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Product and Price Competition with Satiation Effects

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Consumers become satiated with a product when purchasing too much too quickly. How much is too much and how quickly is too quickly depends on the characteristics of the product relative to the time interval between consumption periods. Knowing that, consumers allocate their budget to products that generate less satiation effects. Retailers should then choose to sell products that induce minimal satiation, but usually this is operationally more costly. To study this trade-off, we provide an analytical model based on utility theory that relates customer consumption to price and satiation, in the context of multiple competing retailers. We determine the purchasing pattern over time and provide an explicit expression to determine the consumption level in steady state. We derive market shares and show that they take the form of an attraction model in which the attractiveness depends on price and product satiation. We use this to analyze the competition between firms that maximize long-term average profit. We characterize the equilibrium under three scenarios: (i) price-only competition, (ii) product-only competition, and (iii) price and product competition. The results reveal the interplay between a key marketing lever (price) and the firm's ability to offer products that generate less satiation. In particular, we show that when a firm becomes more efficient at reducing satiation, its competitor may benefit if competition is on product only, but not if it is on price and product. We also find that when satiation effects are not managed, a firm's profit may be significantly reduced while a strategic competitor can largely benefit.

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1. Introduction

For many products, consumers tend to make different purchasing decisions over time. For example, most people would usually avoid eating the exact same meal in the exact same restaurant every day. This observed pattern has been called *variety-seeking* behavior, and it is associated with the utility derived from the change itself (McAlister 1982, Kahn et al. 1986). There have been many attempts in the literature to explain this behavior through analytical decision models. In particular, recent research (Baucells and Sarin 2007) has proposed that satiation effects because of past consumption of a good might reduce the utility derived from new consumption. Satiation will thus cause a consumer with a given budget to avoid consuming too much too quickly of any given good. This results in a "diversification" of spending, which provides an aggregate, continuous version of variety-seeking consumption.

Clearly, the propensity to seek variety to reduce satiation depends on consumers' intrinsic preferences, but it also depends on the characteristics of the product and the time interval between consumption periods. For example, basic consumption goods can be consumed on a frequent (e.g., daily) basis and they do not usually generate much satiation. However, there are other types of products for which satiation effects are significant, such as particular food items or fashionable clothes. In industries where satiation effects are important, firms must realize that the product offering, understood in a generic sense that includes product design, store customer experience, etc., has an influence on customers' choice over time. Hence, selecting an appropriate product with a low satiation effect can allow retailers to improve the value proposition of their business. In other words, by incorporating the satiation effect into product decisions, firms can generate higher sales.

Depending on the industry, the specific way by which low satiation can be achieved will be different. For example, a restaurant can reduce the satiation of its products not only by increasing the number of dishes on the menu but also by changing the menu itself more often (Bernstein et al. 2008). In apparel retailing, part of the success of fast fashion firms such as Zara and H&M relative to the incumbents such as the Gap can be attributed to more frequent assortment rotation, which generates the feeling of novelty among consumers (Keeley and Clark 2008). Specifically, these firms introduce new products on a regular basis, e.g., biweekly in the case of Zara, which reduces the satiation perceived by their customers. As a result, Zara receives more visits to its stores than the competition: 17 visits per year per consumer on average, versus 3.5 in the industry (see Ghemawat and Nueno 2003). In general, the levers by which a firm can reduce satiation effects include variety (assortment breadth) and novelty (assortment rotation) among others. More importantly, product strategies that satiate less are operationally more costly (e.g., changing part of the menu on a weekly basis, introducing new products in stores frequently), and strategies that are easier to implement typically satiate more (e.g., a low-cost fixed menu, an apparel store with stable product offering).

Understanding how companies should manage the satiation generated by their products is precisely the objective of this paper. This needs to be closely coordinated with pricing decisions in the same way that the coordination of price and variety can shape the overall attractiveness of a store or brand (Hoch et al. 1999, Cachon and Kök 2007). On the one hand, it is possible that by choosing products that satiate less, a retailer would be able to increase both its margin and the total revenue by charging a higher price and still attracting a larger market share. On the other hand, offering less satiating products will also increase costs. Hence, we consider price and product decisions simultaneously, and we specifically integrate marketing and operational aspects to acknowledge that product decisions reside at the interface of both functional areas, and through product choice operations can indirectly affect the pricing strategy (Tang 2009). Furthermore, these two decisions also have an important external impact because they modify the strategic interaction between different firms: a change in product/price by a given retailer might trigger a competitive response from a competitor. We thus use game-theoretical tools to identify equilibrium situations in a competitive setting.

Our paper makes methodological as well as managerial contributions. From a theoretical standpoint, our approach to incorporate a behavioral element (i.e., satiation) as a controllable lever is a departure from the usual formulations found in the literature. Indeed, we consider an intertemporal utility maximization problem that explicitly accounts for satiation dynamics. We determine the purchasing pattern over time and we derive steady-state consumptions. We use these to obtain market shares, and we show that they take the form of an attraction model that reduces to the Chamberlin–Dixit–Stiglitz demand model when the time interval between consumption periods is large enough. Although we do not explicitly delve into product design issues, we do consider that different product decisions will result in different costs and satiation levels. In that respect, our model formulates a theory of product choice in a competitive setting, driven by the satiation effect. We show the existence of equilibrium in pure strategies, which is a nontrivial task because the strategy space is two-dimensional and the profit functions are not concave.

In terms of managerial insights, we find the following:

- There exists a unique pure-strategy Nash equilibrium in product and prices for the n-retailer game. Hence, for every market configuration, there can only be one outcome. This fundamental property sets the ground for future empirical validation (see Allon et al. 2010).
- When firms are symmetric, the equilibrium is symmetric, and selecting the symmetric product is a (nearly) dominant strategy for a firm that recognizes satiation. In other words, for product selection, managers need not worry about whether their competitor is strategic. Also, as the number of competitors increases, product satiation goes up. In other words, in industries that are more competitive, firms should place less emphasis on the product lever because the consumer can mitigate satiation by diversifying consumption across a larger number of firms.
- When firms are asymmetric, the equilibrium is asymmetric. Therefore, if firms in practice have different prices, our model predicts that they must have different operational capabilities. If firm i is able to reduce its product satiation, e.g., because of process improvement, then competitors should always decrease their prices. Firm i should increase its own price, unless product changes have a substantial impact on consumers' real income, in which case it might be worthwhile to decrease the price.
- In general, firms that ignore satiation tend to price incorrectly and are worse off. Also, firms should aim at developing capabilities to offer less satiating products more efficiently. However, because all firms have the same incentive, they should be aware that major improvements might be needed to guarantee an increase in profits. Indeed, depending on the current cost structure and the magnitude of the changes, all firms can be better off after a "product war." This is in clear contrast with price wars that are always detrimental to profits.

The rest of this paper is organized as follows. In §2 we review the literature. In §3 we present the consumption model with satiation and characterize how customers split their budget among different retailers. In §4 we analyze the strategic game in which retailers compete and choose product and price strategically. We conclude in §5. The analytical proofs are available in the appendix.

2. Literature Review

There are mostly two areas of research relevant to our work: first, models that describe variety-seeking behavior and, second, competitive models on price and product. Here we briefly describe the literature in these two directions. There is a third stream that looks at variety and assortment planning at a tactical level, which we do not review here, but we refer the reader to Ho and Tang (1998), Ramdas (2003), Lim and Tang (2006), Caro and Gallien (2007), and Kök et al. (2008).

According to Kahn (1995), there are at least three motivating factors that induce variety seeking: (i) customers get bored or satiated with their most recent purchase, (ii) customers prefer to change because of external constraints, and (iii) customers switch brands in an attempt to diversify and hedge against uncertainty in their preferences. Whether a customer seeks or avoids variety is idiosyncratic to the individual but also depends on the type of product and the time elapsed between successive purchases (Chintagunta 1998). To model variety-seeking behavior, one approach has used Markov chains to represent a customer's purchasing pattern; see Givon (1984), Kahn et al. (1986), and Feinberg et al. (1992). Another modeling approach considers a consumer utility that depends on attribute/brand inventory levels, and satiation occurs when the level exceeds a threshold; see McAlister (1982) and McAlister and Pessemier (1982). Gilboa and Schmeidler (1997) relate the utility of consuming an item to the number of previous consumptions and derive long-run average relative frequencies, very much in the spirit of our work. More recently, Baucells and Sarin (2007) find the optimal consumption levels in a discounted utility model with satiation effects and observe that consumers progressively spend more in products that generate less satiation. They extend this model to include habituation effects in Baucells and Sarin (2010).

The work mentioned above focuses mainly on explaining customer behavior and managing variety for a single firm. When multiple firms are considered, a competitive analysis is required. We are specifically interested in competition on price and product. For that, two major approaches have been developed in the literature based on whether competition is localized or not. Anderson et al. (1992) provide a general overview of product differentiation under competition and how the localized and nonlocalized approaches can be combined. Most of work on localized competition builds on the address models by Hotelling (1929) and Lancaster (1979), in which firms compete in a virtual space of product characteristics. This approach has recently been used to study the competitive advantage of standard versus customized product; see Cavusoglu et al. (2007), Alptekinoğlu and Corbett (2008), Mendelson and Parlaktürk (2008), and Xia and Rajagopalan (2009). In the nonlocalized approach, a firm faces competition from all the other firms in the industry instead of just its direct neighbors. A recurrent feature of this approach is that market shares can be written as an attraction model, which provides analytical tractability (see Federgruen and Yang 2009). Several models based on the multinomial logit fall in this category and have been used to study the effects of customer search (Cachon et al. 2008), demand substitution (Hopp and Xu 2008), and hierarchical choice (Kök and Xu 2011) under competition. Besbes and Saure (2010) consider pricing decisions in addition to assortment choice under competition. Models based on a representative consumer correspond to another case of nonlocalized competition. In particular, the seminal works by Chamberlin (1933) and Dixit and Stiglitz (1977) use a representative consumer to study the optimal level of variety in an industry, though competition is on price only. Finally, there is a stream of work that has studied price and quality competition in which quality refers to in-stock probability (see Bernstein and Federgruen 2004), waiting time (see Allon and Federgruen 2007), or service reliability (see Federgruen and Yang 2009). These papers represent cases of nonlocalized competition in which demand is usually taken as a primitive of the model instead of deriving it from a utility maximization problem.

In this paper, we derive demand from a consumption model. We consider a representative consumer as in the derivation of the Chamberlin–Dixit–Stiglitz (hereafter, CDS) demand model. However, our starting point is somewhat different. We build on the satiation model from Baucells and Sarin (2007), with two main variations: first, we consider an infinite horizon with a fixed budget per period to derive steady state consumptions; second, we consider n competing products and focus on the strategic interaction between the retailers selling the goods. In contrast with the literature on price and product competition, we integrate a behavioral aspect of consumer choice (i.e., satiation) and for that we explicitly consider intertemporal consumption. Interestingly, we obtain market shares in the form of an attraction model in which the attractiveness is a function of price and product satiation. Because the CDS model and most representative consumer models only depend on price, their use to study product decisions has been limited. The survey by Lancaster (1990, p. 191) makes this clear by stating that such models "provide no basis for a theory of product choice and product design." Our model explicitly incorporates product choice. Therefore, we see it as a first step to build the aforementioned theory. Finally, note that the qualitative study in Caro and Martínez-de-Albéniz (2009) uses a simpler version of the present model in which retailers only compete on one dimension, product choice (which is modeled as assortment rotation), and prices are taken as given.

3. A Multiperiod Utility Model with Satiation

Our objective in this section is to shed light on how price and product decisions drive consumption over time. The results constitute the building blocks for the competitive analysis of §4.

