{2|I_2}}{\partial x^i_2}$ is non-decreasing in c^i_2, and $-\frac{\partial \pi^i_{2|I_2}}{\partial x^j_2}$ is non-negative, non-decreasing in c^i_2. Hence, $\frac{\partial r^i_1}{\partial y^i_1}$ is non-decreasing in c^i_2, which implies that a slow retailer will carry more inventory initially than a quick retailer, given any inventory at the (slow) competitor: $s^{i,SS}_1(y^j_1) \ge s^{i,QS}_1(y^j_1)$.

We can now prove Equation (15). First, we use the following property of s^i_1.

- In the demand signal scenario, $-1 \le \frac{ds^i_1}{dy^j_1} \le 0$ (see Proof of Theorem 2).

- In the mid-season replenishment scenario, $\frac{ds_1^i}{dy_1^j} \geq -1$ and when $(q_1^i)^{-1}(s_1^i(y_1^j)) \geq (q_1^j)^{-1}(y_1^j)$, $\frac{ds_1^i}{dy_1^j} \leq 0$ (see Proof of Theorem 3).

We claim that the total inventory is higher in the S-S competition compared to the Q-S competition. Starting from the S-S equilibrium, we first shift the best-response curve of i downward, and then j's best-response downward, and reach the Q-S equilibrium. In the first shift, the intersection of the two curves moves towards a smaller y_1^i (as i's curve is shifted downward), and, since the best-response of j has slope higher than -1, the equilibrium y_1^j cannot increase more than the decrease of y_1^i. Hence, in the first shift, the total inventory cannot increase. In the second shift, the same change occurs. Figure 20 illustrates this conclusion. This completes the proof. ■

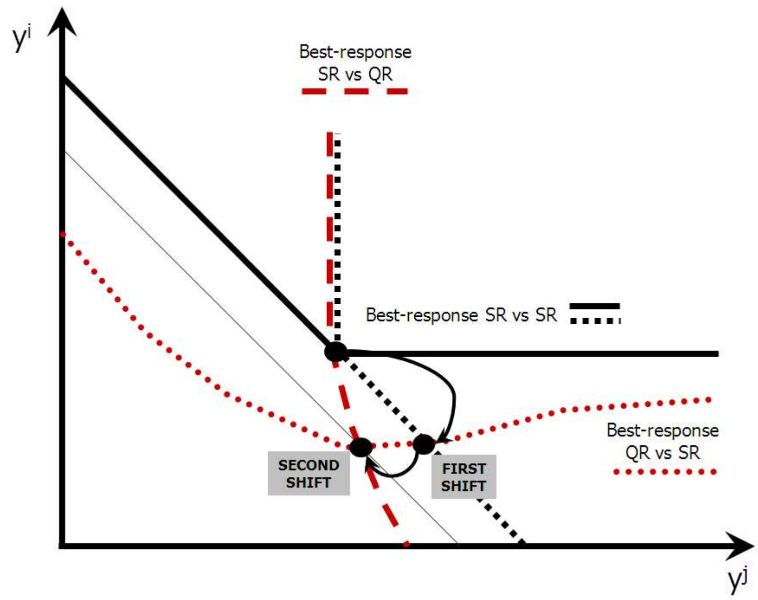

Figure 20: Shape of the best-response functions when one of the retailers (i) is fast or slow, and the other (j) is slow.

C.3 Proof of Proposition 4

Proof. When the two retailers are symmetric and the demand splitting linear, Theorem 4 shows that, in equilibrium, both retailers will place inventory as if there was no spill-over between them. As a result, both in the S-S and Q-Q scenarios, the retailers' profit can be calculated by solving a model without competition. In particular, in the S-S scenario, the retailer's two-period problem is identical to a single-period problem with demand equal to $D = D_1 + D_2$. Letting $c = c^{\text{SR}} = c^{\text{QR}}$, the profit of retailer i can thus be written as

$$\pi_1^{i,SS} = \max_{y^i} \left\{ -cy^i + p\mathbb{E} \min \left[y^i, \alpha^i(D_1 + D_2) \right] \right\}.$$

Similarly, in the Q-Q scenario, the profit of retailer i can be written as

$$\pi_1^{i,QQ} = \max_{y_1^i} \left\{ -cy_1^i + p\mathbb{E} \min \left[y_1^i, \alpha^i D_1 \right] + \mathbb{E}U_{2|I_2}^i \Big((y_1^i - \alpha^i D_1)^+ \Big) \right\}$$

where $U_{2|I_2}^i(y) = \max_{y_2^i \geq y} \left\{ -c(y_2^i - y) + p\mathbb{E} \min \left[y_2^i, \alpha^i D_{2|I_2} \right] \right\}$.

