Double Counting in Supply Chain
Carbon Footprinting

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Carbon footprinting is a tool for firms to determine the total greenhouse gas (GHG) emissions associated with their supply chain or with a unit of final product or service. Carbon footprinting typically aims to identify where best to invest in emission reduction efforts, and/or to determine the proportion of total emissions that an individual firm is accountable for, whether financially and/or operationally. A major and underrecognized challenge in determining the appropriate allocation stems from the high degree to which GHG emissions are the result of joint efforts by multiple firms. We introduce a simple but general model of joint production of GHG emissions in general supply chains, decomposing the total footprint into processes, each of which can be influenced by any combination of firms. We analyze two main scenarios. In the first scenario, the social planner allocates emissions to individual firms and imposes a cost on them (such as a carbon tax) in proportion to the emissions allocated. In the second scenario, a carbon leader voluntarily agrees to offset all emissions in the entire supply chain and then contracts with individual firms to recoup (part of) the costs of those offsets. In both cases, we find that, to induce the optimal effort levels, the emissions need to be overallocated, even if the carbon tax is the true social cost of carbon. This is in contrast to the usual focus in the life-cycle assessment (LCA) and carbon footprinting literatures on avoiding double counting. Our work aims to lay the foundation for a framework to integrate the economics- and LCA-based perspectives on supply chain carbon footprinting.

Key words: carbon footprint; supply chain; sustainable operations; emissions allocation

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1. Introduction
As concern over climate change continues to rise, “carbon footprinting” is becoming ever more widespread. This involves estimating the total greenhouse gas (GHG) emissions for a company, a product (with emissions measured over its entire life cycle), or a supply chain (in part or in full). For instance, total GHG emissions in the Coca-Cola system in 2009 were 5.4 million metric tons (Coca-Cola Company 2010), and the carbon footprint of a 330 ml glass bottle of Coca-Cola was 360 g.¹

Beyond estimating their footprint, some firms go further and pledge to reduce emissions. In 2010, Walmart announced a goal to eliminate 20 million metric tons of GHG emissions from its global supply chain by the end of 2015; this represents one and a half times the company’s estimated global carbon footprint growth over the same period.² Tesla has committed to reduce the carbon footprint of the products it sells by 30% by 2020, jointly with its suppliers, and to help customers reduce their own footprints by 50% (Tesco 2012). Unilever announced that it plans to halve the footprint of its products by 2020.³ Yet other firms go further still, pledging to become carbon neutral, which means investing in emission reduction projects elsewhere and having an independent party verify that those projects are sufficient to compensate for (or “offset”) the firm’s residual emissions.⁴

⁴ “Offsetting” emissions means investing in projects that reduce carbon emissions elsewhere, for instance, by paying an offset provider to invest in renewable energy projects that would otherwise not occur, or to plant trees.
Costa Rica will develop a carbon-neutral supply chain for bananas and pineapples from Costa Rica to North America and Europe, in keeping with Costa Rica’s goal to become the first carbon-neutral country by 2030 (McPhaul 2007). Finsbury Green in Australia sells carbon-neutral paper under its FreshZero brand, offsetting all supply chain emissions. Tesco aims to be a zero-carbon business by 2050 (Tesco 2012). Brazil’s Natura Cosméticos offsets not only its own emissions but those of its entire supply chain, which is all the more noteworthy given that the supply chain accounts for 95% of those emissions (Natura 2009). Method, a cleaning products firm, offsets all emissions of its first-tier suppliers; when suppliers reduce their emissions, Method passes the savings from the reduction in offsets needed on to them. Sunstar Hotels is carbon neutral, and two-thirds of the emissions it offsets occur at its suppliers. In other instances, firms might be required to offset part of their supply chain’s emissions, as in the context of a border tariff (Keskin and Plambeck 2011). Finally, many larger firms have to pay for their carbon emissions through cap-and-trade schemes as in the European Union, California, and elsewhere. In this paper, we focus on when such carbon-related payments (whether from carbon taxes, emissions trading, carbon neutrality, or something else) lead to optimal emissions abatement levels.

Many GHG emissions (or reductions in emissions) are the result of joint effort by multiple parties: (i) In the construction sector, advanced framing of roof trusses reduces lumber wasted relative to cutting trusses on site, but the supplier incurs higher engineering costs and the builder incurs various soft costs associated with needing to plan ahead more carefully (NAHB Research Center 2006). (ii) Water-based paints cause less GHG emissions during manufacture and application than solvent-based paints. However, they are more costly to manufacture, and the customer needs to use different application equipment because of the corrosive effects of water-based paints. Both supplier and customer have to agree to incur higher costs to switch from solvent-based to water-based paints (Nayak and Kumar 2006). (iii) Eastman Chemical can deliver its products in a molten state that is less carbon intensive but more costly and requires capacity and demand coordination with customers (see Koomen 2012 and §6, in which we use Eastman Chemical for a numerical illustration).

The Carbon Disclosure Project 2011 supply chain report found that 86% of respondents have a collaborative process in place to jointly reduce carbon footprints with suppliers (up from 49% the year before), but suppliers face difficulties in allocating their emissions to their multiple customers (CDP 2011). A World Economic Forum (2009) report lists opportunities for “supply chain decarbonization,” a number of which require collaboration between customers and suppliers, such as “despeeding the supply chain,” “optimized networks,” and “packaging design initiatives.” Although joint production can occur anywhere, it is likely to be particularly common in indirect goods and services, which do not become part of the final product or service. A CAPS Research (2003) report finds that nine major companies spent between 8% and 68% of their total purchases on indirect goods and services, such as “engineering and manufacturing equipment and services (nonproduction related),” “facilities maintenance,” “industrial supply,” “logistics freight,” “travel,” etc.

Central to footprint reduction efforts is a system for measuring and reporting carbon footprints. Several international standards exist, such as the GHG protocol, ISO 14064, and Publicly Available Specification 2050, which serve slightly different purposes but which are generally consistent with one another, and all of which are closely related to life-cycle assessment (LCA).

A critical question underlying LCA and the carbon accounting standards is who is responsible for the emissions associated with a final product or service? A typical product goes through numerous manufacturing and transportation stages operated by a number of companies in a supply chain. Each firm can invest in reducing emissions in its own operations but can also help lower the emissions at upstream or downstream companies by changing a product’s dimensions, form, flexibility, strength, required storage conditions, durability, etc. Some firms can affect others’ emissions by collaboration, coordination, information sharing, or even simply by leveraging their economic power. A manufacturer sharing advance demand information might smooth its suppliers’ operations and hence reduce the need for fast transportation, resulting in lower emissions.

When a number of firms jointly affect total emissions, they face a challenge in greening their supply chain in the absence of regulation: How should responsibility for emissions be allocated to the various firms to encourage jointly optimal emissions abatement effort? To tackle this question, we introduce a simple but general model of joint production of GHG emissions abatement effort. To tackle this question, we introduce a simple but general model of joint production of GHG emissions abatement effort.

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emissions in which the total footprint of the supply chain is decomposed into separate footprint components corresponding to individual processes, each of which can be influenced by any combination of any number of firms. We examine conditions under which firms exert optimal abatement effort levels. We do this first from the perspective of a social planner that allocates emissions to individual firms and then charges those emissions, and then from the perspective of a “carbon leader,” a firm that (for whatever reason) offsets all supply chain emissions and then recoups (part of) the costs of doing so from its supply chain partners. The social planner and carbon leader cases use the same model, with only two differences. First, the carbon leader offsets emissions at a price $p$, which may be below the societal cost $p^2$. Second, the carbon leader can credibly commit to a given effort level, whereas in the social planner case there is no firm that can do so. One could also view the carbon leader case as a supply chain in which the social planner identifies a firm that can credibly commit, and then charges that firm for all supply chain emissions.

The contribution of this paper is twofold. First is the modeling framework itself, combining carbon footprints with joint production in a supply chain, relaxing the usual (implicit) assumption that carbon footprints are uniquely linked to individual firms. The second contribution lies in the results we derive with this framework. First, we emphasize that although carbon neutrality may be a worthy goal, it does not usually lead to optimal emissions abatement efforts. Second, to induce firms to choose the optimal abatement efforts, double counting of emissions is usually necessary, and optimizing over the carbon price in general does not fully compensate for not being allowed to double count emissions. Third, if there is a credible leader in the supply chain, then the optimal abatement efforts can be achieved by delegating double counting to the (carbon) leader.

