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Improving Supplier Compliance Through Joint and Shared Audits with Collective Penalty

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When suppliers (i.e., contract manufacturers) fail to comply with health and safety regulations, buyers (retailers) are compelled to improve supplier compliance by conducting audits and imposing penalties. As a benchmark, we first consider the ***independent*** audit-penalty mechanism in which the buyers conduct their respective audits and impose penalties independently. We then examine the implications of two new audit-penalty mechanisms that entail a collective penalty. The first is the ***joint*** mechanism under which buyers conduct audits jointly, share the total audit cost incurred, and impose a collective penalty if the supplier fails their joint audit. The second is the ***shared*** mechanism in which each buyer conducts audits independently, shares its audit reports with the other buyers, and imposes a collective penalty if the supplier fails any one of the audits. Using a simultaneous move game-theoretic model with 2 buyers and 1 supplier, our analysis reveals that both the joint and the shared mechanisms are beneficial in several ways. First, when the wholesale price is exogenously given, we establish the following analytical results for the joint mechanism in comparison to the independent mechanism: (a) the supplier's compliance level is higher; (b) the supplier's profit is lower while the buyers' profits are higher; and (c) when the buyers' damage cost is high, the joint audit mechanism creates supply chain value so the buyers can offer an appropriate transfer-payment to make the supplier better off. Second, for the shared audit mechanism we establish similar results but under more restrictive conditions. Finally, when the wholesale price is endogenously determined by the buyers, our numerical analysis shows that the above key results continue to hold.

Key words: Supply Chain Risk, Supplier Compliance, Audits, Collective Penalty, Socially Responsible Operations

1. Introduction

Low labor costs in the East have encouraged many firms to source their products from countries like Bangladesh, China, Indonesia, and Vietnam. However, without strong commitment from buyers and consistent law enforcement by governments, some suppliers (i.e., contract manufacturers) ignore basic health and safety standards at their factories. Over the past decade, Bangladesh has been a popular low cost country for many western companies (e.g., Walmart, H&M, Mango, and Adidas) to source apparel products. However, the tragic collapse of the Rana Plaza building in 2013, which occurred due to the negligence of a supplier, has raised serious concerns about worker-safety standards in supply chains. Donaldson (2014) commented that there is a perception that 20% of the factories in Bangladesh are unsafe in terms of building structure safety, fire safety, electrical safety, and the like. Besides Bangladesh, developing countries such as China, Cambodia, and Vietnam are facing similar challenges from non-compliant suppliers with unsafe factories (Fuller and Bradsher 2013, Demick 2013, Wong and Fung 2015).

While the international brands are not directly and legally responsible for the safety standards employed in their suppliers' factories, they face a "sourcing dilemma". If they do not source from these countries, millions of poor workers will go unemployed because garment exports constitute a substantial portion of the countries' exports in many developing countries such as Bangladesh (Tang 2013). On the other hand, if they continue to source from these countries, the international brands are under public pressure to improve worker-safety standards at their suppliers' factories. To address these challenges, many companies often adopt an ***independent*** audit-penalty mechanism in which they independently conduct audits of their suppliers' factories and impose individual penalties when non-compliance is detected. For example, PVH Corp. (the parent company of brands such as Calvin Klein and Tommy Hilfiger) increased its efforts in auditing its supplier factories. Since 2012, PVH audited 84% of its tier-1 suppliers at least once per year and reported the non-compliant health and safety issues on its website (`www.pvhcsr.com`). Despite its prevalence, the independent mechanism has two drawbacks: (a) the penalty imposed by a single buyer may not be severe enough to ensure that the supplier complies with the required safety standards, especially when the supplier has many buyers, and (b) the audit process can be costly and time consuming.

In this paper, we consider two new audit-penalty mechanisms: joint and shared. These audit-penalty mechanisms are based on a collective penalty and can potentially reduce the drawbacks mentioned above using different auditing procedures. Specifically, the ***joint*** mechanism is conducted by a "consortium" of buyers who share the total audit cost, and the supplier is subjected to a collective penalty if it fails the joint audit. In contrast, the ***shared*** mechanism consists of audits conducted independently by buyers who then share their findings among themselves. In doing so, a supplier's non-compliance is exposed to all the buyers when the supplier fails even one audit, and the supplier will then be subjected to a collective penalty. The collective penalty under both these mechanisms can be more severe than the penalty imposed by each buyer independently and this mitigates the first drawback. Furthermore, the buyers gain savings in the joint and shared mechanisms. In the joint mechanism they gain savings through sharing the audit cost, whereas in the shared mechanism, given the advantages of information sharing, the buyers save on auditing by lowering their individual audit levels. This mitigates the second drawback.

We present a unified framework to analyze the independent, joint, and shared mechanisms. Such analysis provides a better understanding of the approaches recently employed by retailers to improve supplier compliance in their supply chain. Two well publicized approaches are the Accord on Fire and Building Safety in Bangladesh (`bangladeshaccord.org`) instituted by the European retailers and the Alliance for Bangladesh Work Safety (`bangladeshworkersafety.org`) set up by the North American retailers.[1] More details and discussion on the differences between these initiatives can be found in Greenhouse and Clifford (2013), Economist (2013), and Jacobs and Singhal (2015). From our perspective, the joint audit mechanism captures two key aspects of these initiatives: (i) instituting common work place safety standards through a joint audit, and (ii) imposing a collective penalty on a non-compliant supplier. Thus our framework provides a basis to develop a better understanding of the Accord and the Alliance. Furthermore, since these initiatives have affirmed to share information about suppliers and impose collective penalties on non-compliant suppliers, their interactions can be analyzed by the shared mechanism.

Figure 1 summarizes the three audit-penalty mechanisms. As shown in the figure, while the joint and shared mechanisms impose the same collective penalty, they differ in terms of the auditing process: joint versus independent audits. On the other hand, the independent and shared mechanisms use the same audit process but they differ in terms of the penalty they impose: individual versus collective penalty. Therefore, it is unclear which mechanism

[1] The Accord is a legally binding agreement signed in May 2013 by 166 apparel corporations from 20 countries in Europe, North America, Asia and Australia, along with numerous Bangladeshi unions and NGOs (e.g., Workers Rights Consortium, International Labor Organization). The goal of the Accord is to improve workplace safety of over 2 million workers at 1,800 factories (Kapner and Banjo 2013). To reduce the exposure to broad legal liability, U.S. retailers formed the Alliance in 2013, a non-legally binding, five-year commitment to improve safety in Bangladeshi ready-made garment factories. The Accord is committed to provide funds to improve building safety whereas the Alliance is not committed to finance safety improvements.

is more effective from the buyers' perspective. This serves as the motivation to examine the following three key questions in this paper:

1. Which of the three mechanisms results in a higher supplier compliance?
2. Which mechanism results in a higher payoff to the supplier?
3. Which mechanism is the most effective from the buyers' perspective?

To study these questions, we develop a simultaneous move game-theoretic model with 3 players (2 buyers and 1 supplier) to capture the essence of the independent, joint, and shared mechanisms. For each of these mechanisms, the buyers select their audit levels and the supplier selects its compliance level simultaneously.

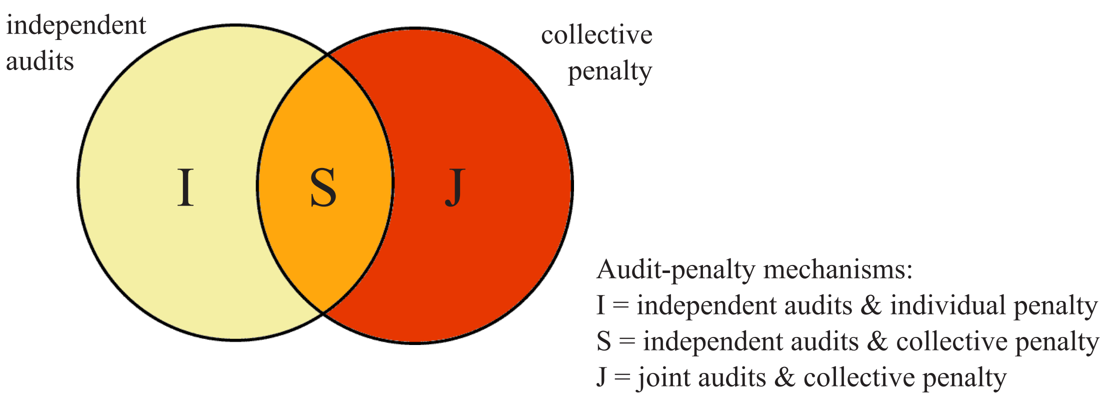


Figure 1 The independent (I), shared (S), and joint (J) audit-penalty mechanisms.