3.1. Model Formulation

Consider an individual consumer—which we will refer to as she—who maximizes her utility over T periods, e.g., months. This consumer has an aggregate budget per period for a category (e.g., apparel) that can be spent at multiple retailers or in an outside good that represents consumption in other categories. We assume that all the budget must be spent in each period. One could integrate a savings decision into our model, but this would unnecessarily complicate it. We assume that future consumption is less valuable to the consumer and is discounted at a rate $\delta < 1$ per period. Let i denote a particular firm, with $i = 0$ the outside good, and let n denote the total number of competing firms. We denote x_{it} the quantity bought at retailer i in period t, and p_i the price per unit at retailer i.

Consumption of product $i > 0$ generates satiation; i.e., consuming large quantities of the good may reduce the utility obtained by the customer in subsequent periods. To model this effect, we follow Baucells and Sarin (2007) and assume that the contribution of the current consumption is an increment over the satiation level achieved from previous consumption. Formally, let y_{it} be product i's satiation level at the beginning of period t, which can be seen as a consumption stock level that remains from the previous period. The incremental utility derived from consuming x_{it} in period t is defined as $u_i(y_{it} + x_{it}) - u_i(y_{it})$, where $u_i(z)$ is an increasing and concave function that represents the utility generated by a consumption stock level z.

Of course, the satiation level y_{it} is related to previous consumption $x_{i1}, \ldots, x_{it-1}$. As in Baucells and Sarin (2007), we assume that this relationship takes the form of exponential smoothing:

$$y_{it+1} = \gamma_i(y_{it} + x_{it}), \tag{1}$$

where $\gamma_i \in [0, 1)$ is called the satiation retention factor at retailer i. One can observe from this formulation that $y_{it} = \sum_{\tau=1}^{\infty} \gamma_i^{\tau} x_{it-\tau}$. In other words, the impact of past consumption $x_{it-\tau}$ on the current satiation level y_{it} decays exponentially. For the outside good, we assume that it does not generate satiation, i.e., $\gamma_0 = 0$, although none of the results would be affected if it did.[1]

After specifying the utility derived from each product, the consumer's problem (CP) can thus be defined as

$$\begin{aligned}
\text{(CP)} \quad \max \quad & \sum_{t=1}^{T} \delta^{t-1} \left(\sum_{i=0}^{n} (u_i(y_{it} + x_{it}) - u_i(y_{it})) \right) \\
\text{s.t.} \quad & \sum_{i=0}^{n} p_i x_{it} \le W_t \quad \text{for } t = 1, \ldots, T \\
& y_{it+1} = \gamma_i(y_{it} + x_{it}) \quad \text{for } i = 0, \ldots, n, \\
& \qquad\qquad t = 1, \ldots, T \\
& x_{it} \ge 0 \quad \text{for } i = 0, \ldots, n,\ t = 1, \ldots, T.
\end{aligned}$$

Here W_t is the consumer's budget for period t. Alternatively, using a dynamic programming formulation with $U_t(y_t)$ the "utility-to-go" from period t onward, we can write

$$U_t(y_t) = \max_{\substack{\sum_{i=0}^{n} p_i x_{it} \le W_t \\ x_{it} \ge 0}} \left\{ \sum_{i=0}^{n} (u_i(y_{it} + x_{it}) - u_i(y_{it})) + \delta U_{t+1}(\gamma \cdot (y_t + x_t)) \right\} \tag{2}$$

and $U_{T+1}(y_{T+1}) \equiv 0$, where $(\gamma \cdot (y_t + x_t))_i = \gamma_i(y_{it} + x_{it})$.

To avoid end-of-horizon effects, we focus on the infinite horizon case $T = \infty$ with a stationary per period budget $W_t \equiv W$. As a result, U_t is stationary, i.e., $U_t \equiv U$, and satisfies the following Bellman equation (see Bertsekas 2000):

$$U(y) = \max_{\substack{\sum_{i=0}^{n} p_i x_i \le W \\ x_i \ge 0}} \left\{ \sum_{i=0}^{n} (u_i(y_i + x_i) - u_i(y_i)) + \delta U(\gamma \cdot (y + x)) \right\}. \tag{3}$$

3.2. Model Discussion

In the model formulation above, the consumer must allocate her budget among several products. Note that here we use the term "product" in the general sense of Chamberlin (1933), which includes real/physical characteristics as well as any intangible factors that can influence customer preferences (e.g., assortment rotation and store management policies). Therefore, one could apply the model to a wider generic product

[1] The outside good is an aggregation of all other product categories in which the consumer could spend her budget. Assuming $\gamma_0 = 0$ means that there is always an external option that the consumer has not consumed before (or that was consumed a long time ago).

(e.g., a Zara T-shirt), a brand, or even an entire store. Given this broad definition, we assume that each firm offers only one (extended) product, so throughout the paper we refer to products and firms interchangeably. This assumption is shared by all competition models à la Chamberlin; see Anderson et al. (1992) for further discussion.

For a particular product, the satiation effect is incorporated through the retention factor γ_i. Though this parameter is an abstract modeling device, it is similar to a discount factor and can be elicited easily (see Baucells and Sarin 2007). An important remark is that the value of γ_i is not only tied to the product's characteristics but also depends on the length of the interval between time periods. In fact, regardless of the type of product, if the time between consumption periods is large enough, then there is no reminiscence of past consumption and γ_i is close to zero. Another remark is that even though products can be perfectly ranked according to their satiation factors, product differentiation in our model occurs horizontally. Indeed, at equal prices, even a product with a high satiation factor can have a positive market share because of the decreasing marginal utilities. Also, each product can be seen as a substitute of every other one, so the competition we analyze later is by construction nonlocalized (see Moorthy 1998 for a model with vertically differentiated products and localized competition).

Clearly, we are interested in situations in which satiation effects are present ($\gamma_i > 0$). This is true in many cases when products are purchased regularly. Take, for instance, food (daily) or apparel (weekly or monthly in women's apparel; e.g., Zara consumers visit a store 17 times per year). Indeed, eating a pizza primavera may generate a large utility at first, but when this has been the same meal 10 days in a row, the incremental utility of the 11th pizza is (usually) quite low. Similarly, purchasing a T-shirt from a given store once may provide a large utility, but purchasing one every week is (again, usually) less satisfying because the store has only so many designs, which tend to be similar (they are consistent with a given store image). When satiation is not relevant (i.e., $\gamma_i = 0$ for all i), our utility model starts afresh in each period. In that respect, we do not consider other intertemporal effects such as reinforcement behavior (Kahn et al. 1986), customer loyalty (Gans 2002), or habit formation (Baucells and Sarin 2010), which are beyond the scope of the paper.

A few other modeling choices are worth discussing. First, we consider an infinite horizon. Hence, we focus on the long-term, steady state behavior of the consumer. Although the optimal transient consumption path will be computed, ultimately we are interested in understanding how price and satiation influence the share of the budget that each retailer captures in the long run. Second, the price and satiation factor at each retailer are known to the customer and remain constant throughout the horizon. This assumption is mainly made for tractability. It is also consistent with the strategic focus of the game analyzed in §4, where p_i and γ_i represent the price and product positioning chosen by firm i. These are decisions that involve marketing, product development, store location/layout, and supply chain design, which are usually kept fixed for long periods. Thus, we do not consider any tactical price or product changes (this would require a much more complex analysis). Third, we have included in the model an outside good to allow for consumption in other categories. This can be interpreted as the no-purchase option in the multinomial logit model or as the reservation price in the Hotelling–Lancaster approach. We assume that the outside good does not satiate ($\gamma_0 = 0$), to represent the case when the consumer has plenty of opportunities to spend her budget elsewhere. Finally, we have ignored cross-satiation effects, i.e., when the satiation level of product i is affected by the past consumption of product $j \neq i$, which might exist but have not been reported in the literature.

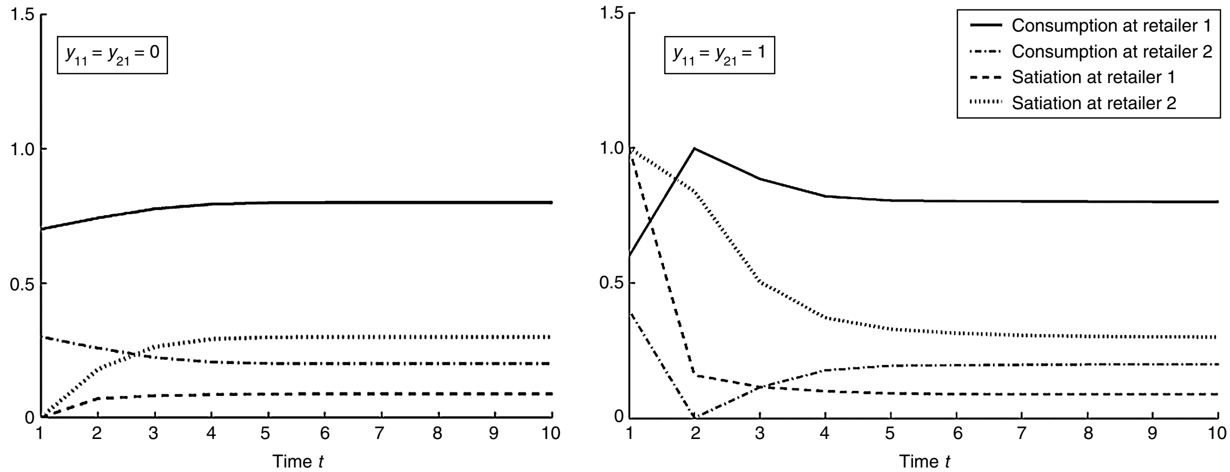
3.3. Optimal Consumption Paths

To simplify the exposition, we can reformulate problems (2) and (3) using the (stationary) incremental utility function, which is defined as $v_i(z_i) \equiv u_i(z_i) - \delta u_i(\gamma_i z_i)$. Letting $z_t = x_t + y_t$ and $V_t(y_t) = U_t(y_t) + \sum_{i=0}^n u_i(y_{it})$ in Equation (2), with $V_{T+1}(y_{T+1}) = \sum_{i=0}^n u_i \cdot (y_{iT+1})$, and then taking the limit $T \to \infty$, we can rewrite Equation (3) as

$$\text{(SP)} \quad V(y) = \max_{z_0,\ldots,z_n} \left\{ \sum_{i=0}^n v_i(z_i) + \delta V(\gamma \cdot z) \right\}$$
$$\text{s.t.} \quad \sum_{i=0}^n p_i z_i \le W + \sum_{i=0}^n p_i y_i$$
$$z_i \ge y_i \quad \text{for } i = 0, \ldots, n,$$

where (SP) stands for stationary problem. Let (SP-R) be a relaxation of (SP) without the constraints $z_i \ge y_i$, $i = 0, \ldots, n$. With these definitions we can begin to characterize the optimal consumption decision. First, if $v_i(z_i)$ is concave in z_i for all i, then an inductive argument shows that $V(y)$ is jointly concave in y. For $v_i(z_i)$ to be concave, we need $u_i''(z_i) - \delta\gamma_i^2 u_i''(\gamma_i z_i) \le 0$, which holds for all δ and γ_i if $z^2 u_i''(z)$ is nonpositive and nonincreasing in $z \ge 0$. The latter is satisfied for many utility functions, such as power, logarithmic, quadratic, etc. The concavity of $V(y)$ allows us to characterize the optimal consumption decision from first-order optimality conditions. The following proposition shows that these conditions can be further simplified. We omit the proof for the sake of space, but it is available from the authors.