2. Background and Literature
This paper draws on and aims to contribute to two main bodies of literature: that on sustainable supply chain, and that on LCA and carbon footprinting in supply chains. We first review the general background and literature on sustainable supply chains in §2.1, then we do the same for LCA and carbon footprinting in §2.2.

2.1. Sustainable Supply Chains
Within operations management, the notion of “greening the supply chain” has been gaining popularity, as illustrated by the reviews in Kleindorfer et al. (2005) and Corbett and Klassen (2006). Kolk and Pinske (2004) provide a framework for ways firms can address climate change, either focusing internally on their own supply chain or going beyond that, and either merely compensating for their emissions or aiming to innovate. Our framework expands on theirs by embedding it in an analytical model in the context of joint carbon production.

The extensive literature review (citing 190 articles) by Seuring and Müssler (2008) suggests that carbon footprinting specifically has hardly been studied in the supply chain field so far. Benjaafar et al. (2013) show how carbon footprint parameters can be added to various optimization models. Cholette and Venkat (2009) find that supply chain configuration can make a major difference in carbon intensity of wine distribution. Hoen et al. (2012) find that plausible regulations and carbon costs will have minimal effect on the efficiency with which a transportation network is managed, assuming that the physical network does not change. Cachon (2011) concludes that the optimal solution to a stylized supply chain design problem is robust to misspecifications in the carbon cost. Keskin and Plambeck (2011) focus on how to allocate emissions from a process among its coproducts. Earlier work by Corbett and DeCroix (2001) and Corbett et al. (2005) studied contracts to provide incentives for both buyer and supplier to reduce consumption of undesirable materials in the presence of joint production. This focus on joint production (though not in the context of green supply chains) is continued in Roels et al. (2010).

We draw on the mechanism design literature on joint production in teams. We generalize Holmstrom’s (1982) result, about the impossibility of achieving first-best without double counting, to the case of multiple processes. Battaglini (2006) shows that, with a sufficient number of processes, first-best can be achieved, but only using noncontinuous and punitive mechanisms. A key difference between our context and other joint production situations is the emergence of third-party verification mechanisms for carbon footprints, which fundamentally expands the set of practically conceivable allocation mechanisms.

We contribute to the literature on sustainable supply chains by developing a framework for joint production (of GHG emissions) in far more general supply chains than the common one-buyer, one-supplier context and by highlighting that even carbon-neutral supply chains do not lead to optimal emissions reductions.

2.2. Life-Cycle Assessment and Carbon Footprinting
LCA is a method for determining total environmental impacts caused by products and processes, from “cradle to grave” (Reap et al. 2008). Although LCA covers all environmental impacts, much attention has recently focused on climate change and hence on “carbon footprinting,” which can be thought of as LCA
limited to GHG emissions. In addition to various forms of carbon trading or other regulations, firms face increasing pressure from organizations such as the CDP (formerly Carbon Disclosure Project, see www.cdproject.net) to disclose their emissions.

In trading schemes, avoiding double counting of GHG emissions is crucial, and therefore the existing literature on LCA focuses heavily on how to allocate (reductions in) emissions to each of the parties separately. For that reason, the GHG protocol separates emissions into Scope 1 (direct, onsite emissions), Scope 2 (indirect, from energy usage), and Scope 3 (other indirect). On average, only 14% of an industry’s emissions are Scope 1, and only 26% are Scopes 1 and 2 combined (Matthews et al. 2008). Using economic input-output LCA, Huang et al. (2009, p. 8513) show how total GHG emissions are divided among Scope 1, Scope 2, and several subcategories within Scope 3 (commuting, top 10 suppliers, and other upstream emissions) and find that the top 10 suppliers can account for 30%-50% of a sector’s total footprint. Walmart estimates that more than 90% of its emissions comes from its supply chain (including end consumers), rather than its own operations (Birchall 2010). For Natura, employee travel causes Scope 3 emissions, which are Scope 1 for the airline used. The same emissions can fall within Scope 3 for multiple companies: the emissions of the manufacturer of the paper in Natura’s catalogues will count toward Scope 3 for both Natura and the catalogue printer. Nevertheless, allocating emissions to firms is difficult because of joint production of emissions. The following quote from WRI and WBCSD (2011, p. 108) illustrates the challenge: “If GHG reductions take on a monetary value or receive credit in a GHG reduction program, companies should avoid double counting of credits from such reductions. To avoid double crediting, companies should specify exclusive ownership of reductions through contractual agreements.” That is, double counting may be considered appropriate as long as there is no direct link between emissions and payments. We argue that double counting can be necessary even when there is such a link, because of joint production.

In the LCA and carbon footprinting literatures, various guidelines exist on how to allocate shared emissions, such as those of a ship, to the products it carries, by weight, volume, or value. The allocation is more challenging when multiple parties share responsibility for those emissions. Given that LCA is aimed at making product and process design decisions based on an accurate inventory of environmental impacts, it is natural that the LCA literature (see, e.g., Lenzen 2008) seeks to avoid double counting of impacts.

More recently the LCA literature has started investigating how to reconcile allocating responsibility for impacts while avoiding double counting. Lenzen et al. (2007), building on Gallego and Lenzen (2005), propose a scheme by which producers and consumers share responsibility for emissions in such a way that adding total emissions across all producers and consumers yields the correct economywide emissions. Our work contributes to this literature by showing that avoiding double counting is desirable for aggregate reporting purposes but not for setting incentives. If the intention is to aggregate emissions across sub-systems to yield a correct estimate of systemwide emissions, then allocating responsibility is not necessary. Allocating responsibility is usually done to design regulatory mechanisms or incentive schemes; we argue that for such applications, whenever joint production is present, avoiding double counting is not desirable.

3. Model

We study a supply chain that is already operating, so our model does not involve the decision of participating or not. The current operation represents the baseline scenario or “business as usual,” in which firms might already be exerting some abatement effort. We assume that firms have exhausted all carbon abatement initiatives that are profitable in the absence of external incentives; the question why firms sometimes fail to implement actions with a positive payback is outside the scope of the paper. We also assume that the abatement efforts, although costly, do not affect the firms’ revenues from their core operations. We do allow firms to make payments contingent on emissions, as in Rayo (2007), but that does not affect the amount of business (in terms of revenue) they do with each other. This assumption is consistent with the joint production examples described in §1, in which the process changes do not affect the quantity produced by the firms. As is common in LCA studies, the scope of what is included in the supply chain can vary depending on the context. One can decide how far upstream and downstream to go, and whether to include raw materials and/or consumers.

3.1. Notation and Model

Matrices are written in boldface, except when all components are equal to a scalar $\alpha$, in which case we just write $\alpha$ to denote a matrix of appropriate dimensions. Inequalities for matrices are componentwise. We use the superscript $T$ to denote transpose. For functions, “increasing” and “decreasing” are understood in the nonstrict sense. Unless noted, all functions are assumed to be differentiable. We let $n \in \mathbb{N} = \{1, \ldots, N\}$ denote the firms and $i \in \mathcal{I} = \{1, \ldots, I\}$ denote the processes, which can be joint across multiple firms.

Our model of joint production follows Battaglini (2006). We consider the realistic structure that the
total footprint emanating from a process has multiple components, each of which can be affected by one or more firms if they exert costly effort. Assume that firm \( n \) can choose any of \( m_n \) alternative carbon abatement actions. Any action in \( \{1, \ldots, m_n\} \) can influence any subset of the set of processes \( \mathcal{I} \). The influence of any given action on process \( i \) can be direct (changing transportation from truck to rail) or indirect (working with a transportation provider to use lighter packaging). The carbon abatement efforts by firm \( n \in \mathcal{N} \) associated with each action \( m \) are given by \( e_n^m = [e_{n,1}^m, \ldots, e_{n,m}^m] \), assumed to be nonnegative and bounded. We refer to firm \( n \)'s profit as \( V_n(e_n) \), which we assume is concave and componentwise decreasing. Since we assume that the abatement efforts do not affect revenues, we can write \( V_n(e_n) = V_n - c_n(e_n) \), where \( V_n \) is a constant and \( c_n(e_n) \) is the carbon abatement cost which is convex and strictly increasing. We define the baseline effort level as equal to zero \( (e_n = 0, \forall n \in \mathcal{N}) \).