When the wholesale price is exogenously given and remains the same across all three mechanisms, our key findings are as follows. First, the joint mechanism improves supplier's compliance. Second, compared to the independent mechanism, the joint mechanism yields a higher profit to the buyer but a lower profit to the supplier. Third, when the buyers' damage cost is higher than the supplier's compliance cost, the supplier can always be made better off under the joint mechanism through a transfer-payment by the buyers. We establish similar results (with smaller impact) for the shared mechanism under more restrictive conditions. Therefore, when a collective penalty is combined with joint audits, the joint mechanism (instead of shared mechanism) offers more opportunities to create supply chain value.

Likewise, when the wholesale price is *endogenously* determined by the buyers, our numerical results show that most of the key structural results derived in the exogenous wholesale price model continue to hold. In particular, we find that, relative to the independent mechanism, the joint mechanism can be Pareto improving so that both the buyers and the supplier are better off. Additionally, we find that the joint mechanism dominates the other two mechanisms in terms of supplier's compliance level and buyers' profits. By combining our analytical and numerical results, we conclude that the joint mechanism is an effective mechanism for improving supplier's compliance level and the buyers' profits. This result provides a more formal justification for the value of the Accord and the Alliance that are designed to make suppliers increase their compliance levels.

Our paper belongs to a new research stream in supply chain risk management that examines three types of supply chain disruptions (Sodhi et al. 2012). The first type is due to disruptions caused by natural disasters (e.g., Japan's Tōhoku earthquake and tsunami, Thailand's major flood, etc.) and human induced disasters (e.g., the terrorist attacks on 9/11). Sodhi and Tang (2012) provide a comprehensive discussion on this type of supply chain disruptions. The second type of disruption is caused by major financial crises (e.g., Asian currency devaluations in 1997, the sub-prime financial crisis in 2008) that can disrupt supplier's operations (Babich et al. 2007). Our paper deals with the third type of supply chain disruptions that are caused by an "intentional act" committed by the supplier. Well-publicized examples include Mattel's lead tainted toys in 2007, melamine tainted milk in 2008, and Baxter's adulterated Heperin in 2008. The research in this area examines issues of product adulteration that occur when suppliers use unsafe materials to produce products that can cause physical harm to consumers (Babich and Tang 2012, Rui and Lai 2015).

Such supplier non-compliance issues have forced many western firms to take action to improve supplier compliance. In this setting, Plambeck and Taylor (2015) use a game-theoretic model with a single buyer and a single supplier to explore the interactions between the buyer's audit level and the supplier's compliance and deception effort. By examining the equilibrium outcomes (supplier's compliance level, supplier's deception effort, and buyer's audit level) they show that when a supplier deceives the auditors by hiding certain critical information, the buyer's actions could motivate the supplier to cause more harm.

In the context of environmental violations, Kim (2015) examines the interactions between a regulator's inspection policy and a firm's non-compliance disclosure timing decisions. By considering the case when environmental violations are stochastic, this work shows that there are conditions under which periodic inspections can be more effective than random inspections. Orsdemir et al. (2015) investigate how vertical integration can be used as a strategy to ensure compliance. They examine the scenario of two supply chains, one of which is vertically integrated, and highlight that the presence of a supply chain partnership plays a key role in determining supplier compliance. They argue that, in the absence of a partnership, overly tight scrutiny of violations can backfire and degrade compliance when negative reporting externalities are high. However, tighter scrutiny encourages compliance in the presence of partnership. Moreover, if the positive externalities are high, the integrated and compliant firm will cease to share responsibly sourced components with its competitors thus hurting the industry-wide compliance. More recently, Fang and Cho (2015) consider a setting with joint and shared audits in which multiple buyers engage in a cooperative game in the presence of externalities by which the violation of one buyer can affect the profit of other buyers.

While our paper also deals with the issue of supplier compliance, it is fundamentally different from the existing literature on supply chain risk management in three ways. First, the papers listed above primarily focus on the strategic interaction between one buyer and one supplier. Instead, we examine and compare three different mechanisms (independent, joint, and shared) by capturing the strategic interactions among two buyers and one potentially non-compliant supplier. Second, we consider the issue of a non-compliant supplier and employ the notion of "collective penalty" imposed by both buyers when such a non-compliant supplier fails the joint audit under the joint mechanism, or one of the audits under the shared mechanism. Our contribution is to examine the implications of a collective penalty facilitated by the joint and shared mechanisms. Third, in comparison to Fang and Cho (2015), our paper has a different motivation. Our work is geared towards comparing three audit-penalty mechanisms and understanding when they can increase supplier compliance and supply chain profits in a non-cooperative setting. In particular, our model and results emphasize the tension between buyers and the supplier, whereas Fang and Cho (2015) mostly study the cooperation among buyers when the supplier is indifferent between auditing schemes. Though our research is motivated by workplace safety, it also applies to other regulations that require auditing to verify compliance.

This paper is organized as follows. In Section 2 we present our modeling framework and the resulting equilibrium outcomes, and in Section 3 we compare the results across all the three mechanisms. In Section 4, we extend our analysis to the case when the wholesale price is endogenously determined by the buyers. In Section 5 we discuss implications for the the Alliance and the Accord. We present our conclusions in Section 6. All proofs are provided in the Appendix D.

2. The Model

Consider a supply chain comprising of two buyers ($i = 1, 2$) and one supplier s. For ease of exposition, we focus our analysis on the case when the buyers are identical so that buyer i sells one unit of its product at price p and pays the supplier a wholesale price w. We denote the supplier's unit cost by c. Since our focus is on the audit-penalty mechanism, we consider p, w and c to be exogenous so that the values of these parameters do not depend on the mechanism adopted by the buyers. In other words, the strategic intent of different mechanisms is to encourage the supplier to improve its compliance level, but not to increase selling prices, or reduce wholesale prices (e.g., Van Mieghem 1999), or do both. This seems reasonable in the context of outsourcing agreements between western firms and suppliers located in developing countries because reducing the wholesale price would create public concern about the firm's moral and ethical standards. However, in Section 4 we extend our analysis to the case when the wholesale price is endogenously determined by the buyers under each mechanism.

We use a simultaneous move game to model the dynamics between the buyers and the supplier for all the three mechanisms. Specifically, each buyer i simultaneously selects its audit level z_i, $i = 1, 2$, and incurs an audit cost of αz_i^2, where $\alpha > 0$ and $z_i \in [0, 1]$ (in the joint mechanism the buyers choose z_i but reach a joint audit level z through a process that will be explained later). Here, z_i represents the probability that buyer i's audit will be effective in detecting non-compliance (if it exists). This notion of audit probability is commonly used in the literature (e.g., Babich and Tang 2012, Orsdemir et al. 2015). While the buyers select their audit levels, the supplier simultaneously selects its compliance level x and incurs a compliance cost γx^2, where $\gamma > 0$ and $x \in [0, 1]$. Here, x represents the probability that the supplier complies with the workplace safety regulations. In practice, the supplier might face other decisions besides compliance. However, we focus exclusively on the compliance decision in order to have a parsimonious model that serves our research goal. Incorporating other decisions is left for future work.

The simultaneous move framework is justifiable when the supplier cannot observe the buyer's audit level. However, if this is observable, then a sequential move framework would be the more appropriate in which the buyers will first select their audit levels simultaneously in each mechanism. By anticipating the buyers' audit levels, the supplier selects its compliance level. For completeness, we also analyzed the sequential game model and found that the key results are consistent with those in the simultaneous game model. We refer the interested reader to Caro et al. (2015).

To facilitate analytical comparisons, we assume that the audit cost α remains the same across all the three mechanisms, even though the same approach can be applied to examine the case when audit cost depends on the audit mechanism chosen. We also assume a convex auditing cost αz_i^2 since one would expect the buyers to prioritize the most cost-effective activities. Moreover, this assumption is quite standard whenever each marginal increase in effort is more costly, e.g., see Plambeck and Taylor (2015).