Figure 1 **Consumption Level at Two Retailers over Time with a Budget of** $W = 1$, $\delta = 0.99$, $u_i(z_i) = 2\sqrt{z_i}$, **and** $p_i = 1$ **for** $i = 1, 2$



Notes. The satiation retention factors are $\gamma_1 = 0.1$ and $\gamma_2 = 0.6$. The initial satiation is $y_{11} = y_{21} = 0$ in the left figure and $y_{11} = y_{21} = 1$ in the right one.

Proposition 1. *Let $\gamma_i < 1$ for all i such that the initial satiation level is positive; i.e., $y_{i1} > 0$. Assume that $v_i(z_i)$ is concave and $v_i'(z_i)$ is convex in z_i for all i. Then the optimal policy for the customer's problem* (CP) *converges to an optimal solution of* (SP-R).

The proof of this result is technical and long, but the main idea is to show that the customer's optimal consumption policy is a unique sequence that converges to a stationary solution, and for $t \to \infty$, the nonnegative constraints $x_{it} \geq 0$ are inactive. In fact, if the initial satiation is zero for all i, then $x_{it} \geq 0$ is inactive for all $t \geq 1$. Therefore, (CP) has a unique stationary solution that is optimal for (SP) as well as (SP-R). A direct implication is that we can ignore the Lagrange multipliers of the constraints $z_i \geq y_i$ in the first-order conditions of (SP). We use this fact in the theorem below.

Note that Proposition 1 needs v_i to be concave so that $V(y)$ is concave and the structure of the problem is well behaved. In addition, v_i' needs to be convex, which is equivalent to having $u_i'''(z_i) - \delta\gamma_i^3 u_i'''(\gamma_i z_i) \geq 0$. This is true when $z^3 u_i'''(z)$ is nonnegative and nondecreasing in $z \geq 0$, which, again, is satisfied for power, logarithmic, or quadratic utility functions.[2]

We can now state the main theorem of this section (the remaining proofs are in the appendix).

Theorem 1. *Under the conditions of Proposition* 1, *the optimal consumption policy (x_t, y_t) for the customer's problem* (CP) *converges to a stationary solution (x_∞, y_∞) such that $x_{i\infty} = (1-\gamma_i)z_{i\infty}$ and $y_{i\infty} = \gamma_i z_{i\infty}$, where $z_{i\infty}$ is uniquely defined by the first-order conditions*

$$\frac{v_i'(z_{i\infty})}{p_i(1-\delta\gamma_i)} = \mu \quad \text{for } i = 0, \ldots, n, \tag{4}$$

and μ ensures that $\sum_{i=0}^{n} p_i(1-\gamma_i)z_{i\infty} = W$.

[2] Note that the results here in §3 can be extended to the case in which $v(z_i)$ and $v'(z_i)$ are concave and convex, respectively, for $z_i \in [0, z_i^*]$, which is the interval where $v(z_i)$ is increasing. This includes the exponential utility.

Theorem 1, and in particular Equation (4), shows that in steady state the consumer will diversify consumption because of the decreasing marginal utilities and the satiation effects ($\gamma_i > 0$). This contrasts with the static multinomial logit and the Hotelling–Lancaster model in which an individual consumer will choose only one variant and consumption is diversified in the aggregate because of heterogeneity in tastes. Theorem 1 also complements the results in Baucells and Sarin (2007) because it supports the observation that consumption tends to stabilize around an "equilibrium level" in the periods that are not at the beginning (or at the end, if the horizon is finite). The same is observed by Popescu and Wu (2007), though in a dynamic pricing model in which current prices influence future demand. Figure 1 illustrates the convergence of the consumption decisions for different initial satiation levels.

It can be shown that $x_{i\infty}$ decreases as the number of competing retailers n increases (higher competition) and the firm's satiation factor γ_i increases. On the other hand, it increases as the competitors' satiation factor γ_j, $j \neq i$, increases. Under the regularity condition that $zv_k'(z)$ is nondecreasing for all k—which is verified for power, logarithm, and quadratic utilities—it can also be shown that the stationary consumption $x_{i\infty}$ decreases in firm i's price p_i and increases in the competitors' prices p_j, $j \neq i$.

3.4. An Attraction Model for Power-Type Utility Functions

For the remainder of the paper, we assume that the utility is of the power type: $u_i(z) = z^{1-\beta}/(1-\beta)$, with $0 \leq \beta < 1$. This family of utilities, or positive monotone transformations of it, has been widely used because of its mathematical tractability (see Baucells and Sarin 2007). The parameter β gives some degree of flexibility to model different attitudes with respect to satiation: the larger β, the more sensitive is the customer's marginal utility to changes in the satiation level because of previous consumptions. In particular, if β equals zero, then the utility is linear and the consumer will spend all her budget at the retailer with the lowest price p_i, regardless of whether the item generates satiation on the consumer or not. That would be the case of customers who are *satiation indifferent*. On the other hand, large values of β imply that customers perceive a large utility for initial consumption but the incremental utility from additional consumption is minimal. Therefore, they quickly become satiated, and the consumption inventory level must substantially decrease for them to derive utility again.

For a utility function of the power type, we can solve Equation (4) to obtain

$$x_{i\infty}=\frac{1}{p_i}\frac{p_i^{1-1/\beta}(1-\gamma_i)((1-\delta\gamma_i^{1-\beta})/(1-\delta\gamma_i))^{1/\beta}}{\sum_{k=0}^{n}p_k^{1-1/\beta}(1-\gamma_k)((1-\delta\gamma_k^{1-\beta})/(1-\delta\gamma_k))^{1/\beta}}W. \tag{5}$$

When the market is homogeneous (i.e., the budget W and the utility functions u_i are the same for all consumers), retailers compete for the budget of a representative consumer, and thus W in Equation (5) can be seen as the total market size of the category per period. A remarkable fact is that when $\gamma_i=0$ for all i, then the steady state consumption in Equation (5) reduces to the well-known CDS demand model.[3] This shows that when satiation effects are not significant, possibly because the time interval between consumption periods is large enough, then our model reduces to a common demand form found in the literature.

Assuming a constant per-unit production cost c_i, retailer i's long-term average profit can be written as $\pi_i=(p_i-c_i)x_{i\infty}$. Given a power utility function, Equation (5) allows us to rewrite the profit of retailer i as

$$\pi_i=\frac{p_i-c_i}{p_i}\frac{a(p_i,\gamma_i)}{\sum_{k=0}^{n}a(p_k,\gamma_k)}W, \tag{6}$$

where $a(p_i,\gamma_i):=p_i^{1-1/\beta}b(\gamma_i)$ and $b(\gamma_i):=(1-\gamma_i)\cdot((1-\delta\gamma_i^{1-\beta})/(1-\delta\gamma_i))^{1/\beta}$. The profit (6) can be interpreted as the gross margin times dollar market share times market size. Interestingly, the market share $m_{i\infty}:=a(p_i,\gamma_i)/(\sum_{k=0}^{n}a(p_k,\gamma_k))$ follows an attraction model with individual attraction $a_i=a(p_i,\gamma_i)$, separable in p_i and γ_i and decreasing in these variables.

Such log-separable attraction structure is called *multiplicative competitive interaction* (see, for instance, Bernstein and Federgruen 2004) and is the result of the consumer's optimal consumption problem. Note that whereas usually market shares are expressed in terms of units sold, we instead find a market share in dollar terms, but this does not change the essence of the problem. In general, attraction models have a large appeal in the literature because they are tractable and have an intuitive interpretation (see Federgruen and Yang 2009). Given their popularity, many studies take the attraction model as exogenously given. In contrast, we derive it endogenously and we show that it extends the CDS demand model by making it dependent on a important product attribute such as satiation and not only on price.

4. Retailer Decisions on Price and Product

In this section, we build on the satiation model to study how firms set prices and select products. The results that follow assume a power-type utility function. As a robustness check, we numerically tested all the results using a quadratic utility. Note that for this function, the dollar market share $p_i x_{i\infty}$ does not have an attraction form. However, in our experiments, all the results continued to hold, with one exception described at the end of §4.1. The full study is available upon request.

4.1. Price Decisions Only

We start the competitive analysis assuming that product characteristics are fixed; i.e., γ_i cannot be changed by retailer i. This is the case in a market where the competitors are stuck with different technologies or capabilities that yield different levels of satiation. For instance, consider the string of amusement parks in Southern California. What each park can offer is quite established, so competition is mostly on price (in practice, it calls for revenue management). In this paper, we are interested in finding the Nash equilibria of the pricing game in pure strategies: the game has a (pure) Nash equilibrium if there exists $(p_1^{eq},\ldots,p_n^{eq})$ such that no single firm can improve its profit π_i by changing unilaterally p_i away from p_i^{eq}.

For a given vector of competitor prices, p_{-i}, π_i is first increasing and then decreasing in p_i, i.e., quasi-concave. Existence of equilibrium follows and it is unique, which can also be established from Gallego et al. (2006). More importantly, it can be described as follows.

[3] Other authors refer to it as the constant elasticity of substitution (CES) model. The exact equivalence between Equation (5) and the demand expression (7) in Dixit and Stiglitz (1977) is obtained by setting $\gamma_i=0$ for all i in (5) and $s(q)=q^{1-1/\beta}/(q^{1-1/\beta}+p_0^{1-1/\beta})$ in Dixit and Stiglitz (note that our β parameter is equivalent to $1-\rho$ in the paper by Dixit and Stiglitz).

Theorem 2. *There exists a unique pure-strategy Nash equilibrium* $(p_1^{\text{p-only}}, \ldots, p_n^{\text{p-only}})$ *in the price-only game. It is characterized by the unique solution to*

$$\frac{p_i - c_i}{c_i}(1 - m_{i\infty}) = \frac{\beta}{1-\beta}. \qquad (7)$$

We can derive a number of observations from the theorem. First, when $\beta = 0$, customers are indifferent to satiation, and as a result retailers engage in Bertrand competition. Second, when two firms are identical, i.e., i and j are such that $c_i = c_j$ and $\gamma_i = \gamma_j$, then they will be priced identically in equilibrium. In particular, when firms are symmetric (all equal, i.e., $c_i = c$) then their prices are also identical, equal to

$$p_i^{\text{p-only}} = \frac{c}{1-\beta}\left(1 + \frac{\beta}{n-1}\right) \quad \text{for all } i, \qquad (8)$$

when no outside good is present (i.e., $p_0 = \infty$). The symmetric price is decreasing in n (more competition leads to lower prices) and increasing in β (more satiation-sensitive customers will see higher prices). Because of the multiplicative interaction in our attraction model, the symmetric price (8) is the same as in the original CDS model without satiation effects (see Equation (7.9) in Anderson et al. 1992). Note that when $p_0 < \infty$, the symmetric price is lower than the value in (8).

It can be shown that as γ_i increases, $p_i^{\text{p-only}}$ decreases whereas $p_j^{\text{p-only}}$ increases for $j \neq i$. Therefore, a firm that sells a product with high (low) satiation but fails to take it into account—e.g., uses the CDS model without satiation—will tend to overprice (underprice). Note that this fact hinges on the attraction form of the dollar market share (see Equation (5)). Indeed, when γ_i increases, the consumer spends more on the other products and the attraction form makes sure that the budget remains balanced. Therefore, there is no income effect, and the pure substitution effect forces firm i to lower its price in equilibrium. If $p_i x_{i\infty}$ does not have an attraction form, then some of the consumer's income might remain unused after substitution takes place. This provides incentives to all firms, including firm i, to increase prices. For quadratic utility, we observed that the net effect is that $p_i^{\text{p-only}}$ also increases, so if firm i ignored satiation it would systematically underprice.