Let \( \mathbf{e}^T = [e_1^T, \ldots, e_N^T] \in [0, 1]^M \subset \mathbb{R}^M \) be the effort vector, where \( M = \sum_{n=1}^{N} m_n \) and \( A \) is a sufficiently large bound to guarantee interior solutions to the ensuing optimization problems. The resulting total footprint caused by process \( i \in \mathcal{I} \) as a function of the collective effort \( \mathbf{e} \) exerted by all firms is \( f_i = f_i(\mathbf{e}) \). We assume \( f_i \) to be convex and componentwise decreasing. Let \( \mathbf{f}^T = [f_1, \ldots, f_N] \in \mathbb{R}^N \) be the total footprint vector function. We assume footprints to be nonnegative, though this could be relaxed.

An important feature of our model is a mapping between the firms and the processes (and hence emissions) that they influence. Let \( b_{n,i} \) equal one if and only if firm \( n \) influences the footprint of process \( i \) and zero otherwise. Formally, \( b_{n,i} = 1 \Leftrightarrow \sum_{n=1}^{N} \partial f_i / \partial e_{n,j} < 0 \). Let \( \mathbf{B} \) denote the \( N \times I \) matrix of indicators \( b_{n,i} \). For any process \( i \), \( \sum_{n=1}^{N} b_{n,i} \) represents the number of firms that influence process \( i \). Similarly, for any firm \( n \), the sum \( \sum_{i=1}^{I} b_{n,i} \) represents the number of processes that firm \( n \) influences. Firms that have no influence on any process or that have processes that are not influenced by any firm are irrelevant, so with no loss of generality, we assume that each row and each column of \( \mathbf{B} \) adds up to at least one; i.e., \( \sum_{i=1}^{I} b_{n,i} \geq 1, \forall n \in \mathcal{N} \) and \( \sum_{n=1}^{N} b_{n,i} \geq 1, \forall i \in \mathcal{I} \).

Consider the following simple example, with \( I = 2 \) processes and \( N = 3 \) firms. Assume process 1 is influenced by firms 1 and 2, and process 2 by all three firms. Then

\[
\mathbf{B}^T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.
\]

We will refer back to this simple setting to illustrate several other definitions later. We say that the supply chain exhibits joint carbon production if \( \mathbf{B} \) has at least one column that adds up to more than one, i.e., if there exists \( i^* \in \mathcal{I} \) such that \( \sum_{n=1}^{N} b_{n,i} \geq 2 \). We believe that most, if not all, supply chains exhibit joint carbon production; without that, the footprint allocation problem is far simpler. Note that \( \mathbf{B} \) is invariant under outsourcing decisions or other redrawing of firms’ organizational boundaries, which is one way that our modeling framework is more general than traditional contracting models.

### 3.2. Social First-Best Solution

We model the societal cost of carbon as \( p_s > 0 \) per unit of GHG emissions. “Societal cost” refers to the monetary equivalent of the environmental impact created by the emissions, in line with Bovenberg and Goulder (1996), who show that the optimal (carbon) tax is equal to the marginal damage (unless the tax is redistributed to households, in which case it can be much lower). How to determine \( p_s \) (and whether it is even well defined) is outside the scope of our work. There is a wide range of estimates of \( p_s \) as shown by Tol (2005), whose meta-analysis of 28 estimates found a median of $14 per ton CO\(_2\)e and a mean of $93. This includes Plambeck and Hope’s (1996) estimate of $21 per ton, with a 90% uncertainty range of $10–$48 per ton. Tol (2008) extends this to include 211 estimates. Weitzman (2009) uses probabilistic reasoning to argue that \( p_s \) can be arbitrarily large. As Pearce (2003) argues, it is better to include a parameter even in the presence of large uncertainty than to ignore it altogether.

Let the social value of a supply chain be the sum of firm profits minus the social cost of emissions, i.e., \( \sum_{n=1}^{N} V_n(e_n) - p_s \sum_{i=1}^{I} f_i(\mathbf{e}) \). We do not include the utility consumers derive from whatever the supply chain produces because of our assumption that output does not change. We define the social first-best as the effort levels that maximize the social value of the supply chain, obtained by solving the following concave problem:

\[
 z^s = \max_{\mathbf{e}} \sum_{n=1}^{N} V_n(e_n) - p_s \sum_{i=1}^{I} f_i(\mathbf{e}). \tag{1}
\]

Let \( \mathbf{e}^* \) denote the effort levels that solve (1), which for simplicity we assume are unique. Let \( \mathbf{f}^* \) denote the first-best emissions, i.e., \( f_i^* = f_i(\mathbf{e}^*), \forall i \in \mathcal{I} \).

### 3.3. Carbon-Based Payments

The social first-best is the result of optimizing directly over all firms’ effort levels. In most practical settings, supply chain decisions are decentralized and firms’ efforts can only be controlled indirectly by setting appropriate incentives, such as through carbon-based payments. In this section, we model how firms choose their effort levels when such a payment scheme exists, whether imposed by a social planner as part of a regulatory program or initiated by a carbon leader as part.
of a voluntary effort. In §§4 and 5, we characterize the optimal carbon-based payment schemes from the perspective of a social planner and of a carbon leader, respectively.

We assume there exists a carbon price \( p > 0 \), which could be the social cost of carbon \( p^s \), an arbitrary carbon tax, the cost of permits in a cap-and-trade system, or the cost of carbon offsets. The cost of emissions for the whole supply chain is \( p \sum_{i=1}^{n} f_i \). Let \( h_n(f) \) be the total carbon-based payment made by firm \( n \) to the social planner—if it exists—and to the other firms, which may also be negative if the firm receives incentives instead. We write \( h_n \) as a function of footprint and not effort because footprints can be audited, whereas efforts usually cannot be verified and therefore are not contractible. If the carbon-based payment rule is linear, then \( h = pA f + k \), where \( A \) is an \( N \times 1 \) matrix with elements \( a_{n,i} \), and \( k \) is a constant \( N \)-vector. This is how carbon footprinting is thought of in practice, but for greater generality we allow for nonlinear payment rules. A carbon-based payment rule \( h_n \) satisfies footprint balance if the aggregate marginal payment is equal to \( p \) for all processes; that is,

\[
\text{footprint balance: } \sum_{i=1}^{N} \frac{\partial h_n}{\partial f_i} = p, \quad \forall i \in I, \forall f_i.
\]

(2)

If the aggregate marginal payment is greater than \( p \) for some process \( i \), i.e., \( \sum_{n=1}^{N} \frac{\partial h_n}{\partial f_i} > p \), we say there is double counting of footprint since two or more firms pay for the same marginal emissions. (We use “double counting” to refer to any degree of multiple counting.) With a linear allocation rule, footprint balance is equivalent to \( \sum_{n \in I, i \in I} a_{n,i} f_i = \sum_{i \in I} f_i, \forall f \geq 0 \). This implies that the total allocated emissions are equal to the total emissions generated, hence the term “footprint balanced.” In our previous example, if firm 1 fully pays for the emissions of process 1, although firms 2 and 3 equally pay for the emissions of process 2, then a footprint-balanced payment rule is given by

\[
A^T = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}.
\]

Conversely, if firms 2 and 3 both fully pay for the emissions of process 2, then double counting occurs, and

\[
A^T = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 1
\end{pmatrix}.
\]

Note that the definitions of footprint balance and double counting use the prevailing price of carbon \( p \), which can be different from the societal cost \( p^s \).

We assume that firms have reservation profits \( \pi_n, n \in N \). For the payment scheme \( h_n(f) \) to be viable, it must satisfy the following participation constraint:

\[
\text{individual rationality: } V_n(e_n) - h_n(f(e)) \geq \pi_n, \quad \forall n \in N.
\]

(3)

The specific value of \( \pi_n \) is irrelevant for our analysis but it could be equal to zero to represent the limited liability condition imposed in Battaglini (2006).