When a slow retailer competes against a quick retailer, with the same cost, given any initial order of the QR retailer, there might be a non-negative spill-over from j to i in periods 1 and/or 2. Hence, the profit of i is

$$\pi_1^{i,SQ} = \max_{y^i} \left\{ -cy^i + p\mathbb{E} \min \left[y^i, \alpha^i(D_1 + D_2) + SPILLOVER^{j \to i} \right] \right\} \geq \pi_1^{i,SS}.$$

The inequality is in fact strict when $\left(q_1^j\right)^{-1}(y_1^j) < \left(q_1^i\right)^{-1}(y_1^i)$, as there is a non-zero probability of positive spill-over in this case.

Similarly, when a quick retailer competes against a slow retailer, with the same cost, given any initial order y^j of the slow retailer, the profit of i is

$$\pi_1^{i,QS} = \max_{y_1^i} \left\{ \begin{array}{c} -cy_1^i + p\mathbb{E} \min \left[y^i, \alpha^i D_1 + SPILLOVER_1^{j \to i} \right] \\ +\mathbb{E}\hat{U}_{2|I_2}^i \Big((y_1^i - \alpha^i D_1 - SPILLOVER_1^{j \to i})^+ \Big) \end{array} \right\},$$

where $\tilde{U}_{2|I_2}^i(y) = \max_{y_2^i \geq y} \left\{ -c(y_2^i - y) + p\mathbb{E} \min \left[y_2^i, \alpha^i D_{2|I_2} \right] + SPILLOVER_2^{j \to i} \right\} \geq U_{2|I_2}^i(y)$. Clearly, $\pi_1^{i,QS} \geq \pi_1^{i,QQ}$.

Finally, we claim that $\pi_1^{i,QQ} \geq \pi_1^{i,SQ}$. The argument relies on combining two facts. First,

$$e_1^{j,QQ} \leq e_1^{j,SQ}. \tag{29}$$

This is true because $e_1^{j,SQ} \geq e_1^{j,SS}$ (since the slow retailer in Q-S competition receives spill-over and in the equilibrium in S-S both retailers act as if there was no spill-over), and $e_1^{j,SS} \geq e_1^{QQ}$ (since the equilibrium values can be computed in a model without competition, and thus a QR retailer carries lower inventory than a SR retailer with the same cost). Second, for any (y_1^i, y_1^j),

$$r_1^{i,QQ}(y_1^i, y_1^j) \geq r_1^{i,SQ}(y_1^i, y_1^j), \tag{30}$$

which is directly implied by $\pi^{i,QQ}_{2|I_2}(x^i_2, x^j_2) \geq \pi^{i,SQ}_{2|I_2}(x^i_2, x^j_2)$. With these two results, we have

$$\begin{aligned}
\pi^{i,QQ}_1 &= r^{i,QQ}_1(e^{i,QQ}_1, e^{j,QQ}_1) \text{ by definition, since there is no initial inventory} \\
&= r^{i,QQ}_1(e^{i,QQ}_1, x) \text{ for any } x \geq e^{j,QQ}_1 \text{ because, in this Q-Q equilibrium, } i \text{ receives no spill-over} \\
&= r^{i,QQ}_1(e^{i,QQ}_1, e^{j,SQ}_1) \text{ because } e^{j,SQ}_1 \geq e^{j,QQ}_1 \text{ from (29)} \\
&\geq r^{i,QQ}_1(e^{i,SQ}_1, e^{j,SQ}_1) \text{ because for any } x \geq e^{j,QQ}_1, r^{i,QQ}_1(e^{i,QQ}_1, x) = \max_y\{r^{i,QQ}_1(y, x)\} \\
&\geq r^{i,SQ}_1(e^{i,SQ}_1, e^{j,SQ}_1) \text{ because } r^{i,QQ}_1 \geq r^{i,SQ}_1 \text{ from (30)} \\
&= \pi^{i,SQ}_1.
\end{aligned}$$

This completes the proof. ■

D. Claims

Claim 1 *For all i and I_2, $\dfrac{\partial^2 \pi^i_{2|I_2}}{(\partial x^i_2)^2} \leq \dfrac{\partial^2 \pi^i_{2|I_2}}{\partial x^i_2 \partial x^j_2} \leq 0$ almost everywhere.*

Proof. Note that these quantities are well-defined almost everywhere because, as seen in the proof of Proposition 1, $\dfrac{\partial \pi^i_{2|I_2}}{\partial x^i_2}$ is continuous, and the points of non-differentiability are at the borders of regions (I)-(IV) only. The first inequality is easily proved using that $\dfrac{\partial^2 r_{2|I_2}}{(\partial y^i_2)^2} \leq \dfrac{\partial^2 r_{2|I_2}}{\partial y^i_2 \partial y^j_2}$. The second inequality comes from the fact that, in each region, the cross-derivative is either zero, in regions (I)-(III), or non-positive in region (IV). ■

Claim 2 *Letting*

$$\phi_2(x^i_2) = \max\left\{0, \frac{f'_2}{f_2}\left(x^i_2\right)\right\}, \tag{31}$$

we have that $\dfrac{\partial^2 \pi^i_2}{(\partial x^i_2)^2} \leq -\phi_2((q^i_2)^{-1}(x^i_2))\left(c^i_2 - \dfrac{\partial \pi^i_2}{\partial x^i_2}\right)$.