4.2. Product Decisions Only

Knowing that satiation has a strong influence on equilibrium prices and market shares, retailers will find quite valuable the possibility of choosing the type of product offered so as to reduce the satiation that it generates on consumers. For example, in apparel, a retailer might choose to increase the frequency at which the stores are being refreshed because this improves the perception of novelty on the customer. Similarly, in the food service industry, a restaurant might decide to change the menu more often. Another example are innovation-driven industries, e.g., smartphones, where obsolescence drives product satiation and prices remain relatively stable across product generations.

To analyze product decisions, we allow the retailers to choose from a continuum of products that spans the entire range of possible satiation retention factors, so the strategy space for γ_i is the interval $[0, 1]$. The operational cost incurred by firm i when it offers a product with a satiation factor γ_i, is denoted $k_i(\gamma_i)$. To make the problem meaningful, k_i should satisfy the following properties. First, k_i should be decreasing in γ_i because it is more expensive to offer a less satiating product. Second, k_i should be convex in γ_i to represent that each marginal reduction in the satiation factor becomes increasingly harder to achieve. Third, $\lim_{\gamma_i \to 0} k_i(\gamma_i) = \infty$, meaning that the operational requirements to offer a nonsatiating product is prohibitively expensive. To simplify the analysis but still offer modeling flexibility, we consider the following functional form for k_i:

$$k_i(\gamma_i) = \frac{f_i(\gamma_i^{-g_i} - 1)}{g_i} \geq 0, \quad \text{for } i = 1, \ldots, n, \qquad (9)$$

where the parameters $f_i, g_i \geq 0$ can be firm dependent.[4] Note that by taking the limit of $g_i \to 0$, $k_i(\gamma_i)$ tends to $-f_i \ln(\gamma_i)$. The profit of retailer i now becomes

$$\pi_i(p_i, \gamma_i, p_{-i}, \gamma_{-i}) = \frac{p_i - c_i}{p_i} m_{i\infty} W - k_i(\gamma_i). \qquad (10)$$

In practice, product decisions are usually more difficult to change than prices, but in some industries with strong price focus (e.g., affordable fashion or the other examples given above), price is taken as given and only product decisions are used. We are thus interested in finding the Nash equilibria in the product strategy space, which exists per the following theorem.

Theorem 3. *There exists a pure-strategy Nash equilibrium* $(\gamma_1^{\gamma\text{-only}}, \ldots, \gamma_n^{\gamma\text{-only}})$ *in the product-only game, which is characterized by*

$$\frac{p_i - c_i}{p_i} m_{i\infty}(1 - m_{i\infty}) = \frac{f_i}{W\psi(\gamma_i)\gamma_i^{g_i}},$$
$$\text{where } \psi(\gamma_i) := -\gamma_i b'(\gamma_i)/b(\gamma_i). \qquad (11)$$

If p_0 *is sufficiently low, then* $m_{i\infty} \leq 1/2$, *for* $i = 1, \ldots, n$, *and the equilibrium is unique.*

[4] A numerical study for other functional forms that satisfy the assumptions gave the same results. Tests for quadratic and exponential costs such that $k_i(0) < \infty$ were also positive. However, for linear costs, the profit function can have local optima, indicating that some notion of strict convexity for $k_i(\gamma_i)$ is needed for our results to hold.

Uniqueness can be guaranteed when in equilibrium $m_{i\infty} \le 1/2$, for $i = 1, \ldots, n$, (note that Allon et al. 2010 have the same condition for price-only competition). Otherwise, there may be multiple equilibria and in each there is at least one firm with more than 50% market share. When firms are symmetric (all equal) and there is no outside good ($p_0 = \infty$), then there is a unique symmetric equilibrium such that $\psi(\gamma_i^{\gamma\text{-only}}) \cdot (\gamma_i^{\gamma\text{-only}})^g = (f/W)(p/(p-c))(n^2/(n-1))$ for all i. Hence, the symmetric product generates more satiation as n increases (more competition leads to less investment to increase market share). Interestingly, in contrast with §4.1, the symmetric satiation level depends on the prevailing prices, even when $p_0 = \infty$. When the price is taken as the one in (8), the corresponding satiation factor is determined by

$$\psi(\gamma_i^{\gamma\text{-only}})(\gamma_i^{\gamma\text{-only}})^g = \frac{f}{W}\frac{n(n-1+\beta)}{\beta(n-1)}. \tag{12}$$

4.3. The Price-Product Equilibrium

We now consider the general situation when firms can change both price and product (i.e., satiation retention factors). We are interested in finding the Nash equilibria in the two-dimensional strategy space: these equilibria are defined as $((p_1^{eq}, \gamma_1^{eq}), \ldots, (p_n^{eq}, \gamma_n^{eq}))$ such that no retailer i can improve its profit π_i by changing unilaterally (p_i, γ_i) away from $(p_i^{eq}, \gamma_i^{eq})$. We first focus on finding the best-response function to the competitors' choice of (p_{-i}, γ_{-i}). Though the profit function (10) is not concave, it is still well behaved, which leads to the following proposition.

Proposition 2. *For any p_{-i}, γ_{-i}, π_i is maximized at the unique $p_i^{BR}, \gamma_i^{BR} < 1$ that satisfy Equations* (7) *and* (11). *As a result, p_i^{BR}, γ_i^{BR} are continuous with respect to p_{-i}, γ_{-i}. Moreover, p_i^{BR} is always increasing in p_k and in γ_k, $k \ne i$. In contrast, given γ_k, γ_i^{BR} is first decreasing and then increasing in p_k; given p_k, γ_i^{BR} is first decreasing and then increasing in γ_k.*

Proposition 2 characterizes the best response of i in both the price and product dimensions. It also shows that firm i always has an incentive to increase its price when its competitors are less aggressive (higher p_k or γ_k). On the other hand, the optimal product choice is nonmonotonic in the competitors' actions. In fact, it has an inverted U-shape. This means that when it comes to satiation, a firm that has enough market share (facing high p_k or γ_k) can afford to match its competitors' actions. Otherwise, it is better to react in the opposite direction because there is a larger benefit in cutting operational costs. This is the best response to unilateral changes. The outcome in equilibrium is discussed in the next two sections after we establish its existence.

We use the continuity of firm i's best response to prove that a pure-strategy Nash equilibrium exists in the two-dimensional game between retailers. This result should not be taken for granted, because in many competitive models on price and product an equilibrium in pure strategies fails to exist (this is mostly the case with the Hotelling–Lancaster model; see Anderson et al. 1992). Note also that because we consider dollar market shares and the operational cost $k_i(\gamma_i)$, the equilibrium results in Federgruen and Yang (2009) do not apply here.

Theorem 4. *There exists a unique pure-strategy Nash equilibrium in the n-retailer game, $n \ge 2$.*

4.3.1. The Symmetric Price-Product Oligopoly.

Consider the symmetric game in which all retailers share the same cost parameters: $c_i = c$, $f_i = f$, $g_i = g$. The following theorem characterizes analytically the symmetric equilibrium.

Theorem 5. *In the symmetric game the unique pure-strategy equilibrium is symmetric and such that $p_i^{eq} = p^{\text{sym}} = p^*(\lambda^{\text{sym}})$ and $\gamma_i^{eq} = \gamma^{\text{sym}} = \gamma^*(\lambda^{\text{sym}})$ for $i = 1, \ldots, n$, where*

$$p^*(\lambda) = \frac{c\beta}{1-\beta}\left(\frac{1}{\lambda}+\frac{1}{\beta}\right) \quad \text{and} \quad \gamma^*(\lambda) \text{ is such that}$$
$$\psi(\gamma)\gamma^g = \frac{f}{W}(1+\lambda)\left(\frac{1}{\lambda}+\frac{1}{\beta}\right), \tag{13}$$

and λ^{sym} is the unique solution of the single dimension fixed point equation

$$\lambda = n-1+\frac{1}{b(\gamma^*(\lambda))}\left(\frac{p^*(\lambda)}{p_0}\right)^{1/\beta-1}. \tag{14}$$

From the theorem, several observations can be made. First, when all firms have the same cost parameters, the unique equilibrium is symmetric. Second, the computation of the equilibrium reduces to solving a single-dimensional fixed point equation. Third, without an outside good ($p_0 = \infty$), the theorem implies that $\lambda = n-1$ and p^{sym} is equal to the symmetric price in Equation (8) for price-only competition (which in turn is equal to the symmetric price in the CDS model). Similarly, the resulting satiation factor γ^{sym} is equal to the symmetric satiation factor in Equation (12). In other words, without an outside good, the choice of price and product decouple because each firm knows that its market share will be $1/n$. Interestingly, when an outside good is present, the decoupling no longer occurs because then the choice of p depends on γ, and vice versa.

Finally, from Equations (13) and (14) we can derive comparative statics, which are summarized in the following corollary.

Corollary 1. *Let $(p^{\text{sym}}, \gamma^{\text{sym}})$ be the symmetric equilibrium. For $n > 1$, if any of the following changes in the parameters occurs—$\uparrow n$, $\uparrow f$, $\uparrow g$, $\downarrow W$, or $\downarrow p_0$—then the equilibrium price decreases and the equilibrium level of satiation increases ($\downarrow p^{\text{sym}}$ and $\uparrow \gamma^{\text{sym}}$).*

As expected, an increase in costs (higher f or g) implies an increase in the satiation factor offered, which in turns requires the firms to decrease the price. The same occurs as the market size W or the price of the outside good p_0 decreases or when the number of firms $n > 1$ increases. This last observation is interesting because it means that more competition leads to more emphasis on price and less on product "freshness," which occurs because consumers can now satisfy their need for variety by consuming across a larger number of retailers. Therefore, in saturated markets, and in the absence of habituation effects, firms will stick to standard offerings and consumers will consume a bit of each every period, just like in a food court.

4.3.2. The Asymmetric Price-Product Duopoly. We now focus on the duopoly and focus on understanding how differences in operational capabilities to offer low-satiation products affect the equilibrium outcome. To focus on the satiation aspect, we assume as before that firms have identical production costs $c_1 = c_2 = c$. In contrast, we assume that $f_1 \le f_2$ and $g_1 \le g_2$; i.e., retailer 1 can offer a product with a given satiation factor at a lower operational cost than retailer 2 can. Therefore, retailer 1 has an operational advantage and will be prone to offer lower-satiation products. For example, in apparel, retailer 1 might be a more flexible firm that can refresh its store frequently without incurring high costs, such as Zara or H&M; retailer 2 might be a traditional retailer with a more rigid supply chain that makes new product introduction operationally more costly. We can present a result analog to Theorem 5 for the asymmetric duopoly.

Theorem 6. *Let $c_1 = c_2 = c$. In the asymmetric duopoly, the unique pure-strategy Nash equilibrium is asymmetric: if $f_1 < f_2$, $g_1 \le g_2$ or $f_1 \le f_2$, $g_1 < g_2$, the equilibrium is such that $\gamma_1^{eq} < \gamma_2^{eq}$ and $p_1^{eq} > p_2^{eq}$. In addition, firm 1's equilibrium profit π_1^{eq} is increasing in f_2, g_2 and decreasing in f_1, g_1; firm 2's equilibrium profit π_2^{eq} is increasing in f_1, g_1 and decreasing in f_2, g_2.*

The theorem shows that the unique equilibrium is asymmetric in which retailer 1 offers a less satiating good. Retailer 1 can thus also charge higher prices and reap higher profits. In contrast, retailer 2 makes lower profits because of its inability to reduce satiation and to attract sales; it is thus forced to emphasize a more aggressive price position.