Conditional on the participation constraint (3), the firms will choose their abatement efforts by maximizing their individual profits:

\[
e_n \in \arg \max \left\{ V_n(e_n) - h_n(f(e)) \right\}, \quad \forall n \in N.
\]

(4)

We show that these constraints impose strong conditions on the payment scheme if a social planner or carbon leader wants to achieve the first-best effort levels.

4. The Social Planner Perspective

In this section, we study the social planner who can decide a footprint allocation rule and impose a cost on the firms in proportion to the emissions allocated to them. We construct the corresponding carbon-based payments \( h \) and let all firms choose their efforts simultaneously. This is the main distinction from the next section, where the carbon leader moves first and can credibly commit to an effort level.

Let \( f_i = \hat{f}_i(f) \) denote the rule by which the social planner allocates emissions to firm \( n \) and let \( f = \hat{f}(f) \) be the footprint allocation vector function. Given any rule \( f \), firms might want to make payments to each other contingent on emissions to reduce the total cost. A firm with high abatement costs might want to motivate a firm with lower costs and with which it shares joint carbon production to exert additional effort. Following Rayo (2007), we do not model each agreement separately, but let \( g_n(f) \) be the aggregate voluntary payment that firm \( n \) makes to all other firms. These aggregate payments can be positive or negative, but since they are internal transfers, the total sum must be balanced; i.e., \( \sum_{n=1}^{N} g_n(f) = 0, \forall f \geq 0 \). Hence, with a social planner, the total net carbon-based payment made by firm \( n \) to the social planner and to the other firms is

\[
h_n(f) = p \hat{f}_n(f) + g_n(f), \quad \forall f \geq 0,
\]

(5)

where \( p \) is the prevailing carbon price. In the linear case \( (h = pA f + k) \) the allocation is linear \( \hat{f} = Af \) and so are the internal payments \( g = pG f + k \) with \( A = \hat{A} + G \). Note that the payment rule \( h \) in (5) double counts if and only if \( \sum_{n=1}^{N} \frac{\partial h_n}{\partial f_i} > 1 \) for some \( i \in I \).

We assume the social planner wants firms to exert their first-best efforts. However, given the emphasis in the GHG protocol and the LCA literature on avoiding double counting, a crucial question is whether there exists a footprint-balanced payment rule \( h \) such that the social first-best efforts yield a Nash equilibrium in the decentralized game (4) with \( p \leq p^s \). With a single process and unidimensional efforts, the following classic result shows that the answer is negative.
Proposition 1 (Holmstrom 1982, Theorem 1). If $m_n = I = 1$, $\forall n \in N$, then there does not exist a carbon-based payment rule $h$ given by (5) that satisfies footprint balance and yields $e^*$ as a Nash equilibrium in the decentralized game (4) with $p \leq p^5$.

Proposition 1 is an impossibility result: footprint balance and first-best efforts cannot coexist. The reason is that to achieve first-best, each firm has to internalize the full benefits of its efforts, but “no double counting” means that these benefits must be split, leading to underinvestment in efforts. The following proposition extends Holmstrom’s single-process result to multiple processes and is a key insight of this paper. (All proofs are provided in Appendix A.)

**Proposition 2.** For any carbon-based payment rule $h$ given by (5) that is differentiable and increasing, if there is joint carbon production, then double counting is necessary to attain $e^*$ as a Nash equilibrium in the decentralized game (4) with $p \leq p^5$.

Since the internal payments $g$ are balanced, Proposition 2 implies that the social planner’s allocation rule $\hat{f}$ must double count to achieve first-best with payments $h$ that are increasing in the footprint. In other words, allowing internal payments $g$ does not help overcome the double-counting problem. Proposition 2 also requires the payment rule to be differentiable, which includes linear rules, the only type we have encountered in practice. Linear rules are also relevant because any nonlinear differentiable rule can be replaced by linear payments that yield the same outcome in the decentralized game (4) (similar to Bhattacharyya and Lafontaine 1995). For the important case of linear rules, we provide a complete characterization of when first-best can be achieved.

**Proposition 3.** For any $p > 0$ consider the linear payment rule $h = pA f + k$, where $A$ is a $N \times 1$ matrix such that $0 \leq p A \leq p^5$. Then $p A \geq p^5 B$ is necessary and sufficient to attain $e^*$ as a Nash equilibrium in the decentralized game (4).

The condition $0 \leq p A \leq p^5$ means that the rule is increasing and no single firm pays more than 100% of the social cost of any footprint. Proposition 3 says that charging the full societal cost to each firm that can influence a process is the only way to achieve first-best with a linear increasing payment rule. It is well known that $A = p^5 / p$ (“charging everything to everyone”) solves the moral hazard problem in teams (e.g., Segerson 1988, McAfee and McMillan 1991). Our result leverages the supply chain structure embedded in the influence matrix $B$. Setting $A = p^5 / p$ is an extreme case of double counting that is not necessary if the supply chain can be decomposed into processes and each firm is only charged for footprints it can affect, allowing more fine-grained allocation of footprint and reducing the amount of double counting needed.

As with Holmstrom’s result, Propositions 2 and 3 should be seen as impossibility results, or a choice between “two evils.” When $p \leq p^5$, to achieve first-best, the social planner has to choose between rules that are reasonable (e.g., linear increasing) but that double count and rules that avoid double counting but that are complex, noncontinuous, and likely to be seen as unfair. The literature provides many examples of these more contrived rules. For instance, Holmstrom (1982) suggests the allocation rule

$$\hat{f}_n(f) = \begin{cases} w_n & \text{if } f \leq f^*, \\ V_n(0)/p & \text{if } f > f^*, \end{cases}$$

with $w_n \leq V_n(e_n^*)/p$, $\forall n \in N$ and $\sum_{n=1}^{N} w_n = f^*$. This rule is increasing so firms prefer a lower footprint, but the rule is nondifferentiable and not footprint balanced in the sense that $\sum_{n=1}^{N} f_n(f) \neq f$, $\forall f \neq f^*$. Moreover, this rule supports the undesirable outcome $e = 0$ as a Nash equilibrium. Holmstrom (1982) also considers bringing in a principal (or “asset holder” in Rayo 2007) that does not engage in joint production and using the allocation rule $\hat{f}_n(f) = f - f^*$ for the firms and $\hat{f}_{N+1}(f) = f - N(f^* - f^*)$ for the principal. This rule is differentiable and footprint balanced, but it is not increasing for the principal who would benefit from the supply chain not achieving first-best and therefore faces a moral hazard problem, as she has an incentive to make a clandestine offer to one of the firms to increase the footprint (see Eswaran and Kotwal 1984). In the case of multiple processes, Battaglini (2006) shows that when there are “enough” processes, one can construct an allocation rule that is footprint balanced and achieves first-best, but again this rule is nondifferentiable and not increasing (see Appendix B).

If the social planner were allowed to set the allocation rule and to optimize over the carbon price $p$ she charges to the firms, she would choose an allocation rule with double counting and set the price equal to $p^5$. If double counting is not feasible, she can try to compensate by charging a carbon price higher than $p^5$.

**Proposition 4.** Let $\hat{f}$ be differentiable and increasing. Let $V_n(e_n) - h_n(\hat{f}(e))$ be supermodular in the effort levels $e$ for all $n \in N$. If $\hat{f}$ must be footprint balanced but the carbon price $p$ can be set freely, then the social planner will choose a price $p \geq p^5$.

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*Holmstrom (1982) does not model internal payments contingent on emissions, but Theorem 1 in that paper can be extended to allow for transfers $g$ because the proof holds for arbitrary sharing rules.*
Can the social planner achieve first-best with a footprint-balanced allocation rule by appropriately selecting \( p \)? In general not, as this requires solving a system with \( 1 + N \times I \) unknowns (the price \( p \) and the derivatives of the allocation rule) and \( 1 + \sum_{n=1}^{m} \) equations (the footprint-balanced and first-order conditions), which might not have a solution when there are more equations than unknowns. When all processes are affected by the same number of firms, so that \( \sum_{n \in S} b_{n,i} \) is constant, or when firms’ actions are unidimensional, i.e., \( m_n = 1, \forall n \), then the planner can achieve first-best by choosing \( p = p^* \sum_{n \in S} b_{n,i} \) and \( f_n(f) = (\sum_{i=1}^{m} b_{n,i} f_i) / (\sum_{n \in S} b_{n,0}) \). Practically, though, it is dubious that charging a \( p \) so much in excess of the social cost \( p^* \) would be any more feasible than double counting emissions.