Regardless of the mechanism adopted by the buyers, all parties face the following risks. First, if a non-compliant supplier is detected by buyer i, the buyer will reject the unit product without payment, and the supplier will incur a goodwill cost g associated with the contract termination imposed by buyer i. Second, if a non-compliant supplier is not detected by buyer i, the buyer will accept the unit product and pays the supplier the wholesale price w. However, there is a chance that this non-compliance will be exposed to the public. In that case, buyer i will incur an expected "collateral damage" d due to the spillover effect of the non-compliant supplier. Throughout this paper, we assume that the collateral damage d is severe enough so that there is an incentive for the buyer to audit its supplier. For this reason, we make the following two assumptions that provide motivation for the supplier to care about compliance and for the buyer to care about auditing:

Assumption 1. *The supplier's goodwill cost g associated with contract termination imposed by buyer i, $i = 1, 2$, is higher than the supplier's profit margin (i.e., $g > w - c$).*

Assumption 2. *The damage cost d of buyer i, $i = 1, 2$, due to a non-compliant supplier is higher than the buyer's profit margin (i.e., $d > p - w \equiv m$).*

After all players have made their (audit or compliance) decisions, the sequence of events is as follows: (i) the supplier produces the product and incurs the production cost c; (ii) the buyers inspect for non-compliance; (iii) trade occurs only if non-compliance is not detected by the buyers; otherwise, g is incurred by the supplier; (iv) the public finds out about any possible non-compliance in which case the buyers incur d and the supplier incurs a discounted penalty ηg, with $0 \leq \eta \leq 1$. For ease of exposition, we analyze the non-cooperative simultaneous game for the case when $\eta = 0$. The analysis associated with the case when $\eta > 0$ is omitted because the results change in the expected direction (i.e., the supplier complies more and the buyers audit less compared to when $\eta = 0$).

2.1. Independent Mechanism (I)

Under the independent mechanism, buyer i selects its audit probability z_i and the supplier selects its compliance level x. Figure 2 depicts the extensive form of the simultaneous game under the independent mechanism. We follow the convention that the dashed line represents information imperfection in the game tree. We begin our analysis with the supplier's problem. From the figure we observe that the supplier will fail buyer i's audit with probability $z_i(1-x)$. By considering the wholesale price w, the goodwill cost g, and the compliance cost γx^2, the supplier's problem for any given audit levels z_1 and z_2 is given by:

$$\begin{aligned}\pi_s(z_1, z_2) &= \max_{x \in [0,1]} \sum_{i=1}^{2} \left[w(1 - z_i(1-x)) - g z_i(1-x) - c\right] - \gamma x^2 \\ &= \max_{x \in [0,1]} 2(w-c) - (w+g)(1-x) \cdot \sum_{i=1}^{2} z_i - \gamma x^2. \end{aligned} \tag{1}$$

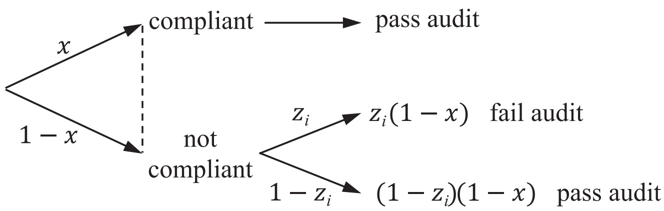


Figure 2 **Independent mechanism extensive-form game: supplier's compliance level x and buyer's audit level z_i.**

To ensure that the supplier has incentive to fully comply, we assume that the supplier's profit margin is high enough so that the supplier's expected profit is non-negative under full compliance (i.e., when $x = 1$). By considering the objective function given in (1), this assumption can be stated as:

ASSUMPTION 3. *The supplier's total profit margin is higher than its full compliance cost so that* $2(w-c) \geqslant \gamma$.

Before determining the supplier's best-response, observe that $\frac{\partial \pi_s}{\partial x}$ evaluated at (1,1) is equal to $2(w+g)-2\gamma x$. Hence, we can interpret the term $r \equiv \frac{w+g}{2\gamma}$ as the supplier's "rate of return on compliance per buyer." By applying Assumptions 1 and 3, it is easy to check that $2g > 2(w-c) \geqslant \gamma$ so that $2w > \gamma$. Thus, we conclude that $r > \frac{1}{2}$. As we shall see later, r will be used in proving and interpreting our results. By considering the first order condition associated with (1), the supplier's best response for any given buyers' audit levels z_1 and z_2 is given by $x^I(z_1, z_2) = \min\{1, r(z_1 + z_2)\}$. (Throughout this paper, we use superscripts I, J and S to denote the outcomes associated with the independent, joint, and shared mechanisms respectively.)

Next, we determine buyer i's best response $z_i(x, z_j)$ for a given supplier compliance level x and buyer j's audit level z_j. We assume that the general public is not aware that the buyers have a common supplier, so the two buyers are treated independently by the public. Following Figure 2 and considering the profit margin $m \equiv (p-w)$, the damage cost d, and the audit cost αz_i^2, the profit of buyer i is given by:

$$\Pi_i(z_i; x, z_j) = m(1 - z_i(1-x)) - d(1-z_i)(1-x) - \alpha z_i^2. \tag{2}$$

From the first order condition we obtain buyer i's best response to be $z_i^I(x, z_j) = \min\{\frac{d-m}{2\alpha}(1-x), 1\}$ for $i = 1, 2$. By considering the supplier's best response $x^I(z_1, z_2)$ and buyer i's best response $z_i^I(x, z_j)$ simultaneously, it can be easily established that the equilibrium compliance and audit levels are given by

$$x^I = \frac{r(d-m)}{\alpha + r(d-m)} \quad \text{and} \quad z^I = \frac{d-m}{2(\alpha + r(d-m))}. \tag{3}$$

Note that $x^I < 1$ and $z^I < 1$ because $r \equiv \frac{w+g}{2\gamma} > \frac{1}{2}$ so we are guaranteed to obtain an interior solution. The characteristics of the equilibrium in Equation (3) are described in the following lemma:

LEMMA 1. *Under the independent mechanism I, the buyer's audit level z^I and the supplier's compliance level x^I given in* (3) *possess the following properties:*

(i) The supplier's compliance level is always higher than the buyer's audit level (i.e., $x^I = 2rz^I > z^I$).

(ii) Both supplier's compliance level x^I and the buyer's audit level z^I are increasing in the buyer's damage cost d, and decreasing in the buyer's audit cost α.

(iii) The supplier's compliance level x^I is decreasing in the supplier's compliance cost γ. However, the buyer's audit level z^I is increasing in γ.

(iv) The supplier's compliance level x^I is increasing in the supplier's goodwill cost g. However, the buyer's audit level z^I is decreasing in g.

(v) The supplier's compliance level x^I is increasing in the wholesale price w. However, the buyer's audit level z^I is increasing in w if, and only if, $w < \sqrt{2\alpha\gamma} - (d - p)$.

Lemma 1 has the following implications. The first statement reveals that the buyer's audit has an "amplifying" effect as it makes the supplier to increase its compliance level by the factor of $2r(> 1)$ (i.e., twice the rate of return on compliance). Consequently, the first statement implies that the buyer can encourage the supplier to comply fully (i.e., $x = 1$) without conducting full audits (i.e., $z_i < 1$). The second statement is intuitive. A higher damage cost d will force the buyers to increase their audit levels that, in turn, will cause the supplier to increase its compliance level. In the same vein, the audit cost has a dampening effect. A higher audit cost will force the buyers to reduce their audit levels that, in turn, leads to a lower compliance of the supplier. The third statement shows the opposite effect of the supplier's compliance cost γ. When the supplier's compliance cost γ increases (i.e., as r decreases), the supplier will lower its compliance level x^I. On anticipating this, the buyer will increase its audit level z^I. To interpret the last statement, it is intuitive that the supplier would increase its compliance level when the buyer offers a higher wholesale price. However, to explain the characteristics of buyer's audit level, we consider the case when w is low so that the supplier's compliance level is low. When this is the case, a buyer can easily expose the supplier's non-compliance without needing to exert a high audit level. However, when w gets larger, the compliance increases and the buyer needs to exert a higher audit level to detect the residual level of non-compliance by the supplier.

By substituting z^I and x^I given in (3) into (1) and (2), and by noting that $x^I = 2rz^I$, the buyer's profit $\Pi^I(z^I)$ and the supplier's profit $\pi_s^I(z^I)$ at equilibrium are given by:

$$\Pi^I(z^I) = m(1 - z^I(1 - 2rz^I)) - d(1 - z^I)(1 - 2rz^I) - \alpha {z^I}^2, \tag{4}$$

$$\pi_s^I(z^I) = 2(w - c) - \gamma + \gamma(1 - 2rz^I)^2 = 2(w - c) - \gamma + \gamma(1 - x^I)^2. \tag{5}$$

2.2. Joint Mechanism (J)

Next, we analyze the simultaneous game for the joint mechanism. For any given joint audit level z selected by the consortium (i.e., both the buyers), the supplier will fail the joint audit with a probability of $z(1-x)$. Upon failing the joint audit, the supplier receives no payment and it will be subject to the collective penalty $2g$ imposed by both the buyers. Hence, the supplier's problem can be written as:

$$\pi_s(z) = \max_{x\in[0,1]}\{[2w(1-z(1-x)) - 2gz(1-x) - 2c] - \gamma x^2\}. \tag{6}$$

Using the first-order condition, the supplier's best response $x^J(z)$ is obtained as:

$$x^J(z) = \min\{2rz, 1\}. \tag{7}$$

Identifying the buyers' best response requires specifying how the joint audit level is selected and how the audit cost is shared. For that, consider buyer i's profit when the joint audit level is z and buyer i pays a proportion θ_i of the auditing cost:

$$\Pi_i(\theta_i; z, x) = m(1-z(1-x)) - d(1-z)(1-x) - \theta_i\alpha z^2. \tag{8}$$

Suppose for a moment that buyer i is able to unilaterally select the joint audit level. Clearly, in that case buyer i would want z to maximize the profit above. From the first order condition, buyer i would want the joint audit level z to be:

$$z = z_i(\theta_i) \equiv \frac{(d-m)(1-x)}{2\alpha\theta_i}. \tag{9}$$

Note that if $\theta_i = \frac{1}{2}$ for $i = 1, 2$, then both buyers would want the joint audit level to be $\frac{(d-m)(1-x)}{\alpha}$, and therefore they would reach consensus automatically. With that in mind, in what follows we assume that the buyers a priori agree to evenly share the audit cost. We make this assumption for ease of exposition. However, in the Appendix B we formally show that $\theta_1 = \theta_2 = \frac{1}{2}$ is indeed the outcome of a non-cooperative game between the two buyers.