Theorem 6 also provides some insights regarding the competitive advantage derived from a lower cost structure f_i, g_i. Consider two cost structures or types denoted H and L for high and low, respectively, with $f_L \le f_H$ and $g_L \le g_H$ and one of the inequalities being strict. Denote $\pi^{T_1T_2}$ the equilibrium profits achieved by a retailer of type T_1 competing against a firm type T_2. From Theorem 6 we have that a retailer's equilibrium profit is decreasing in its own cost. This implies that $\pi^{LH} \ge \pi^{HH}$ and $\pi^{LL} \ge \pi^{HL}$. In other words, regardless of the competitor's type, a firm is better off with a low cost structure. On the other hand, Theorem 6 shows that a firm's profits are increasing in the competitor's cost, which implies that $\pi^{LH} \ge \pi^{LL}$ and $\pi^{HH} \ge \pi^{HL}$. Put differently, a firm is worse off competing against a type-L retailer, regardless of its own type. These observations are summarized in the following corollary.

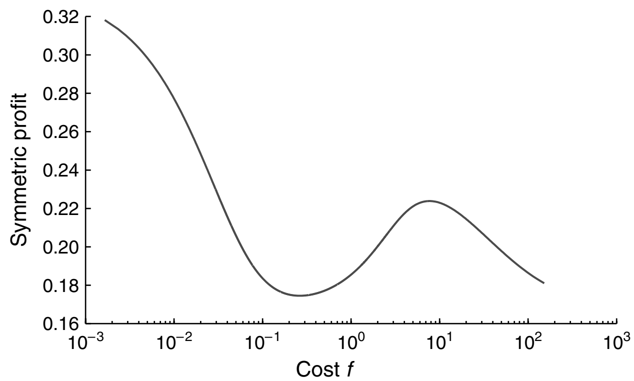
Corollary 2. *Let $f_L \le f_H$ and $g_L \le g_H$. The competitive scenarios can be ordered as follows:*

$$\pi^{LH} \ge \max\{\pi^{LL}, \pi^{HH}\} \ge \min\{\pi^{LL}, \pi^{HH}\} \ge \pi^{HL}. \quad (15)$$

Corollary 2 shows that under price and product competition, if firms are allowed to select their type, then choosing L is a dominant strategy, as long as the production costs remain identical for both firms. In that case, the equilibrium profits would be π^{LL}. Even when $\pi^{HH} > \pi^{LL}$, the firms have incentives to deviate unilaterally and both end up making lower profits, just as in the Prisoner's Dilemma (how $\pi^{HH} > \pi^{LL}$ can occur is discussed later). Interestingly, the order in (15) can change when there is product-only competition. Indeed, it is possible that $\pi^{HL} \ge \pi^{HH}$, which means that a type H firm would prefer to face an asymmetric type L competitor. This observation was confirmed numerically in Caro and Martínez-de-Albéniz (2009), and it can happen when the type H firm responds to the lower satiation product of the type L firm by increasing its own product satiation (recall from Proposition 2 that the best response in γ_{-i} has an inverted U-shape), so the cost savings from less rotation for the type H outweigh the decrease in market share. This situation cannot happen under product and price competition because then the type H firm must also lower its price so revenues are hurt twice—through a lower market share and a decreased price—which is why $\pi^{HH} \ge \pi^{HL}$ in Corollary 2.

Finally, it is worth noting that the profit associated with the symmetric duopoly is not necessarily monotonic in f. Although this may seem surprising, it can be explained because the higher cost might reduce the intensity of competition, rendering higher profits to the firms. In particular, we can apply Theorem 5 and find that $\lambda^{\text{sym}} = 1$ (because $p_0 = \infty$ and $n = 2$). This results in $p^{\text{sym}} = (1+\beta)/(1-\beta)$ and γ^{sym} being increasing in f. As a result the revenue is independent of f, whereas the fixed cost is f times a decreasing function of f. This last product is not monotonic in f. Figure 2 illustrates the symmetric profit as a function of f, with the x-axis presented in log-scale. It can be seen the curve is first decreasing, then increasing and finally decreasing again. Though there are cases in which both firms would be better off having a higher

Figure 2 **Symmetric Equilibrium Profits as a Function of the Operational Cost Parameter f**



Note. Here $f_1 = f_2 = f$, $g_1 = g_2 = 0$, $c_1 = c_2 = 1$, $\beta = 0.5$, $W = 1$, $\delta = 0.99$, and $p_0 = \infty$.

cost structure, i.e., $\pi^{HH} > \pi^{LL}$, if process improvements move the market equilibrium "from peak to peak" in Figure 2, then despite the "product war" both firms would end up with higher profits. This nonlinear behavior contrasts with price wars in which profits always decrease.

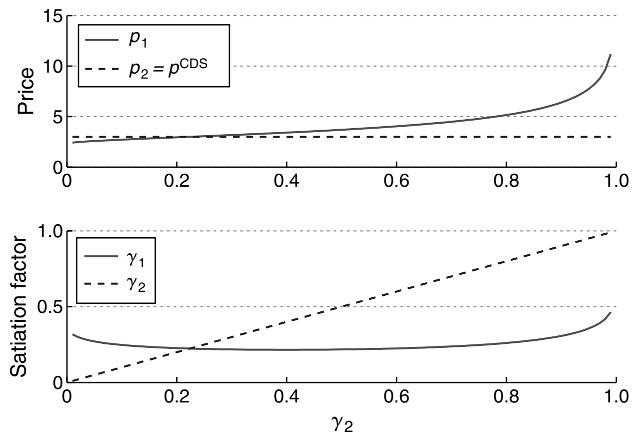
4.4. The Impact of Ignoring Product Satiation

One of the main contributions of this paper is incorporating satiation effects into to a competitive model. In particular, we obtain market shares that generalize the classic CDS model by adding a product dimension captured through a multiplicative term that depends on the satiation factors γ_i, $i = 1, \ldots, n$. Given this new feature, it is natural to ask what happens when a firm ignores it. That is, how much should a firm care about incorporating satiation effects into its product decisions? In this section we answer this question, which in turn helps us to better understand the underlying forces in the model.

Consider the symmetric duopoly. As before, by symmetry we mean that both firms have the same cost structure. Suppose that firm 2 does not act strategically in terms of its product and it believes that competition is only based on prices. In other words, firm 2 ignores satiation and prices according to the CDS model, which again we denote p^{CDS}. In contrast, firm 1 is forward looking and responds strategically in terms of product and price according to Proposition 2. To sharpen the insights, we ignore the outside good ($p_0 = \infty$), but at the end we discuss the impact when it is brought back in.

Firm 1 follows its best response knowing that firm 2 will offer a product with a given satiation level γ_2 and that it will set the price to p^{CDS}, regardless of γ_2. Because firm 2 does not act strategically with respect to product satiation, γ_2 must be treated as a parameter, and therefore we present our results as a function of $\gamma_2 \in [0, 1]$. Figure 3 shows the price and

Figure 3 **Price and Product Outcome in Equilibrium When Firm 1 Optimizes Both Dimensions While Firm 2 Ignores Satiation and Follows the CDS Model**



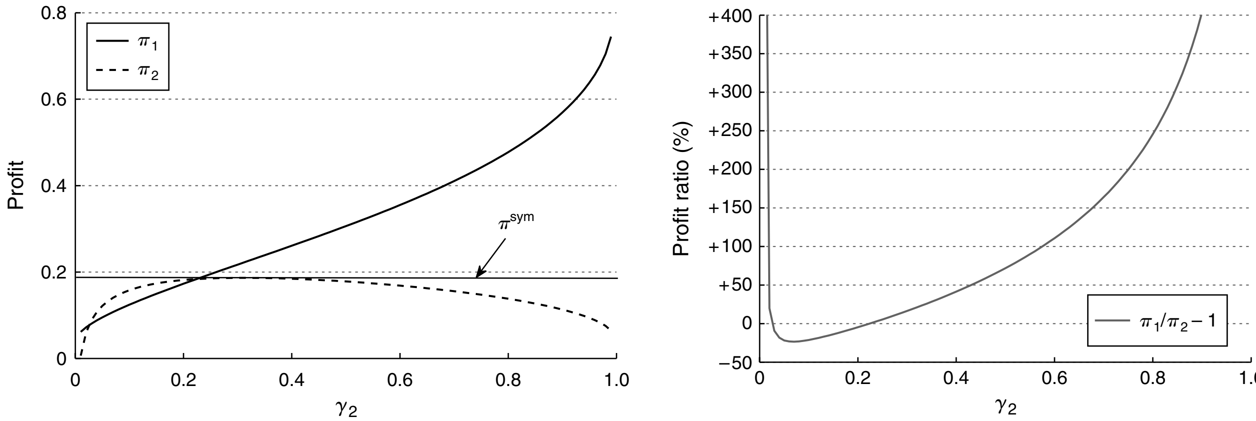
Notes. Here $f_1 = f_2 = 0.1$, $g_1 = g_2 = 0$, $c_1 = c_2 = 1$, $\beta = 0.5$, $W = 1$, $\delta = 0.99$, and $p_0 = \infty$. The curves cross at $\gamma_2 = \gamma^{\text{sym}} = 0.224$.

product outcome in equilibrium for an illustrative instance. From Equation (8), firm 2 prices its product at $p^{\text{CDS}} = 3$, which corresponds to the horizontal line in the top graph. In the bottom graph, firm 2's product satiation is represented by the 45 degree line. Note that in both graphs the curves cross at $\gamma_2 = \gamma^{\text{sym}} = 0.224$, where γ^{sym} is computed from Equation (12). The fact that firm 1's best response is very flat means that selecting γ^{sym} is a safe bet for a firm that internalizes satiation. In other words, it can choose its satiation level without worrying too much about whether the competitor will react strategically.

Figure 4 shows the firm profits for the same problem parameters as in Figure 3. The left-hand graph shows the absolute profits π_1, π_2. As a reference, we include an horizontal line with the symmetric profit $\pi^{\text{sym}} = 0.184$, which corresponds to the profit the firms would make if both were strategic in price and product. Without an outside good, both firms achieve π^{sym} when $\gamma_2 = \gamma^{\text{sym}}$ because the price in the symmetric game p^{sym} is equal to p^{CDS}; see §4.3.1. For $\gamma_2 < \gamma^{\text{sym}}$, both firms have profits lower than π^{sym}, meaning that both are worse off compared to the fully strategic game. However, when γ_2 is not too low, firm 2 does better than firm 1. Indeed, by being myopic, firm 2 commits to $\gamma_2 < \gamma^{\text{sym}}$. That forces firm 1 to lower its price, which undermines its profits.

Interestingly, when γ_2 is slightly greater than γ^{sym}, firm 2 can have profits (slightly) greater than π^{sym}. For instance, for $\gamma_2 = 0.313$ we have that π_2 is 1.6% higher than π^{sym}. In other words, there is a small range where firm 2 benefits from committing (myopically) to the price from the CDS model. The right-hand graph in Figure 4 plots the profit ratio $\pi_1/\pi_2 - 1$, which represents firm 1's profit advantage of having a competitor that ignores satiation. The graph shows

Figure 4 Absolute (Left) and Relative (Right) Profits When Firm 1 Optimizes Both Dimensions While Firm 2 Ignores Satiation and Follows the CDS Model



Notes. Parameter values are the same as in Figure 3. The horizontal line in the left-hand graph corresponds to $\pi^{sym} = 0.184$.