5. Decentralized Supply Chains with a Carbon Leader

So far we have focused on scenarios with a social planner and shown that, to achieve first-best, she has to choose between double counting emissions or rules that are impractical or implausible. An alternative scenario is emerging in practice, where a single firm pays for all the supply chain emissions at some carbon price \( p \geq 0 \). We refer to this firm as the carbon leader and denote it \( n = N \). This carbon price \( p \) might be different (and most likely lower; see Buchanan 1969, Fowle et al. 2012) than the societal cost \( p^* \). It might be the price that the carbon leader pays to offset emissions. Natura reported total emissions (Scopes 1–3) of 188,051 tons for 2008 (Natura 2009). Although the cost of Natura’s offsets is confidential, if one assumes a price range of $10–20 per ton of CO\(_2\)e, their total costs of offsets would be $1.8–$3.7 million. Alternatively, the carbon price \( p \) might be the result of a border adjustment tariff, as in Ismer and Neuhoff (2007) and Keskin and Plambeck (2011). How to determine \( p \) is extensively discussed elsewhere (e.g., Tol 2005, 2008); our purpose is to highlight the incentive challenges that remain even if there is agreement on the “right” value of \( p \).

Since we are motivated by firms like Natura that voluntarily offset the supply chain emissions, we assume that the carbon leader can commit to move first as a Stackelberg leader. To reduce the cost of offsets, the carbon leader can make payments to the other firms in the supply chain to give them an incentive to reduce emissions. Every firm \( n \neq N \) must be able to verify the carbon leader’s effort, which is possible if, for instance, the footprint is strictly decreasing in \( e_n \). Using the same notation as before, let \( g_n \) denote the internal payment made by firm \( n \neq N \); \( g_n \leq 0 \) since firm \( n \neq N \) must be compensated by the carbon leader. The carbon leader pays the other firms \( \sum_{n \neq N} g_n(e_n) \) and incurs the cost \( p \sum_{i=1}^{m} f_i \) of all supply chain emissions. Hence, with a carbon leader the total carbon-based payments are \( h_n = g_n \) for \( n \neq N \) and \( h_N = p \sum_{i=1}^{m} f_i - \sum_{n \neq N} g_n \). Note that this payment rule is footprint-balanced according to our definition in (2). We could allow firms (other than the carbon leader) to make payments contingent upon emissions to each other, as in §4, but they will not choose to do so, as we will show that they earn their reservation profits in equilibrium and exert their optimal effort levels, so there is no opportunity for further efficiency improvement.

We consider two cases. First, let the carbon leader contract on efforts, paying \( \{g_n(e_n)\} \) to firm \( n \). Some firms do work so closely with their supply chain partners that they can to some extent observe their effort, but it is unlikely that a carbon leader could do this for the entire supply chain. This case, \( P_f \), mostly serves as a benchmark, analogous to the social planner’s first-best. The carbon leader maximizes its profits by solving

\[
P_f: \max_{\{S_1, \ldots, S_{N-1}, e_N\}} V_N(e_N) - p \sum_{i=1}^{I} f_i(e) + \sum_{n \neq N} g_n(e_n) (6)
\]

subject to \( V_n(e_n) - g_n(e_n) \geq \bar{\pi}_n, \forall n \neq N \),

\[
e_n \in \text{argmax}\{V_n(e_n) - g_n(e_n)\}, \forall n \neq N \tag{7}
\]

Equation (6) is the carbon leader’s objective function, which includes the carbon offsets at price \( p \) and the payments to the other firms. Equations (7) and (8) are the participation and incentive compatibility constraints, equivalent to (3) and (4) but excluding the carbon leader. The reservation profits \( \bar{\pi}_n \) can be made equal to \( V_n(0) \) to ensure that the other firms engage in the carbon leader’s greening initiative.

Since the carbon leader can contract on efforts, it will pay the other firms the minimum possible to induce their participation, dictated by Equation (7), which means setting \( g_n(e_n) = V_n(e_n) - \bar{\pi}_n = -\bar{c}_n(e_n), \forall n \neq N \). Firm \( n \neq N \) will then be indifferent with any effort level, so we assume that it will choose the level determined by the carbon leader. Replacing \( g_n(e_n) = V_n(e_n) - \bar{\pi}_n \) in objective function (6), the problem reduces to maximizing \( \sum_{n} V_n(e_n) - p \sum_{i=1}^{m} f_i(e) - \sum_{n \neq N} \bar{\pi}_n \). Since the last term is a constant, the carbon leader contracting on efforts is equivalent to the social planner’s first-best problem (1) but with carbon price \( p \) instead of \( p^* \).

In the second case, efforts cannot be verified, so the carbon leader can only make payments \( g_n \) contingent on emissions \( f \). We call this case \( P_f \) and the carbon leader’s problem:

\[
P_f: \max_{\{S_1, \ldots, S_{N-1}, e_N\}} V_N(e_N) - p \sum_{i=1}^{I} f_i(e) + \sum_{n \neq N} g_n(f(e)) \tag{9}
\]
subject to \( V_n(e_n) - g_n(f(e)) \geq \pi_n, \quad \forall n \neq N \), \( e_n \in \arg\max[V_n(e_n) - g_n(f(e))], \quad \forall n \neq N \), \( e_n \in \mathbb{R}^+ \)

where Equations (9)–(11) serve the same purpose as Equations (6)–(8).

One might expect the carbon leader to be worse off than when it can contract on efforts, but that is not the case, because the carbon leader acts as a Stackelberg leader and its commitment induces the other firms to follow suit. Mathematically, the social planner case is equivalent to that with a carbon leader that can credibly commit to a given carbon abatement effort. The next proposition shows that the carbon leader, by means of payments contingent on emissions, can induce the same efforts and profits as when it can contract on efforts, if the carbon leader is allowed to double count.\(^\text{10}\)

**Proposition 5.** Let \( z^* \) be the optimal value of \( P_z \) and let \( e^* \) and \( f^* \) be the corresponding optimal efforts and emissions, respectively. Suppose the carbon leader can only make payments contingent on emissions, as in \( P_z \). Then, the carbon leader can achieve profits \( z^* \) and can induce efforts \( e^* \) by compensating firm \( n \neq N \) according to the linear payment rule:

\[
g_n(f) = p \sum_{i \in \mathcal{I}} b_{n,i} f_i + k_n, \quad \text{where} \quad k_n = V_n(e_n^*) - p \sum_{i \in \mathcal{I}} b_{n,i} e_n^* - \pi_n. \tag{12}
\]

Under this rule, each firm is compensated for the costs of exerting \( e_n^* \), the carbon leader’s preferred effort levels, and is charged the carbon price \( p \) per unit by which the footprints it influences deviate from the optimum. If a footprint component is influenced by multiple firms, each is charged the full cost of deviating from the optimal level of that footprint component. Proposition 5 has a positive message: the carbon leader’s greening initiative will be as effective as if it could commit directly on effort. The forces behind Proposition 5 are interesting. Consider two firms and a single process. The net payment to firm 1 is \( g_1(f) = pf - k_1 \). Replacing \( g_1 \) in (9), the carbon leader’s objective function becomes \( V_2(e_2) \), where \( k_1 \) can be ignored because it is constant. Since \( V_2(e_2) \) is decreasing, one might think that the carbon leader would choose not to exert any effort. However, the carbon leader must entice the other firm to participate. This means it must optimally in the total supply chain profits.\(^\text{11}\) Hence, the carbon leader must lead by example and exert an effort to make the other firms exert an effort as well.