Given $\theta_i = \frac{1}{2}$, we can derive buyer i's best response from (9), and together with the supplier's best response in (7) we can solve the simultaneous equilibrium as:

$$x^J = \frac{2r(d-m)}{\alpha + 2r(d-m)} \quad \text{and} \quad z^J = \frac{d-m}{\alpha + 2r(d-m)}. \tag{10}$$

An interior solution is guaranteed since $x^J < 1$ and $2r > 1$ implies that $z^J < 1$. Lemma 4 in Appendix A is analogous to Lemma 1 and shows that the joint mechanism equilibrium in Equation (10) exhibits the same characteristics as stated in the independent mechanism equilibrium given in Lemma 1 (i.e., Equation (3)).

By using (6), (7), (8) and (10) along with $\theta_1 = \theta_2 = \frac{1}{2}$, the equilibrium profits of the buyers and supplier under the joint mechanism can be written as:

$$\Pi^J(z^J) = m(1 - z^J(1 - 2rz^J)) - d(1 - z^J)(1 - 2rz^J) - \frac{1}{2}\alpha\, {z^J}^2, \tag{11}$$

$$\pi_s^J(z^J) = 2(w - c) - \gamma + \gamma(1 - 2rz^J)^2 = 2(w - c) - \gamma + \gamma(1 - x^J)^2. \tag{12}$$

2.3. Shared Mechanism (S)

In this section, we analyze a simultaneous game to examine the third mechanism: the shared mechanism. In this mechanism, each buyer conducts its own audit independently, but shares its findings with the other buyer so that a non-compliant supplier will be exposed to both buyers if it fails either of the buyers' audits. Figure 3 provides the extensive-form game of the shared mechanism. For any given audit levels z_1 and z_2, the supplier with compliance level x will fail buyer i's audit with probability $[z_i(1 - x) + z_j(1 - z_i)(1 - x)]$ for $i = 1, 2$, and $j \neq i$. By noting that the supplier will fail buyer i's audit with probability $z_i(1 - x)$ under the independent mechanism (Figure 2), we can conclude that, through sharing audit reports, the shared mechanism enables buyer i to identify a non-compliant supplier with an "additional probability" of $z_j(1 - z_i)(1 - x)$. This additional probability plays an important role in analyzing the shared mechanism.

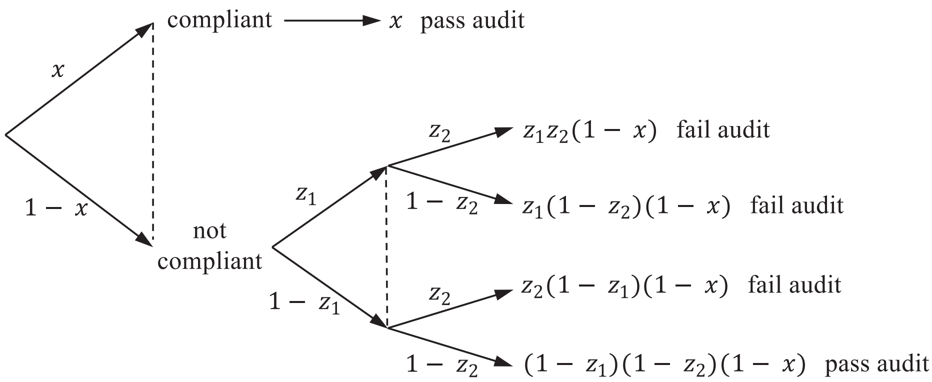


Figure 3 **Shared mechanism extensive-form game: buyer i's audit level z_i ($i = 1, 2$) and supplier's compliance level x.**

Under the shared mechanism, supplier's profit can be written as

$$\pi_s(x; z_1, z_2) = 2(w - c) - 2(g + w)(z_1 + z_2 - z_1 z_2)(1 - x) - \gamma x^2 \tag{13}$$

and buyer i's $(i=1,2)$ profit can be written as

$$\Pi_i(z_1;z_2,x) = m\left[1-(z_1+z_2-z_1z_2)(1-x)\right] - d(1-z_1)(1-z_2)(1-x) - \alpha z_i^2. \quad (14)$$

The best responses of the supplier and the buyers are given by:

$$x(z_1,z_2) = 2r(z_1+z_2-z_1z_2) \quad \text{and} \quad z_i(x,z_j) = \frac{(d-m)}{2\alpha}(1-z_j)(1-x), \quad (15)$$

where $i=1,2$ and $i \neq j$. By solving the above three equations simultaneously, we characterize the equilibrium in Lemma 2.

LEMMA 2. *Under the shared mechanism S, the buyer's audit level z^S and the supplier's compliance level x^S can be characterized as follows:*

(i) The buyer's audit level z^S is the unique root $z \in (0, 1-\sqrt{\frac{2r-1}{2r}})$ of the following cubic equation:

$$V(z) \equiv 2rz^3 - 6rz^2 + \left(1 + 4r + \frac{2\alpha}{d-m}\right) z - 1 = 0. \quad (16)$$

(ii) The supplier's compliance level is $x^S = 2rz^S(2-z^S)$ and $x^S \in (0,1)$.

Lemma 5 in Appendix A shows that the shared mechanism equilibrium (as implicitly defined in Lemma 2) exhibits the same characteristics as stated in Lemma 1. Finally, the supplier and the buyer profits under the shared mechanism are given by:

$$\Pi^S(z^S) = m\left[1-(2z^S-(z^S)^2)(1-x^S)\right] - d(1-z^S)^2(1-x^S) - \alpha {z^S}^2, \quad (17)$$

$$\pi_s^S(z^S) = 2(w-c) - \gamma + \gamma(1-x^S)^2, \quad (18)$$

where z^S and x^S are the equilibrium audit and compliance levels as given in Lemma 2.

3. Comparison of Equilibrium Outcomes Across Mechanisms

To gain a deeper understanding about the results derived in the last section, we now compare the equilibrium decisions across all three audit-penalty mechanisms. Then we compare the buyers' and the supplier's profits across the mechanisms.

3.1. Comparison of buyers' audit and supplier's compliance levels

We compare the equilibrium decisions across the three mechanisms in Proposition 1.

PROPOSITION 1. *Across all three mechanisms, the buyers' audit levels satisfy: $z^S < z^I < z^J$. Additionally, the supplier's compliance levels satisfy the following:*

(i) $x^J > x^I$ *and* $x^J > x^S$.

(ii) $x^S > x^I$ *if and only if* $\alpha \geq \widetilde{\alpha} \equiv \max\{(d-m)(\widetilde{r}-r), 0\}$, *where* $\widetilde{r} \equiv \dfrac{1}{\sqrt{5}-1} (\approx 0.81)$.

Proposition 1 has the following implications. First, relative to the independent mechanism, the buyer can afford to audit less under the shared mechanism because all the audit findings are shared. On the other hand, relative to the independent mechanism, the buyer can afford to increase their joint audit level under the joint mechanism because the joint audit cost is shared by the two buyers. This explains the first statement.

Statement (i) in the second statement indicates that because the joint audit level is higher (i.e., $z^J > z^I$), the supplier must commit to a higher compliance level under the joint mechanism in response to the increased audit level and the higher (collective) penalty for non-compliance. Hence, $x^J > x^I$. Next, while both the joint and shared mechanisms impose the same collective penalty, the buyers in the consortium maintain a higher audit level under the joint mechanism. In response, the supplier must commit to a higher compliance level under the joint mechanism. Thus, $x^J > x^S$.