Proof. We prove the claim for each one of the four regions in Figure 1. We use the notation of Equation (17) and let $\kappa^i_2(y^i_2) = (q^i_2)^{-1}(y^i_2)$ and $\kappa^j_2(y^j_2) = (q^j_2)^{-1}(y^j_2)$.

(I) $\dfrac{\partial^2 \pi^i_2}{(\partial x^i_2)^2} = 0 \cdot \left(c^i_2 - \dfrac{\partial \pi^i_2}{\partial x^i_2}\right)$.

(II) $\dfrac{\partial^2 \pi^i_2}{(\partial x^i_2)^2} = 0 \cdot \left(c^i_2 - \dfrac{\partial \pi^i_2}{\partial x^i_2}\right)$.

(IV) We leave case (III) for the end. Here, using Equation (17), we have

$$\beta^i(y^i_2, y^j_2) = 1_{\kappa^i_2 \geq \kappa^j_2} \int_{y^i_2 + y^j_2}^{\infty} f_2(u)du + 1_{\kappa^i_2 < \kappa^j_2} \int_{\kappa^i_2}^{\infty} f_2(u)du$$

and thus, noting that $\dfrac{d\kappa_2^i}{dy_2^i} = \dfrac{1}{\left(q_2^i\right)'(\kappa_2^i)}$,

$$\frac{\partial \beta^i}{\partial y_2^i} = -1_{\kappa_2^i \geq \kappa_2^j} f_2(y_2^i + y_2^j) - 1_{\kappa_2^i < \kappa_2^j} \frac{f_2\left(\kappa_2^i\right)}{\left(q_2^i\right)'(\kappa_2^i)}.$$

Since f_2 is log-concave, i.e., $\log(f_2)$ is concave, F_2 is also log-concave, and for all v in the support of f_2,

$$\frac{f_2(v)}{F_2(v)} \geq \max\left\{0, \frac{f_2'(v)}{f_2(v)}\right\},$$

see Bagnoli and Bergstrom (2005) or Martínez-de-Albéniz (2004).

Thus, we have, using that $(q_2^i)' \leq 1$ (because $(q_2^j)' = 1 - (q_2^i)' \geq 0$),

$$\begin{aligned}
-\frac{\partial \beta^i}{\partial y^i} &\geq 1_{\kappa_2^i \geq \kappa_2^j} f_2(y_2^i + y_2^j) + 1_{\kappa_2^i < \kappa_2^j} f_2\left(\kappa_2^i\right) \\
&\geq 1_{\kappa_2^i \geq \kappa_2^j} \max\left\{0, \frac{f_2'}{f_2}(y_2^i + y_2^j)\right\} \int_0^{y_2^i + y_2^j} f_2(u)du \\
&\quad +1_{\kappa_2^i < \kappa_2^j} \max\left\{0, \frac{f_2'}{f_2}(\kappa_2^i)\right\} \int_0^{\kappa_2^i} f_2(u)du \\
&= \max\left\{0, \frac{f_2'}{f_2}\left(y_2^i + y_2^j\right), \frac{f_2'}{f_2}\left(\kappa_2^i\right)\right\} \\
&\quad \times \left(\begin{array}{l} 1_{\kappa_2^i \geq \kappa_2^j}\left(1 - \displaystyle\int_{y_2^i + y_2^j}^{\infty} f_2(u)du\right) \\ +1_{\kappa_2^i < \kappa_2^j}\left(1 - \displaystyle\int_{\kappa_2^i}^{\infty} f_2(u)du\right) \end{array} \right) \\
&\quad \text{(where to obtain the max, we used that } \frac{f_2'}{f_2} \text{ is non-increasing)} \\
&\geq \max\left\{0, \frac{f_2'}{f_2}\left((q_2^i)^{-1}(y_2^i)\right)\right\} \left(\frac{c_2^i}{p_2^i} - \beta^i\right).
\end{aligned}$$

This can be rewritten as

$$\frac{\partial^2 \pi_2^i}{(\partial x_2^i)^2} \leq -\max\left\{0, \frac{f_2'}{f_2}\left((q_2^i)^{-1}(x_2^i)\right)\right\} \left(c_2^i - \frac{\partial \pi_2^i}{\partial x_2^i}\right).$$