The payment rule (12) is linear in the sum of all the emissions that firm \( n \) can influence. The supply chain as a whole pays for the cost of carbon only once, but the carbon leader double counts emissions internally so each other firm is fully internalizing the consequences of its effort. Interestingly, while double counting occurs internally, from a systemwide perspective the payment rule \( h \) is footprint balanced by construction in this case. Hence, the carbon leader can implement a footprint-balanced scheme, even when there is a single process. This scheme does not conflict with the impossibility results in Holmstrom (1982) and §4 because the carbon leader plays first and is excluded from the incentive compatibility constraint (11) in the second stage. In that sense, our result is similar to the sequential rule in Strausz (1999), but with multiple processes and with all other firms playing simultaneously in the second stage.

Proposition 5 assumes that the carbon leader has full information, including the other firms’ cost functions. However, even with information asymmetry—following Theorem 4 in McAfee and McMillan (1991)—Proposition 5 would still hold: i.e., contracting on emissions achieves the same outcome as contracting on efforts, as long as the revelation principle holds. With information asymmetry, neither case results in first-best effort, but contracting directly on effort provides no benefits over contracting only on emissions.

The carbon leader case differs from our earlier social planner case in only two ways. First, the carbon leader pays a carbon price \( p \) that is different from the societal cost \( p^* \). We confirm that if \( p < p^* \), firms will exert less than the first-best effort.

**Lemma 6.** Let \( p < p^* \). If \( V \) and \( f \) are super- and sub-modular, respectively, then \( e^* \leq e^z \).

Second, in the social planner case, we assume no firm can credibly commit to an effort level. If there were such a firm, the social planner could appoint it to be the carbon leader and charge it the full supply chainwide cost of emissions, \( p^* \sum_{i=1}^{\mathcal{I}} f_i \). That carbon leader would then behave exactly as analyzed in this section. In other words, if there is a firm that can commit, the social planner can achieve first-best effort without imposing double counting by delegating that double counting to the appointed carbon leader.

\(^{10}\) A similar result holds for the social planner if she could appoint a carbon leader that could commit.

\(^{11}\) Mathematically, the carbon leader optimizes \( AV_i(e_i) + VC_i(e_i) - pf + g_i(f(1 - \lambda)) - AV_i(0) \), where \( \lambda \) is the Lagrangian multiplier of the participation constraint (10). From the Karush-Kuhn-Tucker conditions, \( \lambda = 1 \) is necessary for optimality. Hence, the carbon leader is effectively optimizing \( \sum_{i=1}^{\mathcal{I}} V_i(e_i) - p \sum_{i=1}^{\mathcal{I}} f_i(e) \) as in \( P_z \).
6. Practical Illustration: Eastman Chemical

In this section, we use data from a case study on Eastman Chemical reported in Koomen (2012) to illustrate the effects of joint production and double counting. In 2009, the European region of a division of Eastman Chemical started selling products in a solid state ("packed"), which can be transported on a regular truck, and in a molten state ("molten bulk"), which requires a heated tank truck. Shipping molten bulk increases transportation emissions but decreases total emissions, as it eliminates several production steps, such as cooling of molten product, making pastilles so it can be stored at room temperature, procuring packaging material and filling them with pastilles, and palletizing. Delivery of molten bulk requires significant coordination (or joint production) between Eastman Chemical and its customers. To handle the molten bulk, both have to invest in capacity, in amounts that have to be coordinated.

Total cost is increasing in molten percentage, primarily because of transportation, which is 1.77 times more expensive for molten than for packed material. Eastman Chemical pays rent for these dedicated tank trucks. For the customers, the main cost is acquiring and maintaining bulk tanks. Since 2005, Eastman Chemical has been subject to the European Union Emissions Trading System, but it also wanted to be "recognized as a company with a genuine and deep-rooted commitment to sustainability." Although Eastman Chemical does not currently offset Scope 3 emissions, we assume for the purpose of this example that it does, making it the carbon leader. Eastman Chemical has been actively studying possible incentives to reduce emissions from joint production, as a carbon leader would do.

Carbon emissions decrease linearly with the proportion of product shipped in molten bulk from Eastman Chemical to its customers. Let the effort \( e_n \) be the molten bulk capacity available at firm \( n \). Because joint effort is required for emissions to decrease, we use a Leontief production function \( f(e) = \Lambda - \gamma \min\{e_N, \sum_{n \neq N} e_n\} \), where \( \Lambda \) and \( \gamma \) are to be estimated. This is an extreme case of the constant elasticity of substitution production function \( (e_N^r/2 + (\sum_{n \neq N} e_n)^r/2)^{1/r} \), with \( r = -\infty \). Because equivalent pre- and postprocessing steps are required on both ends of the distribution system, we assume that all firms—including Eastman Chemical—have the same convex increasing cost function \( C e_n^r \), where \( C \) is a constant. Solving the carbon leader’s problem \( P_I \) yields the following:

\[
e_N = \frac{\gamma p(N - 1)}{2NC} \quad \text{and} \quad e_n = \frac{\gamma p}{2NC}, \quad \forall n \neq N. \tag{13}
\]

By substituting these efforts in the production function, we see that the emissions reduction, expressed as a percentage of the baseline emissions \( \Lambda \), is equal to \( ((\gamma^p p)/(2CA))(1 - (1/N)) \). We express the effort levels (or molten state capacities) as a the effort levels (or molten state capacities) as a percentage of total demand in packed and molten state, so \( e_n \in [0, 1] \). For one particular product, we estimated that \( \Lambda = 566 \text{ ton CO}_2 \text{e} \) and \( C = 17,215 \). The parameter estimation is available from the authors except for the cost, which is confidential and has been disguised. Figure 1(a) shows the emissions reduction as a function of the carbon price \( p \). With a price of \( 30 \text{ per ton} \), emissions reductions of around 10% can be achieved with four customers.

Koomen (2012) reports several potential joint process improvements. If customers share their demand information with Eastman Chemical, the time the product spends in the heated tanks can be minimized. Moreover, Eastman Chemical can then smooth its operations and reduce average inventory. With these improvements, the parameter estimates become \( \gamma = 325.9 \text{ ton CO}_2 \text{e} \) and \( C = 14,875 \) (\( \Lambda \) does not change), and the corresponding emissions reductions are depicted in Figure 1(b). With a carbon price of \( 30/\text{ton} \), the emissions reduction is now 15% with four customers.

We can also use this example to examine the case with a social planner (instead of a carbon leader), who sets the allocation rule and imposes a carbon charge \( p \) on all firms. If the social planner cannot double count emissions but can optimize freely over \( p \), she will choose \( p = 2(1 - (1/N)p)^{5} \). If there is more than one customer, the social planner will charge \( p \geq p^5 \) to correct for the underinvestment resulting from the combination of joint production and the requirement to use a footprint-balanced allocation rule. This is even more evident when \( f(e) \) is differentiable, i.e., when \( r \) is finite in the constant elasticity of substitution function above, because then the social planner would chose a price that is \( N \) times the societal cost \( (p = Np^r) \). In both cases, imposing the optimal price induces the optimal efforts, but this is possible because there is a single joint production process, in line with the discussion at the end of §4.

7. Discussion and Conclusions

What do our findings mean for firms, governments, and NGOs hoping to use voluntary approaches to reduce supply chain carbon footprints? First, firms

\[\text{http://www.eastman.com/Company/Sustainability/Our_Journey/Pages/President_and_CEO.aspx (accessed December 19, 2012).}\]
should not stop at carbon neutrality. Any firm that offsets its emissions, by choice or otherwise, should explore creating incentives to share the rewards of emissions reductions with their suppliers. Moreover, our findings imply that, when conducting its carbon inventory to determine how many offsets to purchase, a carbon leader should (obviously) avoid double counting emissions, but when it comes to providing incentives to suppliers, they should allow for double counting. Vertically integrated firms have even more leeway to implement incentive structures that include double counting. If divisional incentives are tied to carbon footprints, there is no inherent need within a firm for those incentives to add up to 100%. Although a (voluntary) carbon leader may be able to use contracts that induce double counting, a social planner may not be able to do so. We show that the social planner can only partially overcome the resulting underinvestment by setting a carbon price higher than the social cost of carbon.