Statement (ii) is noteworthy because it shows that, relative to the independent mechanism, the shared mechanism can make the supplier to comply more and yet the buyer to audit less. When rate of return on compliance r is high ($r \geq \widetilde{r} \Leftrightarrow \widetilde{\alpha} = 0$ by definition), the supplier will comply more under the shared mechanism because of the collective penalty. However, when the rate of return on compliance is low ($r < \widetilde{r} \Leftrightarrow \widetilde{\alpha} > 0$), the compliance level is driven by the audit cost α of the buyers. If $\alpha < \widetilde{\alpha}$, then the buyers become complacent and try to delegate the responsibility of auditing to each other because the cost of auditing is low. The supplier takes advantage of this behavior and complies less under the shared mechanism. However, when $\alpha \geq \widetilde{\alpha}$, each buyer, realizing that the other buyer alone cannot audit at a greater level due to the high audit cost, seriously takes up the responsibility to audit and this makes the supplier to comply more. Figures 4 and 5 illustrate the results stated in Proposition 1. For all the plots in Section 3 we use the following parameter values: $d = g = 1000, c = 0, p = 1800$, and $w = 900$. (In Appendix C, we provide different plots for the case when $d = 2g = 2000$.)

3.2. Comparison of supplier's profits

Using the equilibrium profits of the supplier as given in (5), (12), and (18), we establish the following result that compares supplier's profits across different mechanisms.

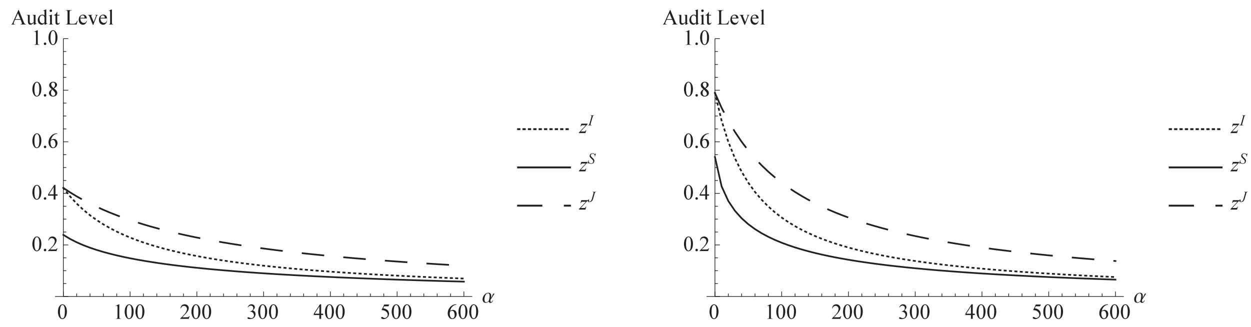


Figure 4 **Audit levels for I, S and J mechanisms with** $\gamma = 800$ **(left) and** $\gamma = 1500$ **(right)**

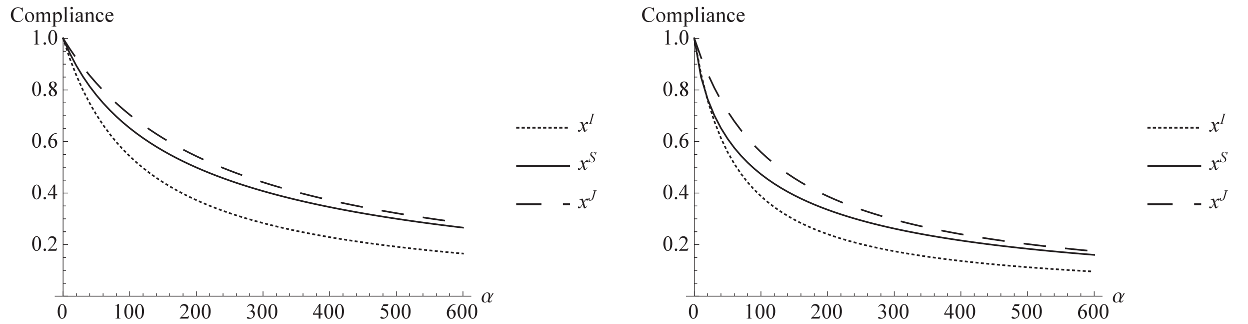


Figure 5 **Compliance levels for I, S and J mechanisms with** $\gamma = 800$ **(left) and** $\gamma = 1500$ **(right)**

PROPOSITION 2. *The supplier's profit possesses the following properties:*

(i) $\pi_s^J(z^J) \leqslant \pi_s^I(z^I)$ *and* $\pi_s^J(z^J) \leqslant \pi_s^S(z^S)$.

(ii) $\pi_s^S(z^S) \leqslant \pi_s^I(z^I)$ *if and only if* $\alpha \geq \widetilde{\alpha}$, *where* $\widetilde{\alpha}$ *is defined as in Proposition 1.*

Because the supplier's profit is driven by the compliance level, the results as stated in Proposition 2 are congruent with Proposition 1. In particular, the supplier has the lowest profit in the joint mechanism due to the collective penalty and the higher compliance level (statements (i) and (ii) in Proposition 1). Figure 6 illustrates the findings of Proposition 2. Here $\widetilde{\alpha} = 0$ for $\gamma = 800$ and $\widetilde{\alpha} = 17.6$ for $\gamma = 1500$ so we observe $\pi_s^S(z^S) \leqslant \pi_s^I(z^I)$ for most values of α.

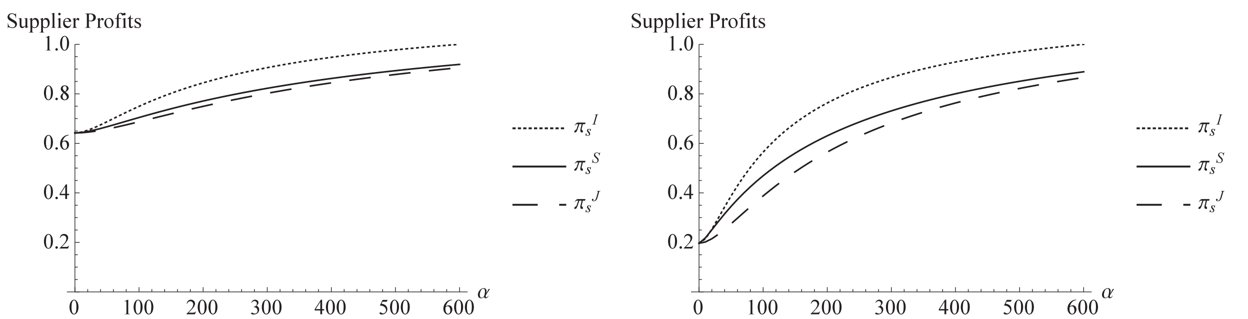


Figure 6 **Supplier's profits (normalized) for I, S and J mechanisms with** $\gamma = 800$ **(left) and** $\gamma = 1500$ **(right)**

3.3. Comparison of buyers' profits

The following result compares the buyers' profits across the different mechanisms.

PROPOSITION 3. *The buyers' profits possess the following properties:*

(i) $\Pi^J(z^J) \geqslant \Pi^I(z^I)$.

(ii) $\Pi^S(z^S) \geqslant \Pi^I(z^I)$ *if and only if* $\alpha \geq \widetilde{\alpha}$*, where* $\widetilde{\alpha}$ *is defined as in Proposition 1.*

Proposition 3 has the following implications. The first statement illustrates that each buyer can obtain a higher profit under the joint mechanism than under the independent mechanism because the buyers share the total audit cost incurred by the consortium while forcing the supplier to comply more. Further, one would intuitively think that the buyers' profits would improve if they can attain higher supplier compliance through lower audit levels. This is the finding in the second statement of the above proposition: when α is large, as shown in Proposition 1, the supplier complies more ($x^S > x^I$) while the buyers audit less ($z^S < z^I$), and therefore they make higher profits under the shared mechanism compared to the independent mechanism.

Proposition 3 does not provide a comparison of the buyers' profit between the joint and shared mechanisms. Our numerical results indicate that $\Pi^J(z^J) \geqslant \Pi^S(z^S)$ as it can be seen in Figure 7. It seems intuitive that the buyers would be better off in the joint mechanism since they can save on the auditing cost while inducing the highest compliance. For a few limiting cases (e.g., $r \to \frac{1}{2}$ and $\alpha \to 0$) one can indeed show analytically that $\Pi^J(z^J) \geqslant \Pi^S(z^S)$, which provides partial support for our numerical observation.