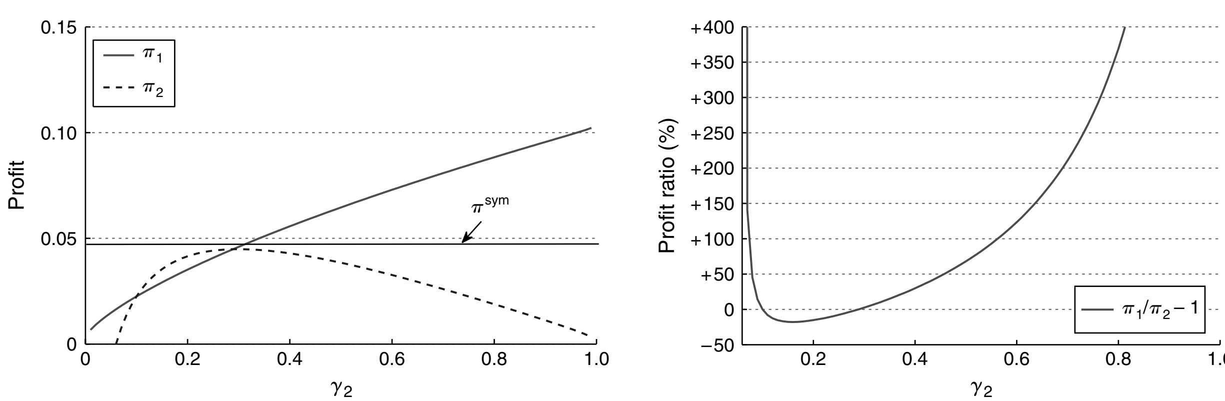
that despite the price and product commitments by firm 2, it is firm 1 that benefits the most and by a large amount. In particular, firm 1 substantially outperforms firm 2 when the satiation level γ_2 is high. This in part explains the situation of apparel retailer the Gap, which has halved its market share from 2003 because of its "dull" clothes, whereas fast-fashion retailers like H&M and Forever 21 have thrived during the same time period (Peterson 2011). Firm 1 in Figure 4 also does better when γ_2 is extremely low, but this is because of our assumption that offering minimal satiation is prohibitively expensive from an operational standpoint; see Equation (9).

Figures 3 and 4 do not consider an outside good ($p_0 = \infty$). When $p_0 < \infty$, the presence of an outside good intensifies competition and both firms must lower their prices. In terms of product, the best response of firm 1 is rotated clockwise, so its product satiation is shifted upward for low values of γ_2 and downward for high values. Note that with an outside good, $p^{CDS} \neq p^{sym}$ and any benefits firm 2 could reap from its price commitment tend to disappear. This is shown in Figure 5 where we plot the same graphs as in Figure 4 but with $p_0 = 5$. First, observe that the profits are lower for both firms. Second, firm 2 is worse off compared to the symmetric equilibrium for all values of γ_2, whereas firm 1 is still better off for a wide range. Third, the profit ratio in the right-hand graph continues to show a large and wide profit advantage for firm 1. In summary, ignoring satiation effects can be even more detrimental under the presence of an outside good.

5. Conclusions

In this paper, we have studied the impact of satiation effects on retail competition. When a customer becomes satiated, she spends a larger fraction of her budget on products that are "fresher," i.e., that generate less satiation. Using a discounted utility approach that accounts for satiation, we find how a consumer should optimally spend her budget among a set of competing retailers. This provides a relatively simple

Figure 5 Absolute (Left) and Relative (Right) Profits When There Is an Outside Good and Firm 1 Optimizes Both Dimensions While Firm 2 Ignores Satiation and Follows the CDS Model



Notes. Parameter values are the same as in Figure 3 except for $p_0 = 5$. The horizontal line in the left-hand graph corresponds to $\pi^{sym} = 0.047$.

demand function with which price and product competition can be analyzed. We show that a competitive equilibrium exists and we characterize it for the symmetric oligopoly and the asymmetric duopoly case.

We consider our work to be a first step in adding behavioral aspects in competitive models. We incorporate satiation effects that induce a consumer to diversify consumption and that provide an alternative explanation for the coexistence of multiple firms engaged in horizontal competition. We also show that ignoring the product dimension in competition may lead to wrong pricing decisions and a large profit reduction. To charge a higher price or to attract a larger market share, the retailer should strive to offer a product that satiates less, which in turn justifies investments in operational improvements to keep costs under control. This illustrates how product decisions are beneficial to retailers that have developed a cost advantage to refresh their stores more often, as in the case of fast-fashion retailers like Zara or H&M.

The model developed here can be extended in a number of directions. First, we focus here on establishing what satiation factor firms should offer that corresponds to selecting the appropriate product. Of course, this is a simplified, high-level view of product choice. In particular, we do not try to explain the mechanisms by which firms can affect the satiation factor. It would thus be interesting to explore how satiation factors are determined, which may depend on industry-specific factors. For example, in apparel, satiation is likely to depend on the customer perception of store "freshness." This can be driven by store or product assortment changes. It would helpful to understand how each operational choice (frequency of product introductions, duration of a product in store, etc.) affects satiation and provide recommendations on how retailers should shape store operations to minimize satiation and hence maximize sales. Second, the market share model derived from the consumer's utility maximization problem is deterministic. In practice sales are affected by demand uncertainty, and hence there may be costs associated with managing it (e.g., safety stock). Including them will introduce scale economies in our model, thereby affecting competitive outcomes. How these change price and product decisions is an interesting question to be addressed. Third, the demand model derived from customer's maximization depends on two competitive decisions: price and satiation factor. In practice, other elements impact the customer's choice, such as habituation (decision analysts have shown) or the vertical differentiation dimension of product choice, in which some products might be perceived as having better quality than others. A relevant question is how these other behavioral elements can be incorporated into retail competition models.

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Appendix. Proofs

Proof of Theorem 1
From the assumptions, $V(y)$ is concave. This can be verified by induction and a limiting argument on T. Hence, the Karush–Kuhn–Tucker conditions are sufficient for optimality in (SP-R). Therefore, if (x_∞, y_∞) is an optimal solution of (SP-R), it must satisfy

$$v_i'(z_{i\infty}) + \delta\gamma_i \frac{\partial V}{\partial y_i}(\gamma \cdot z_\infty) = \mu p_i \quad \text{for } i = 0, \ldots, n, \qquad (16)$$

where μ ensures that $\sum_{i=0}^{n} p_i z_{i\infty} = W + \sum_{i=0}^{n} p_i y_{i\infty}$. In other words, μ ensures that the budget is binding. From the envelop theorem applied to (SP-R) we have that $(\partial V/\partial y_i)(\gamma \cdot z_\infty) = \mu p_i$. Substituting this in (16) and rearranging terms yields Equation (4). Because (4) has a unique solution, the optimum of (SP-R) is unique. Moreover, from Proposition 1 we have that the optimal policy in the customer's problem (CP) must converge to the unique optimum of (SP-R). Therefore, the optimal stationary solution of (CP) is given by Equation (4) as well. The relationships $y_{it+1} = \gamma_i z_{it}$ and $x_{it} = z_{it} - y_{it}$ yield the expressions for $y_{i\infty}$ and $x_{i\infty}$, respectively, which completes the proof. □

Proof of Theorem 2
To prove existence and uniqueness of equilibrium, we first calculate each retailer's best-response function. Since $(1/\pi_i)\partial\pi_i/\partial p_i = (1/p_i)[c_i/(p_i - c_i) - (1/\beta - 1)(1 - m_{i\infty})]$, and the expression in brackets is decreasing in p_i, π_i is first increasing and then decreasing. Hence, π_i is quasi-concave in p_i and continuous in p_{-i}. Existence of equilibrium follows from Theorem 1.2 in Fudenberg and Tirole (1991). Any equilibrium thus satisfies each retailer's optimality condition, i.e., Equation (7).

The equilibrium is in fact unique. Indeed, consider two distinct equilibria $\{p_i^A\}$ and $\{p_i^B\}$, with corresponding attractions $\{a_i^A\}$ and $\{a_i^B\}$ and market shares $\{m_{i\infty}^B\}$ and $\{m_{i\infty}^B\}$. From i's optimality equation, $m_{i\infty}^A \le m_{i\infty}^B$ if and only if $p_i^A \le p_i^B$, which implies $a_i^A \ge a_i^B$. As a result, all retailers that have a larger market share in B also have a smaller attraction, and vice versa. This is a contradiction unless $m_{i\infty}^A = m_{i\infty}^B$, and hence the equilibrium is unique. □

Proof of Theorem 3
First, for $0<\beta<1$, ψ as defined in the theorem can be written as

$$\psi(\gamma_i):=-\frac{b'(\gamma_i)\gamma_i}{b(\gamma_i)}=\frac{1}{1-\gamma_i}-\frac{1}{\beta}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right], \quad (17)$$

and it can be shown to be nonnegative and increasing. The profit function (10) can also be expressed as

$$\pi_i(p_i,\gamma_i,p_{-i},\gamma_{-i}) = \frac{p_i-c_i}{p_i}\frac{W}{1+A_{-i}p_i^{1/\beta-1}/b(\gamma_i)}-\frac{f_i(\gamma_i^{-g_i}-1)}{g_i}, \quad (18)$$

where $A_{-i}:=\sum_{k\neq i}a(p_k,\gamma_k)$. We have that

$$\frac{\partial\pi_i}{\partial\gamma_i}=\frac{\psi(\gamma_i)}{\gamma_i}\left[-\frac{p_i-c_i}{p_i}\frac{WA_{-i}p_i^{1/\beta-1}/b(\gamma_i)}{(1+A_{-i}p_i^{1/\beta-1}/b(\gamma_i))^2}+\frac{f_i\gamma_i^{-g_i}}{\psi(\gamma_i)}\right].$$

Consider γ_i where $\partial\pi_i/\partial\gamma_i=0$, i.e., Equation (11). Since $g_i\geq 0$,

$$\begin{aligned}\frac{\partial^2\pi_i}{\partial\gamma_i^2} &= \frac{\psi(\gamma_i)}{\gamma_i}\left[\frac{p_i-c_i}{p_i}\frac{WA_{-i}p_i^{1/\beta-1}/b(\gamma_i)}{(1+A_{-i}p_i^{1/\beta-1}/b(\gamma_i))^2}\left(-\frac{b'(\gamma_i)}{b(\gamma_i)}\right)\right.\\ &\quad\left.\cdot\left(-1+\frac{2A_{-i}p_i^{1/\beta-1}/b(\gamma_i)}{1+A_{-i}p_i^{1/\beta-1}/b(\gamma_i)}\right)-\frac{f_ig_i\gamma_i^{-1-g_i}}{\psi(\gamma_i)}-\frac{f_i\gamma_i^{-g_i}\psi'(\gamma_i)}{\psi(\gamma_i)^2}\right]\\ &= \frac{\psi(\gamma_i)(p_i-c_i)}{\gamma_ic_i}\frac{WA_{-i}p_i^{1/\beta-1}/b(\gamma_i)}{(1+A_{-i}p_i^{1/\beta-1}/b(\gamma_i))^2}\\ &\quad\cdot\left[\left(-\frac{b'(\gamma_i)}{b(\gamma_i)}\right)\left(1-\frac{2}{1+A_{-i}p_i^{1/\beta-1}/b(\gamma_i)}\right)-\frac{g_i}{\gamma_i}-\frac{\psi'(\gamma_i)}{\psi(\gamma_i)}\right]\\ &\leq \frac{(p_i-c_i)}{\gamma_i^2c_i}\frac{WA_{-i}p_i^{1/\beta-1}/b(\gamma_i)}{(1+A_{-i}p_i^{1/\beta-1}/b(\gamma_i))^2}[\psi(\gamma_i)^2-\gamma_i\psi'(\gamma_i)].\end{aligned}$$