Even in the absence of an optimal allocation rule (which would require double counting), firms with an interest in overall supply chain efficiency should at a minimum include the full cost of all GHG emissions that they can influence when they decide where to focus their efforts. The fact that double counting is unlikely to be implemented on a large scale in practice should not preclude firms from identifying where their efforts may have the greatest effect. If the greatest return on firm 1’s effort is on emissions currently allocated to firm 2, then firm 1 could explore mechanisms to share the costs and benefits of reducing emissions with firm 2. Without at least allowing double counting in a pro forma fashion, many valuable opportunities for joint improvement will go unexploited. This is of course analogous to the double-marginalization problem in price-setting, which in turn is a manifestation of a fundamental problem of decentralized decision making in general.

To summarize, even though we realize that regulators and supply chains are unlikely to implement incentive mechanisms based on extensive double counting, failing to allow for double counting in supply chain carbon footprinting will misinform firms about the best improvement opportunities available. Moreover, given that the practical carbon price \( p \) (if any) will likely be well below the marginal damage, allowing double counting would be at least a partial remedy against the otherwise inevitable underinvestment.

Finally, our modeling framework also applies to other contexts. The literature on quality and warranties deals with an analogous situation where (possibly joint) efforts by a buyer and a supplier can reduce external failures. Baiman et al. (2000, 2001), Balakrishnan and Radhakrishnan (2005), and Zhu et al. (2007) study one-buyer, one-supplier systems where both parties’ quality may be unobservable and examine how different warranty contracts affect their efforts to improve quality. Similarly, Chao et al. (2009) compare different forms of cost sharing on a single buyer and supplier’s efforts to reduce costs associated with product recalls. In their “no cost-sharing” scenario, the manufacturer internalizes all costs, even though the supplier may sometimes be at fault, analogous to our carbon leader scenario. Our modeling approach would help extend the one-buyer, one-supplier setting that dominates in the quality and warranty literature to more general supply chains.

This paper aims to provide a first step towards dealing with the challenge of joint carbon production in supply chains. Several future extensions are worth exploring. First, empirical work or field research could be used to better understand what types of emissions reduction opportunities are likely to be missed in practice due to the emphasis on avoiding double counting. Second, one could build on our modeling framework and explore cooperative game theory in allocating carbon footprints, also building on Shubik (1962) and others. Third, one could conduct numerical experiments to explore the interaction between a supply chain’s structure and opportunities to reduce its carbon footprint.
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Appendix A. Proofs
We first provide a condition for achieving first-best in a decentralized supply chain.

**Lemma 7.** Consider a decentralized supply chain in which firms make carbon-based payments according to the rule $h$. A necessary condition to achieve first-best is

$$\sum_{i, j} \frac{\partial h_i}{\partial e_{n,j}} \frac{\partial f_i}{\partial e_{n,j}} - p^* = 0,$$

$$\forall n \in N, j \in \{1, \ldots, m_n\}. \quad (A1)$$

*If $h$ is increasing and convex, then (A1) is also sufficient.*

**Proof.** Suppose that the supply chain can achieve first-best. Then, in equilibrium, the effort levels in the decentralized game must be the same as in the centralized solution; i.e., the first-order conditions of Equations (1) and (4) yield the same efforts. Differentiating (1) and (4) with respect to $e_{n,j}$ and equating the common term $\partial V_n/\partial e_{n,j}$, we obtain the necessary condition:

$$p^* \sum_{i, j} \frac{\partial f_i}{\partial e_{n,j}} = \sum_{i, j} \frac{\partial h_i}{\partial e_{n,j}} \frac{\partial f_i}{\partial e_{n,j}},$$

$$\forall n \in N, j \in \{1, \ldots, m_n\}. \quad (A2)$$

The above equation can be rewritten as Equation (A1), where the second equality follows from the definition of $h_{n,j}$ and the fact that $h_{n,j} = 0$ implies $\partial f_i/\partial e_{n,j} = 0$ for $j \in \{1, \ldots, m_n\}$. That proves the necessary part. For sufficiency, if $h$ is convex increasing, then the payoff functions in (4) are concave in effort (from the composition of convex functions; see Bazaraa and Shetty 1979). From Theorem 1.2 in Fudenberg and Tirole (1991) there exists a pure strategy Nash equilibrium in the decentralized game, with efforts that satisfy the first-order conditions of (4):

$$\frac{\partial V_n}{\partial e_{n,j}} = \sum_{i, j} \frac{\partial h_i}{\partial e_{n,j}} \frac{\partial f_i}{\partial e_{n,j}} = p^* \sum_{i, j} \frac{\partial f_i}{\partial e_{n,j}},$$

$$\forall n \in N, j \in \{1, \ldots, m_n\}, \quad (A3)$$

where the last equality follows from (A2). Hence, the efforts in the decentralized game also satisfy the first-order conditions of problem (1), which by assumption have $e^*$ as their unique solution. So $e^*$ is a Nash equilibrium in the decentralized game. □

A.1. Proof of Proposition 2
Joint carbon production implies that there exist a process $i \in J$ and firms $n_1, n_2 \in N$ such that $b_{n_1,i} = b_{n_2,i} = 1$. This implies that there exist actions $j_1 \in \{1, \ldots, m_{n_1}\}$ and $j_2 \in \{1, \ldots, m_{n_2}\}$ such that $\frac{\partial f_j}{\partial e_{n_1,j_1}} < 0$ and $\frac{\partial f_j}{\partial e_{n_2,j_2}} < 0$. Let $h$ be a differentiable and increasing carbon-based payment rule that supports $e^*$ as a Nash equilibrium in (4). Suppose that $h$ does not allow double counting, then by definition

$$\sum_{n=1}^N \frac{\partial h_i}{\partial e_{n,j}} < p, \quad \forall i \in J, \quad (A4)$$

which, together with the fact that $h$ is (componentwise) increasing and $p \leq p^*$, implies that $\frac{\partial h_i}{\partial e_{n,j}} < p^*$, $\forall n \in N$, $i \in J$. Then, from Equation (A1) it follows that

$$\frac{\partial f_i}{\partial e_{n,j}} \left(\frac{\partial h_i}{\partial e_{n,j}} - p^*\right) = 0, \quad \forall i \in J, n \in N, j \in \{1, \ldots, m_n\}. \quad (A5)$$

For process $i^*$ we must have $\frac{\partial h_i}{\partial f_i} = \frac{\partial h_{i^*}}{\partial f_{i^*}} = p^*$, because $\frac{\partial f_i}{\partial e_{n,j}} < 0$ and $\frac{\partial f_j}{\partial e_{n,j}} < 0$, which implies $\sum_{n=1}^N \frac{\partial h_i}{\partial e_{n,j}} > p^* \geq p^*$, in contradiction with (A4). Therefore, $h$ must double count. □

A.2. Proof of Proposition 3
Suppose the decentralized supply chain attains the first-best solution. Then, from Lemma 7, Equation (A1) must hold. Together with $p A \leq p^*$, this implies (A5). To show that $p A \geq p^* B$, consider the two possible values for $b_{n,j}$. If $b_{n,j} = 1$, then there exists $j \in \{1, \ldots, m_n\}$ such that $\frac{\partial f_j}{\partial e_{n,j}} < 0$, which from (A5) implies that $\frac{\partial h_i}{\partial f_i} = p^* = p^* B$. If $b_{n,j} = 0$, then $\frac{\partial h_i}{\partial f_i} \geq p^* b_{n,j}$ because by assumption $A \geq 0$. Hence, $p A \geq p^* B$ is necessary. For the sufficient part, consider first the case $b_{n,j} = 1$. Then, $\frac{\partial h_i}{\partial f_i} \geq p^* b_{n,j}$, together with $p A \leq p^*$ implies that $\frac{\partial h_i}{\partial f_i} = p^*$. In contrast, if $b_{n,j} = 0$, then by definition $\frac{\partial h_i}{\partial f_i} = 0, \forall j \in \{1, \ldots, m_n\}$. Hence, (A5) holds, which implies (A1) and the result follows from Lemma 7. □