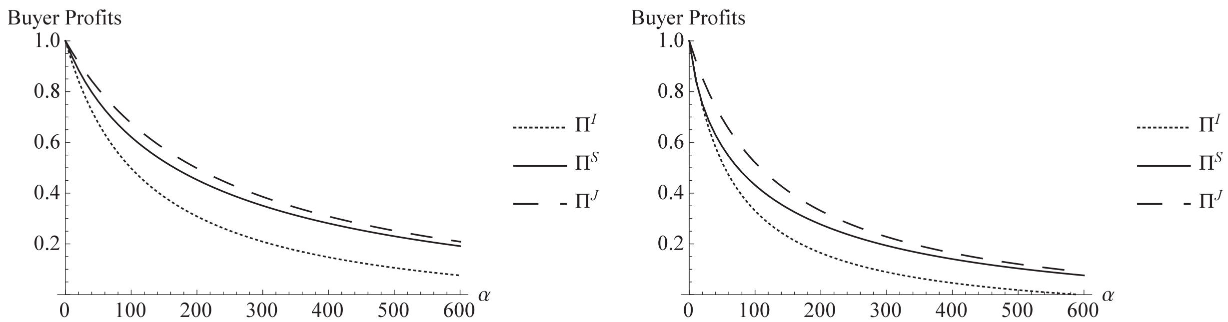


Figure 7 Buyers' profits (normalized) for I, S and J mechanisms with $\gamma = 800$ **(left) and** $\gamma = 1500$ **(right)**

3.4. Comparison of supply chain profits

From Propositions 2 and 3 we observe that buyers are better off but the supplier is worse off when there is a collective penalty under the joint mechanism. In the context of emerging economies such as Bangladesh, making the supplier substantially worse off could be perceived as being socially unfair and the buyers may face adverse publicity. Therefore, we now examine if the buyers can offer transfer-payments to the supplier so that both the buyers and the supplier are better off.

Consider for instance the joint mechanism versus the independent mechanism. When each buyer i offers a transfer-payment $T(>0)$ to the supplier, all parties will be better off if $\Pi^J - T \geqslant \Pi^I$ for each buyer and $\pi_s^J + 2T \geqslant \pi_s^I$ for the supplier. That is, there exists a transfer-payment T that is Pareto improving if, and only if, the supply chain profit is higher (i.e., $2\,\Pi^J + \pi_s^J \geqslant 2\,\Pi^I + \pi_s^I$). Such Pareto-improving transfer-payment will make the joint mechanism acceptable to both the buyers and the supplier. By considering the buyer's profit given in (4) and (11) and the supplier's profit given in (5) and (12) we obtain the following results:

Proposition 4. *The total supply chain profit under the joint mechanism is higher than that under the independent mechanism if* <u>*any*</u> *of the following conditions hold:*

(i) The audit cost α is sufficiently low.

(ii) The damage costs of each buyer is larger than the compliance cost of the supplier (i.e., $d > \gamma$).

(iii) The total damage cost incurred by the buyers is greater than the compliance cost of the supplier (i.e., $2d > \gamma$) and the cost of non-compliance for each buyer is greater than the cost of non-compliance for the supplier (i.e. $d - m > g + w$).

Proposition 4 provides a set of sufficient conditions ensuring the existence of a transfer-payment $T > 0$ such that the joint mechanism creates supply chain value compared to the independent mechanism. Part (i) in Proposition 4 states that, regardless of the other parameter values, if the audit cost α is low enough, then the savings from the joint audit will outweigh the decrease in the supplier's profit. To see this, note that x^I and x^J tend to one when the audit cost α approaches zero. Since $\frac{z^I}{x^I} = \frac{z^J}{x^J} = \frac{1}{2r}$, it follows that the audit level in the joint and independent mechanisms are equal to $\frac{1}{2r}$ when $\alpha \to 0$. This can be confirmed in Figures 4 and 5. Hence, when α is close to zero, $x^I \approx x^J$ and $z^I \approx z^J$, so $\pi_s^I \approx \pi_s^J$, but $\Pi_i^I < \Pi_i^J$ because the joint mechanism has an audit cost saving of $\frac{\alpha}{2}$ compared to the independent mechanism. By continuity, there must exist a range $(0, \alpha')$, with $0 < \alpha' \leq \infty$, such that $2\,\Pi^J + \pi_s^J \geqslant 2\,\Pi^I + \pi_s^I$, which is statement (i) in Proposition 4.

Parts (ii) and (iii) in Proposition 4 are conditions to ensure that the supply chain will earn net positive savings through compliance. In contrast, if there is a net loss through compliance, then the joint mechanism might lead to lower supply chain profits compared to the independent mechanism. This can only happen when α is large so the audit cost advantage of the joint mechanism has less impact – see the discussion of part (i) of Proposition 4 in the previous paragraph.

The shared mechanism is harder to analyze because we only have an implicit characterization of the audit level z^S as stated in Lemma 2. Our best attempt is summarized in Proposition 5 in Appendix A, which is similar to part (iii) of Proposition 4. Nevertheless, in our extensive numerical study we observed that the results in Proposition 4 – in particular, parts (i) and (ii) – also held true for the shared mechanism as shown, for instance, in Figure 8.

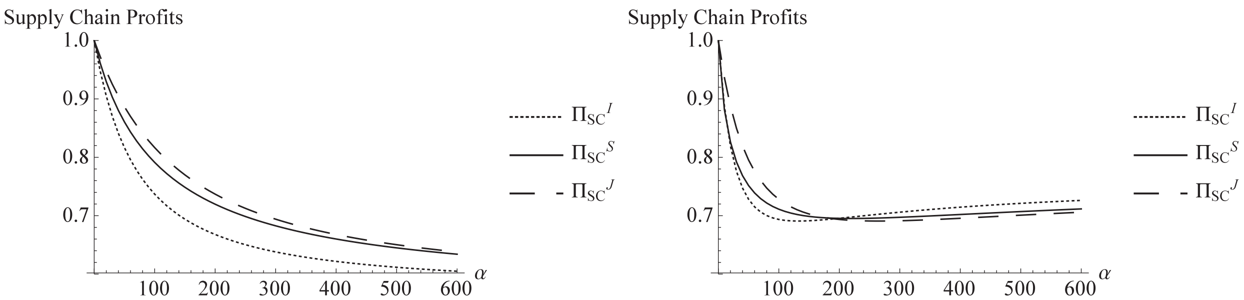


Figure 8 Supply chain profits (normalized) for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

The left plot in Figure 8 has $d > \gamma$, so $2\,\Pi^J + \pi_s^J \geqslant 2\,\Pi^I + \pi_s^I$ for all α per part (ii) of Proposition 4. We observe the same for the shared mechanism. The right plot in Figure 8 has $d < \gamma < 2d$ and $(d-m) < 2(d-m) < g + w$, so neither parts (ii) or (iii) of Proposition 4 apply (and Proposition 5 for the shared mechanism does not apply either). Hence, in the right plot of Figure 8 only part (i) of Proposition 4 applies and the joint mechanism yields a higher supply chain profit for lower values of α (here $\alpha' = 190.83$), but for larger values the independent mechanism is better from a channel perspective. The same can be said for the shared mechanism. Overall, as consumers become more aware of compliance issues, one would expect the collateral damage d to become high enough such that $d > \gamma$, which would ensure that the joint (or shared) mechanism yields a higher profit for any audit cost α.

3.5. Comparison of consumer surplus

The profit comparisons are crucial from the supply chain perspective. However, from the social responsibility perspective, one needs to consider the impact of the mechanisms on consumer surplus.

Consider a typical end consumer who derives an intrinsic utility V from the product and hence, gains a surplus of $V-p$ when consuming one unit of the product. The expected utility of a representative consumer is thus given by $(V-p)\cdot \Pr(Sale)$, where $\Pr(Sale)$ is the probability that trade occurs in equilibrium. We assume that $V>p$, else the consumer would not purchase the product. Thus, to compare the consumer surplus under different audit-penalty mechanisms, it suffices to observe the sale probability $\Pr(Sale)$ under each of the mechanisms.[2] The sale probabilities under the independent, joint and shared mechanisms are given as: $S^I \equiv 1-z^I(1-x^I), S^J \equiv 1-z^J(1-x^J)$, and $S^S \equiv 1-z^S(2-z^S)(1-x^S)$. With these definitions we have the following result:

LEMMA 3. 1. *$S^J > S^I$ if and only if $\sqrt{2}r(d-m) > \alpha$.*
2. *There exists a threshold value α_J such that $S^J > S^S$ if and only if $\alpha < \alpha_J$.*
3. *There exists a value α_I such that $S^I < S^S$ if and only if $\min\{\alpha_I, \widetilde{\alpha}\} < \alpha < \max\{\alpha_I, \widetilde{\alpha}\}$, where $\widetilde{\alpha}$ is defined as in Proposition 1.*

Lemma 3 shows that the independent mechanism has a higher sale probability than the joint mechanism when the audit cost α is large. The sale probability is better in the joint mechanism when the compliance level is relatively high – in fact, much greater than 0.5 – but such high compliance level can only be attained if auditing is not too costly, as shown in Figure 5. Hence, consumer surplus is lower under the joint mechanism when α is large. Note however, that the parameter d implicitly captures how much society values compliance. As d increases, it follows from Lemma 3 that there is a wider range of α for which the sale probability is higher in the joint mechanism than in the independent case. Similar observations can be made for the shared mechanism.