Since

$$\psi(\gamma_i)^2=\left(\frac{1}{1-\gamma_i}\right)^2+\frac{1}{\beta^2}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right]^2-\frac{2}{\beta(1-\gamma_i)}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right]$$

and

$$\begin{aligned}\gamma_i\psi'(\gamma_i) &= \frac{\gamma_i}{(1-\gamma_i)^2}-\frac{1}{\beta}\left[\frac{\delta\gamma_i}{(1-\delta\gamma_i)^2}-\frac{(1-\beta)^2\delta\gamma_i^{1-\beta}}{(1-\delta\gamma_i^{1-\beta})^2}\right]\\ &= -\frac{1}{1-\gamma_i}+\left(\frac{1}{1-\gamma_i}\right)^2+\frac{1}{\beta}\left[\frac{1}{1-\delta\gamma_i}-\frac{(1-\beta)^2}{1-\delta\gamma_i^{1-\beta}}\right]\\ &\quad+\frac{1}{\beta}\left[\left(\frac{1}{1-\delta\gamma_i}\right)^2-\left(\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right)^2\right],\end{aligned}$$

we have

$$\begin{aligned}\psi(\gamma_i)^2-\gamma_i\psi'(\gamma_i) &= \frac{1}{\beta^2}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right]^2\\ &\quad-\frac{2}{\beta(1-\gamma_i)}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}-\frac{\beta}{2}\right]\\ &\quad-\frac{1}{\beta}\left[\frac{1}{1-\delta\gamma_i}-\frac{(1-\beta)^2}{1-\delta\gamma_i^{1-\beta}}\right]\\ &\quad-\frac{1}{\beta}\left[\left(\frac{1}{1-\delta\gamma_i}\right)^2-\left(\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right)^2\right]\leq 0.\end{aligned}$$

This is true because $(1/\beta)(1/(1-\delta\gamma_i)-(1-\beta)/(1-\delta\gamma_i^{1-\beta}))$ is increasing from 1 to $1/(1-\delta)$, and hence always larger than $1/2$, and because

$$\begin{aligned}&\frac{1}{1-\delta\gamma_i}+\frac{(1-\beta)^2}{1-\delta\gamma_i^{1-\beta}}+\left(\frac{1}{1-\delta\gamma_i}\right)^2-\left(\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right)^2\\ &\quad\geq\frac{1}{\beta}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right]\left[\beta+\frac{\beta}{1-\delta\gamma_i}+\frac{\beta-\beta^2}{1-\delta\gamma_i^{1-\beta}}\right]\\ &\quad\geq\frac{1}{\beta}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right]\left[\beta+\frac{\beta}{1-\delta\gamma_i}+\frac{\beta-\beta^2}{1-\delta\gamma_i^{1-\beta}}\right]\\ &\quad\geq\frac{1}{\beta}\left[\frac{1}{1-\delta\gamma_i}-\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\right]^2\end{aligned}$$

since

$$\beta+\frac{1-\beta^2}{1-\delta\gamma_i^{1-\beta}}\geq\beta+\frac{1-\beta}{1-\delta\gamma_i^{1-\beta}}\geq\frac{1-\beta}{1-\delta\gamma_i}.$$

As a result, $\partial^2\pi_i/\partial\gamma_i^2$ is negative, and hence the profit function of retailer i has a unique maximizer. It is continuous on γ_{-i}, which guarantees existence of equilibrium in pure strategies.

Finally, to provide a condition for uniqueness, we notice that Equation (11) relates $m_{i\infty}$ and γ_i: γ_i is increasing in $m_{i\infty}$ when $m_{i\infty}\leq 1/2$, and is decreasing otherwise. Hence, if $m_{i\infty}\leq 1/2$, then $A:=\sum_{k=0}^n a(p_k,\gamma_k)$ is decreasing in $m_{i\infty}$. Assume that there are two distinct equilibria, (p_k^1,γ_k^1) and (p_k^2,γ_k^2), which result in market shares $(m_{1\infty}^1,\dots,m_{n\infty}^1)$ and $(m_{1\infty}^2,\dots,m_{n\infty}^2)$. If for $i=1,\dots,n$, $m_{i\infty}^1\leq 1/2$, then $m_{1\infty}^1<m_{1\infty}^2$ implies that $A^1=\sum_{k=0}^n a(p_k^1,\gamma_k^1)>A^2=\sum_{k=0}^n a(p_k^2,\gamma_k^2)$. This implies that for i $m_{i\infty}^1<m_{i\infty}^2$. However, $1-\sum_{k=1}^n m_{k\infty}^1=m_{0\infty}^1=a(p_0,0)/A^1<a(p_0,0)/A^2=m_{0\infty}^2=1-\sum_{k=1}^n m_{k\infty}^2<1-\sum_{k=1}^n m_{k\infty}^1$. This is a contradiction. The same argument holds when $m_{1\infty}^1>m_{1\infty}^2$. Hence, the equilibrium must be unique when $m_{i\infty}\leq 1/2$ for $i=1,\dots,n$. This occurs for example when p_0 is low enough, in which case $m_{0\infty}\geq 1/2$. □

Proof of Proposition 2
When $\beta=0$, customers are indifferent to satiation and hence the firms engage in price (Bertrand) competition; therefore, the best response is to set $p_i=\max\{c_i,\min_{j\neq i}p_j\}$ (zero gross margin unless the retailer is the lowest cost provider) and $\gamma_i=1$ (no effort in reducing satiation). This is also the unique solution to Equations (7) and (11).

For $0<\beta<1$, consider the profit expression from Equation (18). We start by maximizing π_i with respect to p_i. This implies that Equation (7) is satisfied, given γ_i. This first-order condition can be rewritten as

$$\frac{(1-\beta)p_i^{1/\beta-1}}{\beta}\left(\frac{p_i}{c_i}-\frac{1}{1-\beta}\right)=\frac{b(\gamma_i)}{A_{-i}}. \quad (19)$$

Denote $\hat{p}_i(\gamma_i)$ the solution to Equation (19) given γ_i and A_{-i}. One can observe that $\hat{p}_i(\gamma_i)\geq c_i/(1-\beta)$. It can be verified that b is nonincreasing in γ_i, and hence $\hat{p}_i(\gamma_i)$ is nonincreasing in γ_i. Thus, lower satiation allows the retailer to charge higher prices. Furthermore, implicit differentiation of Equation (19) implies

$$\left(\frac{1-\beta}{\beta\hat{p}_i}+\frac{1}{\hat{p}_i-(c_i/(1-\beta))}\right)\hat{p}_i'=\frac{b'(\gamma_i)}{b(\gamma_i)}=\frac{-\psi(\gamma_i)}{\gamma_i}\le 0.$$

Using the one-to-one relationship between satiation factor and optimal price $\hat{p}_i$, we can now maximize the profit with respect to γ_i. Let

$$\begin{aligned}\hat{\pi}_i(\gamma_i)&=\pi_i(\hat{p}_i(\gamma_i),\gamma_i,p_{-i},\gamma_{-i})\\&=W\left(1-\frac{c_i}{(1-\beta)\hat{p}_i}\right)-\frac{f_i(\gamma_i^{-g_i}-1)}{g_i},\end{aligned}$$

and hence

$$\begin{aligned}\frac{d\hat{\pi}_i}{d\gamma_i}&=\frac{Wc_i\hat{p}_i'}{(1-\beta)\hat{p}_i^2}+\frac{f_i}{\gamma_i^{1+g_i}}=-\frac{1}{\gamma_i}\left(\frac{Wc_i\gamma_i(-\hat{p}_i')}{(1-\beta)\hat{p}_i^2}-\frac{f_i}{\gamma_i^{g_i}}\right)\\&=-\frac{1}{\gamma_i}\left(\frac{Wc_i\psi(\gamma_i)}{(1-\beta)[(1-\beta)/(\beta\hat{p}_i)+1/(\hat{p}_i-c_i/(1-\beta))]\hat{p}_i^2}-\frac{f_i}{\gamma_i^{g_i}}\right)\end{aligned}\tag{20}$$

Interestingly,

$$\frac{\psi(\gamma_i)}{[(1-\beta)/(\beta\hat{p}_i)+1/(\hat{p}_i-c_i/(1-\beta))]\hat{p}_i^2}$$

is nondecreasing in γ_i, because $\hat{p}_i$ is nonincreasing and

$$\frac{\psi}{(1-\beta)/(\beta\hat{p}_i)+1/(\hat{p}_i-c_i/(1-\beta))}$$

is nondecreasing in γ_i. This is indeed true because some algebra yields that

$$\begin{aligned}&\frac{\gamma_i[(1-\beta)/(\beta\hat{p}_i)+1/(\hat{p}_i-c_i/(1-\beta))]}{\psi^2}\\&\cdot\frac{d}{d\gamma_i}\left(\frac{\psi_i(\gamma_i)}{(1-\beta)/(\beta\hat{p}_i)+1/(\hat{p}_i-c_i/(1-\beta))}\right)\\&=\frac{\gamma_i(1/(1-\gamma_i)^2-(\delta/\beta)[1/(1-\delta\gamma_i)^2-(1-\beta)^2\gamma_i^{-\beta}/(1-\delta\gamma_i^{1-\beta})^2])}{(1/(1-\gamma_i)-(1/\beta)[1/(1-\delta\gamma_i)-(1-\beta)/(1-\delta\gamma_i^{1-\beta})])^2}\\&\quad-\frac{(1-\beta)/(\beta\hat{p}_i^2)+1/(\hat{p}_i-c_i/(1-\beta))^2}{[(1-\beta)/(\beta\hat{p}_i)+1/(\hat{p}_i-c_i/(1-\beta))]^2}.\end{aligned}$$

A simple functional analysis shows that

$$\begin{aligned}&[(1-\beta)/(\beta\hat{p}_i^2)+1/(\hat{p}_i-c_i/(1-\beta))^2]\\&\quad/[(1-\beta)/(\beta\hat{p}_i)+1/(\hat{p}_i-c_i/(1-\beta))]^2\le 1\end{aligned}$$

for all $\hat{p}_i$ and all parameters and that

$$\begin{aligned}&(\gamma_i(1/(1-\gamma_i)^2)-(\delta/\beta)\\&\cdot[1/(1-\delta\gamma_i)^2-(1-\beta)^2\gamma_i^{-\beta}/(1-\delta\gamma_i^{1-\beta})^2])/(1/(1-\gamma_i)-(1/\beta)\\&\quad\cdot[1/(-\delta\gamma_i)-(1-\beta)/(1-\delta\gamma_i^{1-\beta})])^2\ge 1\end{aligned}$$

for all γ_i and all parameters.

As a result, $\hat{\pi}_i$ is first increasing and then decreasing, i.e., quasi-concave. As a result, the maximizer of π_i is unique and can be defined by the two first-order conditions. Using Equation (19), the first-order condition in Equation (20) is equivalent to

$$\frac{\beta W}{1-\beta}\frac{c_i(p_i-c_i/(1-\beta))}{p_i(p_i-c_i)}=-\frac{b(\gamma_i)f_i}{b'(\gamma_i)\gamma_i^{1+g_i}}\ge 0.\tag{21}$$

As A_{-i} increases, Equation (21) is unchanged, but the right-hand side of Equation (19) decreases. Note that Equation (21) provides a relationship where γ_i is first decreasing in p_i and then increasing in p_i. This is true because $1/(\psi(\gamma_i)\gamma_i^{g_i})$, where ψ is defined in Equation (17), is decreasing and $c_i(p_i-c_i/(1-\beta))(p_i(p_i-c_i))$ is first increasing and then decreasing. As a result, when A_{-i} is small, p_i^{BR} increases and γ_i^{BR} decreases with A_{-i}; when A_{-i} is large, both p_i^{BR} and γ_i^{BR} increase with A_{-i}. The sensitivity with respect to p_k, γ_k is derived using that A_{-i} is decreasing in p_k and γ_k. □

Proof of Theorem 4

For all i, we know that Equation (21) leads to a best-response γ_i that is bounded below by a positive value $\gamma^{\min}>0$, which is independent of A_{-i}. Consider the price equilibrium of the price-only game in which all retailers set $\gamma_i=\gamma^{\min}$. Let $p^{\max}<\infty$ be the maximum price in this equilibrium across all retailers.