(III) We have

$$\frac{1}{p_2^i} \frac{\partial^2 \pi_2^i}{(\partial x_2^i)^2} = \frac{\partial \beta^i}{\partial y^i} + \frac{\partial \beta^i}{\partial y^j} \frac{ds_2^j}{dy_2^i} = \frac{\partial \beta^i}{\partial y^i}.$$

This is true because, when $\dfrac{ds_2^j}{dy_2^i} \neq 0$, then $\kappa_2^i(y_2^i) \leq \kappa_2^j(s_2^j)$. This implies that $\beta^i(y_2^i, s_2^j) = \displaystyle\int_{\kappa_2^i}^{\infty} f_2(u)du$, and hence $\dfrac{\partial \beta^i}{\partial y^j} = 0$ at (y_2^i, s_2^j). Similarly to case (IV),

$$\frac{\partial^2 \pi_2^i}{(\partial x_2^i)^2} \leq -\max\left\{0, \frac{f_2'}{f_2}\left((q_2^i)^{-1}(x_2^i)\right)\right\} \left(c_2^i - \frac{\partial \pi_2^i}{\partial x_2^i}\right).$$

This completes the proof of the bound. ∎

Claim 3 $\frac{\partial \pi_2^i}{\partial x_2^i} \le c_2^i$; *for all* x^j *such that* $\overline{F}_2\left(\left(q_2^j\right)^{-1}(x_2^j)\right) < 1$, $\frac{\partial \pi_2^i}{\partial x_2^j}(0, x_2^j) > -(p_2^i - c_2^i)$.

Proof. The first part of the claim, $\frac{\partial \pi_2^i}{\partial x_2^i} \le c_2^i$, is a direct consequence of concavity with respect to x_2^i and $\frac{\partial \pi_2^i}{\partial x_2^i}(0, x_2^j) = c_2^i$ (from Proposition 1).

For the second part, we notice that $(0, x_2^j)$ belongs to regions (I) or (II). In region (I), $\frac{\partial \pi_2^i}{\partial x_2^j}(0, x_2^j) = 0$. In region (II), $\frac{\partial \pi_2^i}{\partial x_2^j}(0, x_2^j) = -\beta^j\left(s_2^i(x_2^j), x_2^j\right)$. In this region, x_2^j must be greater than $s_2^j(s_2^i(x_2^j))$. Since s_2^i has slope -1 here, s_2^j must be constant (at the intersection at most one of the best-response curves has slope -1). In other words, $x_2^j \ge q_2^j\left(\overline{F}_2^{-1}\left(\frac{c_2^j}{p_2^j}\right)\right)$.

Notice, from its definition in (17), that $\beta^j(s_2^i(x_2^j), x_2^j)$ is not zero if and only if $\kappa_2^j(x_2^j) < \kappa_2^i\left(s_2^i(x_2^j)\right)$. This is true if and only if the best-response $s_2^i(x_2^j) = \overline{F}_2^{-1}\left(\frac{c_2^i}{p_2^i}\right) - x_2^j$, from Theorem 1. As a result, when $\frac{\partial \pi_2^i}{\partial x_2^j}(0, x_2^j) \neq 0$,

$$
\begin{aligned}
\frac{1}{p_i}\frac{\partial \pi_2^i}{\partial x_2^j}(0, x_2^j) &= -\beta^j\left(\overline{F}_2^{-1}\left(\frac{c_2^i}{p_2^i}\right) - x_2^j, x_2^j\right) \\
&= -1_{\kappa_2^j(x_2^j) \le \overline{F}_2^{-1}\left(\frac{c_2^i}{p_2^i}\right)}\left(\int_{\kappa_2^j}^{\overline{F}_2^{-1}\left(\frac{c_2^i}{p_2^i}\right)} f_2(u)du\right) \\
&= -1_{\overline{F}_2(\kappa_2^j) \ge \frac{c_2^i}{p_2^i}}\left(\overline{F}_2\left(\kappa_2^j\right) - \frac{c_2^i}{p_2^i}\right).
\end{aligned}
$$

Hence, taking into account regions (I) and (II) in the formulation, we can write

$$
\frac{1}{p_i}\frac{\partial \pi_2^i}{\partial x_2^j}(0, x_2^j) = -1_{\frac{c_2^i}{p_2^i} \le \overline{F}_2((q_2^j)^{-1}(x_2^j)) \le \frac{c_2^j}{p_2^j}}\left(\overline{F}_2\left((q_2^j)^{-1}(x_2^j)\right) - \frac{c_2^i}{p_2^i}\right), \tag{32}
$$

which completes the second part of the claim. ∎

Additional References (not cited in the main paper)

Bagnoli, M. and T. Bergstrom. 2005. "Log-Concave Probability and Its Applications." *Economic Theory*, **26**, pp. 445469.

Martínez-de-Albéniz, V. 2004. "Portfolio Strategies in Supply Contracts." Ph.D. dissertation, OR Center, MIT.