[2] Our model assumes that p is exogenous. In practice, some consumer might be willing to pay a premium for responsible sourcing practices.

4. Endogenous Wholesale Price

In this section, we extend our model to the case when the wholesale price w_i and the audit level z_i are endogenously determined by buyer i and when the compliance level x is endogenously determined by the supplier. Since the game for each of the three mechanisms involves 5 different decisions, i.e., $(w_1, z_1; w_2, z_2; x)$, selected by 3 players (2 buyers and 1 supplier), the analysis is complex and the analytical comparisons across all the three mechanisms are no longer tractable. Therefore, we make these comparisons through numerical analysis. To facilitate such analysis, we solve a two-stage game: in the first stage the buyers simultaneously choose the wholesale prices and then the second stage corresponds to the simultaneous game analyzed in Section 3. Note that the Alliance for Bangladesh does not include any provisions for the garment prices, whereas the Accord only states that prices should ensure financial feasibility (see Table 1 in Jacobs and Singhal 2015). In other words, these consortiums do not address pricing and auditing simultaneously, which is consistent with our sequential approach.

To incorporate the issue of endogenous wholesale price to be determined by each buyer, we define two additional terms: (a) buyer i's profit margin $m_i \equiv p - w_i$, $i = 1, 2$; and (b) the supplier's "rate of return on compliance to buyer i's audit" $r_i \equiv \frac{g+w_i}{2\gamma}$. Notice that both terms depend on the wholesale price w_i to be determined by buyer i. In what follows, we first describe how we determine the best-response functions (i.e., the supplier's compliance level and the buyers' audit level) for any given wholesale price vector (w_1, w_2) under each of the three mechanisms. We then explain how we compute the wholesale price and the corresponding profits in equilibrium.

4.1. Independent Mechanism I

By using the same approach presented in Section 2.1, it is easy to check that, for any given wholesale price vector (w_1, w_2), the supplier's profit and the buyers' profit can be written as

$$\begin{aligned}\pi_s(x; z_1, z_2, w_1, w_2) &= \sum_{i=1}^{2}[w_i(1 - z_i(1-x)) - gz_i(1-x) - c] - \gamma x^2 \\ &= \sum_{i=1}^{2}(w_i - c) - (w_i + g)(1-x) \cdot \sum_{i=1}^{2} z_i - \gamma x^2, \qquad (19)\\ \Pi_i(z_i, w_i; x) &= m_i(1 - z_i(1-x)) - \alpha z_i^2 - d(1-z_i)(1-x), \; i = 1, 2. \qquad (20)\end{aligned}$$

On solving the simultaneous game between the supplier and the buyers for a given wholesale price vector (w_1, w_2), we obtain the equilibrium audit and compliance decisions as below:

$$z_i^I(w_1,w_2) = \frac{(d-m_i)}{2\alpha + r_1(d-m_1) + r_2(d-m_2)},\ i=1,2, \tag{21}$$

$$x^I(w_1,w_2) = \frac{r_1(d-m_1) + r_2(d-m_2)}{2\alpha + r_1(d-m_1) + r_2(d-m_2)}. \tag{22}$$

By substituting the above equilibrium into (19) and (20), we obtain the profits of the supplier and the buyers, which we denote by $\pi_s^I(w_1,w_2)$ and $\Pi_i^I(w_1,w_2), i=1,2$, respectively.

By using $\pi_s^I(w_1,w_2)$ and $\Pi_i^I(w_1,w_2)$ and by inducting backward we obtain the equilibrium wholesale prices w_1^I and w_2^I by solving a non-cooperative game between the two buyers as follows. First, we consider the bounds imposed on wholesale prices by Assumptions 1 and 2 (i.e., $\max\{0, p-d\} \leqslant w_i \leqslant \min\{p, g+c\}$) and by Assumption 3 (i.e., $w_1 + w_2 - 2c \geqslant \gamma$). We then compute the best-response function of buyer i (i.e., $w_i^*(w_j)$) numerically by solving the following problem of buyer i for different values of w_j:

$$\begin{aligned}
\mathbf{PI}: \qquad & \max_{w_i} \Pi_i^I(w_i, w_j) \\
\text{subject to}\ & (21), (22), \\
& \max\{0, p-d\} \leqslant w_i \leqslant \min\{p, g+c\} \text{ for } i=1,2, \\
& w_1 + w_2 - 2c \geqslant \gamma, \\
& \Pi_i^I(w_i, w_j) \geqslant 0, \text{for } i=1,2.
\end{aligned}$$

In this problem, the last two constraints correspond to the individually rational constraints associated with the supplier and buyers, respectively. Next, we determine the equilibrium wholesale price w_1^I and w_2^I as the point of intersection of the above derived best-response functions. As the buyers are identical, we observe that $w_1^I = w_2^I \equiv w^{I*}$. Finally, we retrieve the corresponding equilibrium outcomes $(z^{I*}, x^{I*}, \pi_s^{I*}, \Pi^{I*})$ through substitution.

4.2. Joint Mechanism J

For any given wholesale price w_1 and w_2, we can use the same approach as presented in Section 2.2 to determine the supplier's profit as:

$$\pi_s(z) = \max_{x\in[0,1]} \{(w_1+w_2)(1-z(1-x)) - 2gz(1-x) - 2c - \gamma x^2\} \tag{23}$$

where z is the joint audit level adopted by the consortium. The best response of the supplier is obtained as $x^J(z) = \min\{(r_1 + r_2)z, 1\}$.

Now suppose buyer i is able to select unilaterally the joint audit level z. Then, buyer i would choose a joint audit level of

$$z = z_i(\theta_i) \equiv \frac{(d - m_i)(1 - x)}{2\alpha\theta_i} \tag{24}$$

that maximizes its profit

$$\Pi_i(\theta_i; z, x) = m_i(1 - z(1 - x)) - d(1 - z)(1 - x) - \theta_i \alpha z^2. \tag{25}$$

Thus, if $\frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j}$ then both buyers choose the same joint audit level and hence would automatically reach a consensus. Using this fact, we assume that buyer i and buyer j agree a priori to share the audit cost in the ratio $\frac{\theta_i}{\theta_j} = \frac{d-m_i}{d-m_j}$. As before, we make this assumption for ease of exposition and in Appendix B we formally show that $\frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} = \frac{1}{2d-m_1-m_2}$ is the outcome of a non-cooperative game. By using these proportions θ_1 and θ_2, we can determine the equilibrium audit and compliance levels as:

$$z^J \equiv z^J(w_1, w_2) = \frac{(2d - m_1 - m_2)}{2\alpha + (r_1 + r_2)(2d - m_1 - m_2)} \tag{26}$$

$$x^J \equiv x^J(w_1, w_2) = \frac{(r_1 + r_2)(2d - m_1 - m_2)}{2\alpha + (r_1 + r_2)(2d - m_1 - m_2)}. \tag{27}$$

By substituting the equilibrium above into (23) and (25), we can express the supplier's and buyer i's profits as $\pi_s^J(w_1, w_2)$ and $\Pi_i^J(w_1, w_2)$; respectively. We then induct backwards to obtain the equilibrium wholesale prices w_1^J and w_2^J by solving a non-cooperative game between the two buyers. We obtain the best-response function of buyer i by solving the problem **PJ**, which is the same as problem **PI** except that the profit function $\Pi_i^J(w_i, w_j)$ is based on the equilibrium expressions (26) and (27) (instead of (21), (22)). The ensuing procedure to obtain the equilibrium outcomes $(z^{J*}, x^{J*}, \pi_s^{J*}, \Pi^{J*})$ is the same as in explained in Section 4.1.