A.3. Proof of Proposition 4
Consider the payments $h$ as defined by Equation (5). Since $\hat{f}$ is differentiable and increasing, it is straightforward to verify that $V^*(e) - h_0(\hat{f}(e))$ is supermodular in the efforts $e$ and the carbon price $p$. Therefore, the best response of each firm is increasing in the effort levels of the other firms and in the price $p$. (We assume the best response is unique. Otherwise, consider the maximal best response.) Let $e(p)$ denote the equilibrium efforts, which are increasing by Theorem D in Lippman et al. (1987). In what follows, we assume that $e(p)$ is differentiable in $p$ to simplify the proof, but the argument also holds using discrete differences instead of derivatives. Consider the optimal social value $z^*$ in Equation (1) as a function of $p$. We evaluate the derivative at the social cost $p^*$ to get

$$\frac{dz^*}{dp} \bigg|_{p^*} = \sum_{n=1}^N \sum_{j=1}^{m_n} \frac{\partial h_i}{\partial e_{n,j}} \frac{\partial f_j}{\partial e_{n,j}} (e_{n,j}^*) \bigg|_{p^*}$$

$$= \sum_{n=1}^N \sum_{j=1}^{m_n} \frac{\partial h_i}{\partial f_i} \left(\frac{\partial h_i}{\partial e_{n,j}} - p^* \frac{\partial h_i}{\partial f_i} \right) (e_{n,j}^*) \bigg|_{p^*}, \quad (A6)$$

Caro et al.: Double Counting in Supply Chain Carbon Footprinting
where the second equation follows from the optimality conditions in the decentralized game when \( p = p^- \).

If \( h \) is increasing and footprint balanced, then \( \partial h_{\alpha} / \partial f_{\alpha} \leq 0 \). From supermodular game comparative statics, \( z_{\alpha}^* \leq 0 \), \( \forall n \in N \), \( j \in \{1, \ldots, m_{\alpha}\} \). Hence, from (A6) it follows that \( (\partial z^*/\partial p)_{\alpha} \leq 0 \), which means that when the social planner can optimize over the carbon price she will choose \( p \geq p^- \).

A.4. Proof of Proposition 5

We will show that \( e^* \) and \( g^*_n \), defined in Equation (12), constitute a feasible solution to problem \( P_e \) that achieves \( z^* \). First, from the definition of \( e^* \) it follows that \( e^* \) and \( g^*_n \) satisfy the participation constraint (10) for all \( n \neq \mathcal{N} \). Second, since \( f \) is concave, for the incentive compatibility constraint (11) it is sufficient to verify that \( (\partial V_n(e^*)) / (\partial e_{\alpha}) = p \sum_{n=1}^{N} b_{n \alpha}(\partial f_{\alpha}(e^*)) / (\partial e_{\alpha}) \), \( \forall n \neq \mathcal{N}, j \in \{1, \ldots, m_{\alpha}\} \), which must hold from the optimality of \( e^* \) in problem \( P_e \) and the fact that \( b_{n \alpha} = 0 \) implies \( \partial f_{\alpha} / (\partial e_{\alpha}) = 0 \). Finally, if all other firms \( n \neq \mathcal{N} \) exert efforts \( e^*_n \), then the first-best social planner will exert \( e^*_n \), which is an upper bound since \( P_e \) is a constrained version of \( P_e \). Hence, \( e^* \) and \( g^*_n \) are optimal for \( P_e \).

A.5. Proof of Lemma 6

From Proposition 5 we know that problems \( P_e \) and \( P_g \) provide the same solution. Moreover, problem \( P_e \) is equivalent to the social planner’s first-best problem (1), but with \( p \) instead of \( p^- \). Hence, it suffices to analyze the comparative statics of problem (1) with respect to \( p \). From the assumptions of the lemma, the supply chain profit \( \sum_{n=1}^{N} V_n(e_{\alpha}) - p \sum_{n=1}^{N} f_{\alpha}(e_{\alpha}) \) is supermodular in \( e_{\alpha} \). Since \( f_{\alpha} \) is decreasing for all \( i \in \mathcal{I} \), we have \( (p - p') (\sum_{n=1}^{N} f_{\alpha}(e_{\alpha}) - f_{\alpha}(e_{\alpha})) \geq 0 \), for all \( p \geq p' \) and \( e \geq e^* \), which implies that the supply chain profit satisfies increasing differences in \( (e_{\alpha}, p) \). So by Topkis’ monotonicity theorem (see Theorem 10.7 in Sundaram 1996), \( e^* \leq e^* \).


The following result is shown in Battaglini (2006):

**Proposition 8 (Battaglini 2006, Theorem 1)** Let \( 1 \leq m_{\alpha} \leq 1, \forall n \in N \). If \( \sum_{n=1}^{N} m_{\alpha} / (N - 1) < 1 \), then there generally exists an allocation rule \( f^* \) that satisfies footprint Balance and limited liability and achieves first-best in the decentralized game (4) with \( p = p^- \).

This result relies on distinguishing firms that may have failed to exert their first-best effort, and then being able to only penalize that group. Let \( e^* \) and \( f^* \) be the first-best efforts and footprints. Let \( Y_{\alpha} := \{ f \in R^\mathcal{I} \mid \exists e_{\alpha} \in [0, A]^{m_{\alpha}} \text{ s.t. } f = f_{\alpha}(e_{\alpha}, e^*_n), \forall i \in \mathcal{I} \} \) be the emissions that can be achieved if all firms except \( n \) exert the first-best effort and firm \( n \) exerts any effort, and let the intersection \( Y := \bigcap_{n=1}^{N} Y_{\alpha} \) be the set of emissions levels that can be achieved with a unilateral deviation from first-best by any one firm.

For a given footprint \( f \), let \( G(f) := \{ n \in N \mid f \in Y_{\alpha} \} \) be the subset of firms such that the supply chain can achieve \( f \) after a unilateral deviation from the first-best solution by any firm \( n \in G(f) \). If \( Y = \{ f^* \} \), there is no single footprint that could be achieved, irrespective of which firm \( n \) deviates unilaterally.

In that case, if the observed emissions \( f \) are different from \( f^* \), then \( G(f) \) contains the suspected firms that might have deviated from the first-best. If firm \( n \) deviates unilaterally, then \( n \in G(f) \neq \mathcal{N} \). Hence, one can distinguish between firms that for sure did not deviate, \( \mathcal{N} \setminus G(f) \), and those that might have, \( G(f) \), and penalize only the latter. For this to work, \( Y \) must be a singleton; i.e., it must not contain footprint levels other than the first-best \( f^* \). Otherwise, if the actual footprint was in \( Y \) but not equal to \( f^* \), it would be impossible to distinguish between firms that may have deviated and firms that definitely did not, as any firm could have deviated in that case. Let \( \mathcal{F} \) be the set of convex decreasing footprint vector functions and let \( \mathcal{F}_1 \) be the subset containing those vector functions such that \( |\mathcal{F}| = 1 \). Battaglini shows that \( \mathcal{F}_1 \) is dense in \( \mathcal{F} \) if and only if \( \sum_{n=1}^{N} m_{\alpha} / (N - 1) < 1 \). Here “density” means that there might be footprint functions for which \( Y \) is not a singleton, but they can be approximated as closely as necessary by another function for which \( |\mathcal{F}| = 1 \).

To implement Battaglini’s result in the carbon context, let \( \beta_n > 0, \forall n \in N \), be such that \( \sum_{n=1}^{N} \beta_n = 1 \). Adapting the allocation rule to our context yields the following:

\[
p^- f^*_n(f) = \begin{cases} 
V_n & n \in G(f) \text{ and } |G(f)| < N, \\
V_n - \beta_n (\sum_{n=1}^{N} V_n - p \sum_{i=1}^{I} f_i(\mathcal{Y}_{\alpha})) & n \notin G(f) \text{ and } |G(f)| < N, \\
V_n - \beta_n (\sum_{n=1}^{N} V_n - p \sum_{i=1}^{I} f_i) & f \neq f^* \text{ and } |G(f)| = N, \\
V_n (e^*_n) - \beta_n (\sum_{n=1}^{N} V_n (e^*_n) - p \sum_{i=1}^{I} f_i) & f = f^* \text{ and } |G(f)| = N, 
\end{cases}
\]

where \( V_n = V_n(0), \forall n \in N \). This rule is not smooth, and more importantly, it is not monotone increasing. Therefore, an allocation rule of this form is unlikely to be implementable in practice.

References