Given $\gamma^{\min}$ and $p^{\max}$, we claim that if all competitors choose $(\gamma_i,p_i)\in[\gamma^{\min},1]\times[0,p^{\max}]$, then i will choose a best response in this set, too. Indeed, when competitors choose such strategies, $A_{-i}\ge(n-1)b(\gamma^{\min})(p^{\max})^{1-1/\beta}+p_0^{1-1/\beta}>0$. We know from the construction of $\gamma^{\min}$ that i will choose γ_i higher than $\gamma^{\min}$. We also know that retailer i will set a price p_i lower than $p^{\max}$ because the solution p_i of Equation (19) decreases in γ_i and in A_{-i}, and $p^{\max}$ was constructed so as to be the equilibrium in which all players set $\gamma^{\min}\le\gamma_i$.

Consider the multivariate real function F: $\Sigma\to\Sigma$, where $\Sigma\equiv([0,p^{\max}]\times[\gamma^{\min},1])^n$, and for $\sigma\in\Sigma$, the ith component of $F(\sigma)$ is equal to (p_i^{BR},γ_i^{BR}). Because Σ is compact and F is continuous (from Proposition 2), Brouwer's fixed point theorem implies that F must have a fixed point. This fixed point is a pure-strategy Nash equilibrium.

We can now prove uniqueness. The equilibrium conditions from Equations (7) and (11) provide a one-to-one mapping between $m_{i\infty}$ and (p_i,γ_i). Omitting the subindices,

$$\frac{p}{c}=1+\frac{\beta}{(1-\beta)(1-m)}$$

and

$$\psi(\gamma)\gamma^g=\frac{f}{W}\frac{1}{m}\left(\frac{1-\beta}{\beta}+\frac{1}{1-m}\right).$$

Hence,

$$\frac{1}{p}\frac{dp}{dm}=\frac{1/(1-m)^2}{(1-\beta)/\beta+1/(1-m)}$$

and

$$\frac{d\gamma}{dm}\left(\frac{\psi'}{\psi}+\frac{g}{\gamma}\right)=-\frac{1}{m}+\frac{1/(1-m)^2}{(1-\beta)/\beta+1/(1-m)}.$$

Note that p is increasing in m and γ is first decreasing and then increasing in m. Also, $A:=\sum_{k=0}^n a(p_k,\gamma_k)=p^{1-(1/\beta)}b(\gamma)/m$ is decreasing in m because

$$\begin{aligned}&-\frac{1-\beta}{\beta}\frac{1}{p}\frac{dp}{dm}+\frac{b'(\gamma)}{b(\gamma)}\frac{d\gamma}{dm}-\frac{1}{m}=-\frac{1-\beta}{\beta}\frac{1/(1-m)^2}{(1-\beta)/\beta+1/(1-m)}\\&-\frac{-1/m+(1/(1-m)^2)/((1-\beta)/\beta+1/(1-m))}{\gamma\psi'/\psi^2+g/\psi}-\frac{1}{m}<0.\end{aligned}$$

This is negative because $\gamma\psi'/\psi^2 + g/\psi \geq \gamma\psi'/\psi^2 \geq 1$ (see the proof of Theorem 3). Hence, A is decreasing in $m_{i\infty}$.

Assume that there are two distinct equilibria (p_k^1, γ_k^1) and (p_k^2, γ_k^2), which result in market shares $(m_{1\infty}^1, \ldots, m_{n\infty}^1)$ and $(m_{1\infty}^2, \ldots, m_{n\infty}^2)$. Let $A^1 = \sum_{k=0}^n a(p_k^1, \gamma_k^1)$ and $A^2 = \sum_{k=0}^n a(p_k^2, \gamma_k^2)$. If $m_{1\infty}^1 < m_{1\infty}^2$, then we know that $A^1 > A^2$ and hence for all k, $m_{k\infty}^1 < m_{k\infty}^2$. However, $1 - \sum_{k=1}^n m_{k\infty}^1 = m_{0\infty}^1 = a(p_0, 0)/A^1 < a(p_0, 0)/A^2 = m_{0\infty}^2 = 1 - \sum_{k=1}^n m_{k\infty}^2 < 1 - \sum_{k=1}^n m_{k\infty}^1$. This is a contradiction and hence the equilibrium must be unique. □

Proof of Theorem 5

From Equations (19) and (21), in a symmetric equilibrium (p, γ), we must have that

$$\begin{aligned}\frac{(1-\beta)p^{1/\beta-1}}{\beta}\left(\frac{p}{c} - \frac{1}{1-\beta}\right) &= \frac{b(\gamma)}{p_0^{1-1/\beta} + (n-1)p^{1-1/\beta}b(\gamma)} \\ &= \frac{p^{1/\beta-1}}{(p^{1/\beta-1}/p_0^{1/\beta-1})(1/b(\gamma)) + (n-1)} \quad \text{and} \\ \frac{c(p-c/(1-\beta))}{p(p-c)}\,\frac{\beta\psi(\gamma)\gamma^g}{1-\beta} &= \frac{f}{W}. \end{aligned} \tag{22}$$

We claim that these two equations have a unique solution. Define $Z = (1-\beta)p/c - 1$ and $Z_0 = (1-\beta)p_0/c - 1$. The equations above can be expressed as

$$\frac{Z}{\beta}\left[\left(\frac{Z+1}{Z_0+1}\right)^{1/\beta-1}\frac{1}{b(\gamma)} + (n-1)\right] = 1 \quad \text{and}$$
$$\frac{\beta Z\psi(\gamma)\gamma^g}{(Z+1)(Z+\beta)} = \frac{f}{W}. \tag{23}$$

These two equations clearly define a unique γ for each value of $0 \leq Z \leq Z_{\max}$, where $Z_{\max}$ is defined by $(Z_{\max}/\beta)[((Z_{\max}+1)/(Z_0+1))^{1/\beta-1} + (n-1)] = 1$. Denote these values of γ as $\gamma^1(Z)$ and $\gamma^2(Z)$, solutions to the first and second equation in (23), respectively. At equilibrium, $\gamma^1(Z) = \gamma^2(Z)$. We claim that this only occurs for one value of $Z > 0$, provided that $W/f > 1/(\beta(Z_0+1)^{1/\beta-1})$ (when the cost is low enough); otherwise, the only equilibrium is $Z = 0$, $\gamma = 1$, which implies that each retailer makes zero profit.

Indeed, for $Z \approx 0$, a Taylor approximation implies that

$$\frac{\beta(1-\gamma^1(Z))(Z_0+1)^{1/\beta-1}}{Z} \xrightarrow[Z\to 0]{} 1 \quad \text{and}$$

$$\frac{f(1-\gamma^2(Z))}{ZW} \xrightarrow[Z\to 0]{} 1$$

and hence $\gamma^1(Z) > \gamma^2(Z)$ when $W/f > 1/(\beta(Z_0+1)^{1/\beta-1})$. If Z becomes high enough, $\gamma^1(Z)$ reaches zero, and $\gamma^2(Z)$ is increasing in Z and approaches one. As a result, $\gamma^2(Z) > \gamma^1(Z)$ for a large enough Z. This implies that there must be at least one solution to (23). Applying Theorem 4 yields that the unique equilibrium is symmetric. Let $\lambda = n - 1 + (p/p_0)^{1/\beta-1}1/b(\gamma)$ and rearrange the terms in (22) to obtain Equations (13) and (14), which completes the proof. □

Proof of Theorem 6

Consider $f_2 > f_1$ and $g_2 \geq g_1$. In the equilibrium, assume first that $p_1 \leq p_2$. From Equation (7), we must have that $m_{1\infty} \leq m_{2\infty}$. Hence, $p_1^{1-1/\beta}b(\gamma_1) \leq p_2^{1-1/\beta}b(\gamma_2)$ and thus $\gamma_1 \geq \gamma_2$. At the same time, letting $A = \sum_{k=0}^2 a(p_k, \gamma_k)$, Equation (11) (using that $(p_i - c_i)(1 - m_{i\infty})$ is constant) implies that $p_1^{-1/\beta}b(\gamma_1)\psi(\gamma_1)\gamma_1^{g_1} = (1-\beta)f_1A/\beta W < (1-\beta)\cdot f_2A/(\beta W) = p_2^{-1/\beta}b(\gamma_2)\psi(\gamma_2)\gamma_2^{g_2}$. But because $b(\gamma)\psi(\gamma)\gamma^{g_i}$ is nondecreasing in γ (this follows from the inequality $-((\psi'/\psi)/(b'/b))(\gamma) \geq 1$; see the proof of Theorem 5), this implies that $\gamma_1 < \gamma_2$, a contradiction. Hence, in equilibrium $p_1 > p_2$ and $\gamma_1 < \gamma_2$. Similarly, if $f_2 \geq f_1$ and $g_2 > g_1$, the same argument would yield that $p_1^{-1/\beta}b(\gamma_1)\psi(\gamma_1)\gamma_1^{g_1} \leq p_2^{-1/\beta}b(\gamma_2)\psi(\gamma_2)\gamma_2^{g_1} < p_2^{-1/\beta}b(\gamma_2)\psi(\gamma_2)\gamma_2^{g_2}$ and result again in a contradiction.

Finally, note that increasing f_1 or g_1 results in a larger γ_1, p_2 and a smaller p_1, γ_2. Using (7), the equilibrium profits can be rewritten as

$$\pi_i = W\left(1 - \frac{c}{(1-\beta)p_i}\right) - \frac{f_i(\gamma_i^{-g_i} - 1)}{g_i}.$$

Then, we have that

$$\begin{aligned}\frac{d\pi_1}{df_1} &= \frac{\partial\pi_1}{\partial p_1}\frac{dp_1}{df_1} + \frac{\partial\pi_1}{\partial\gamma_1}\frac{d\gamma_1}{df_1} - \frac{\partial k_1}{\partial f_1} \leq 0,\\ \frac{d\pi_1}{dg_1} &= \frac{\partial\pi_1}{\partial p_1}\frac{dp_1}{dg_1} + \frac{\partial\pi_1}{\partial\gamma_1}\frac{d\gamma_1}{dg_1} - \frac{\partial k_1}{\partial g_1} \leq 0,\\ \frac{d\pi_1}{df_2} &= \frac{\partial\pi_1}{\partial p_1}\frac{dp_1}{df_2} + \frac{\partial\pi_1}{\partial\gamma_1}\frac{d\gamma_1}{df_2} \geq 0,\\ \frac{d\pi_1}{dg_2} &= \frac{\partial\pi_1}{\partial p_1}\frac{dp_1}{dg_2} + \frac{\partial\pi_1}{\partial\gamma_1}\frac{d\gamma_1}{dg_2} \geq 0.\end{aligned}$$

Therefore, π_1 increases with f_2, g_2 and decreases with f_1, g_1. A similar analysis shows that π_2 increases with f_1, g_1 and decreases with f_2, g_2, which completes the proof. □

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