4.3. Shared Mechanism S

Akin to (13) and (14), we obtain the supplier's and the buyers' profits as:

$$\pi_s(x; z_1, z_2, w_1, w_2) = \sum_{i=1}^{2} \{(w_i - c) - (w_i + g)(1 - x) \cdot (z_i + z_j + z_i z_j)\} - \gamma x^2, \tag{28}$$

$$\Pi_i(z_i, w_i; z_j, x) = m_i(1 - (z_i + z_j + z_i z_j)(1 - x)) - \alpha z_i^2 - d(1 - z_i)(1 - z_j)(1 - x), \tag{29}$$

so that the best-response functions of the players for any given wholesale price vector (w_1, w_2) are

$$z_i^S = \frac{(d - m_i)(1 - z_j^S)(1 - x^S)}{2\alpha},\ i = 1, 2,\ i \neq j, \tag{30}$$

$$x^S = (r_1 + r_2)(z_1^S + z_2^S - z_1^S z_2^S), \tag{31}$$

where for notational convenience we suppress the arguments (w_1, w_2) of z_i^S and x^S. As before, we obtain the equilibrium wholesale prices by solving the best-response functions of the two buyers simultaneously. The best-response function of buyer i is obtained by solving the problem **PS**, which is analogous to **PI** and **PJ**. The remaining steps to obtain the equilibrium outcomes $(z^{S*}, x^{S*}, \pi_s^{S*}, \Pi^{S*})$ are the same as in the independent and joint mechanisms.

4.4. Numerical Analysis

In this section, we use the approach outlined in sections 4.1, 4.2 and 4.3 to compute the equilibrium outcomes (i.e., $w^{k*}, z^{k*}, x^{k*}, \pi_s^{k*}, \Pi^{k*}$) associated with mechanism k, where $k = I, J, S$. Also we used the same parameter values as in Section 3 (except the fact that the wholesale price w_i is now computed instead of exogenously given). The following figures summarize our results.

First, since the buyers impose a collective penalty under the joint and shared mechanisms, one would expect the buyers to offer a higher wholesale price under these mechanisms than under the independent mechanism to incentivize the supplier. This intuition is confirmed in Figure 9, but only when the buyer's audit cost α is sufficiently high. This is because when audit costs are low, the buyers can afford to audit at a higher level, which in turn increases supplier's compliance without the need to offer higher wholesale prices.

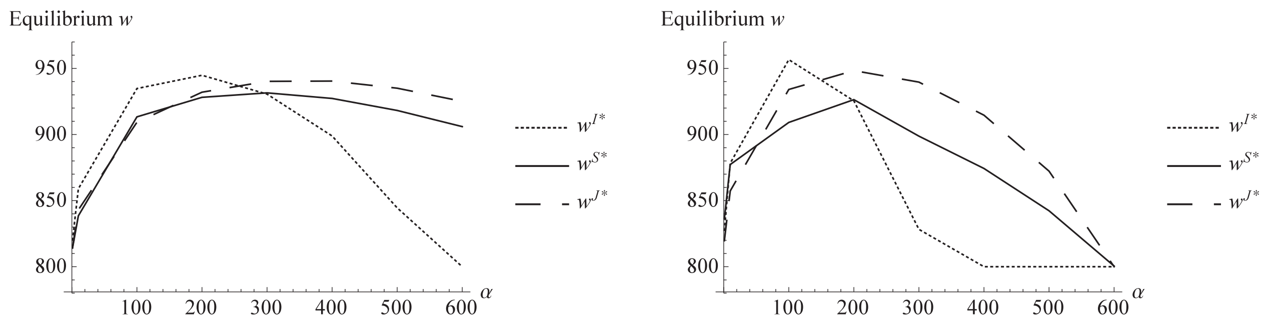


Figure 9 Equilibrium wholesale price w for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

Second, when the wholesale price is endogenously determined by the buyers, Figures 10 and 11 indicate that the results stated in Proposition 1 continue to hold for the case when the buyer's audit cost α is low. More importantly, we confirm that the joint and the shared mechanisms can make the supplier more compliant. However, contrary to the finding made in Proposition 1, when α is high and the wholesale prices are endogenous, we notice that the buyers audit more under the shared mechanism than what they would otherwise do under the independent mechanism. Additionally, as depicted in Figure 9, when α is high, the buyers also offer a higher wholesale price to encourage a higher supplier compliance under the shared mechanism. Thus, the buyers use higher audit levels and higher wholesale prices as two levers to increase supplier's compliance under the shared mechanism when the wholesale price is endogenously determined.

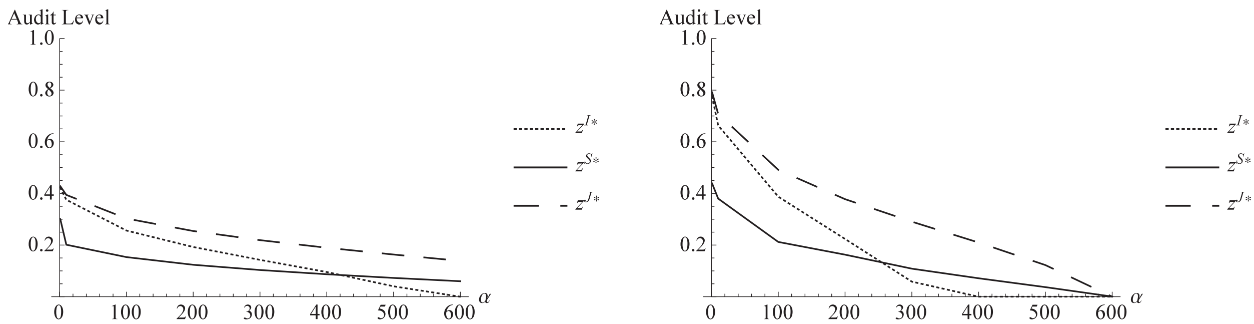


Figure 10 Audit levels when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

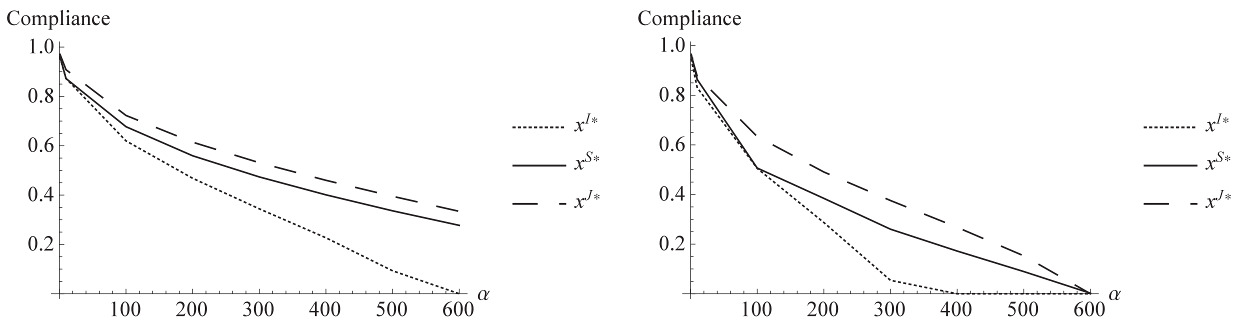


Figure 11 Compliance levels when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

Third, Figures 12 and 13 indicate that, among all three mechanisms, the buyers earn the most and the supplier earns the least under the joint mechanism. This finding is consistent with Propositions 2 and 3. Hence, from the buyer's perspective, the joint mechanism still dominates the other two mechanisms. Note from Figure 12 that the supplier always makes a positive profit when $\alpha > 0$ under all three mechanisms. In contrast, Figure 13 shows that the buyers' profit vanishes when the audit cost α is significantly high, and this happens sooner than with exogenous w because the competitive pressure makes the buyers' profit decrease faster.

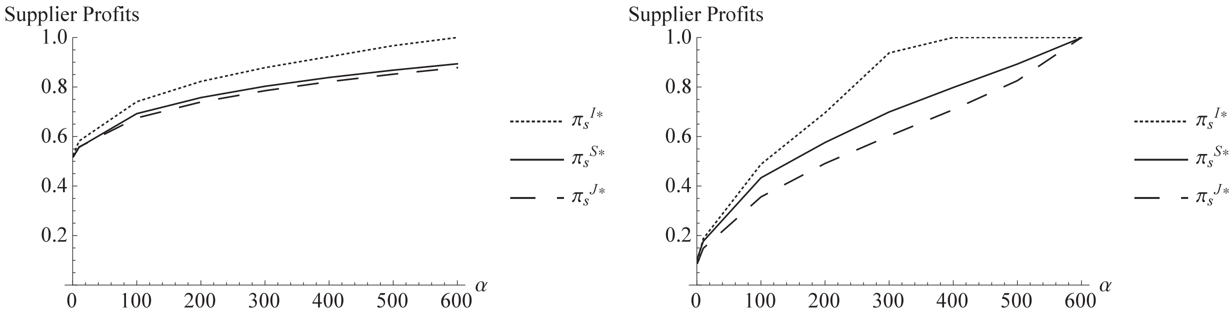


Figure 12 **Supplier's profits when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)**

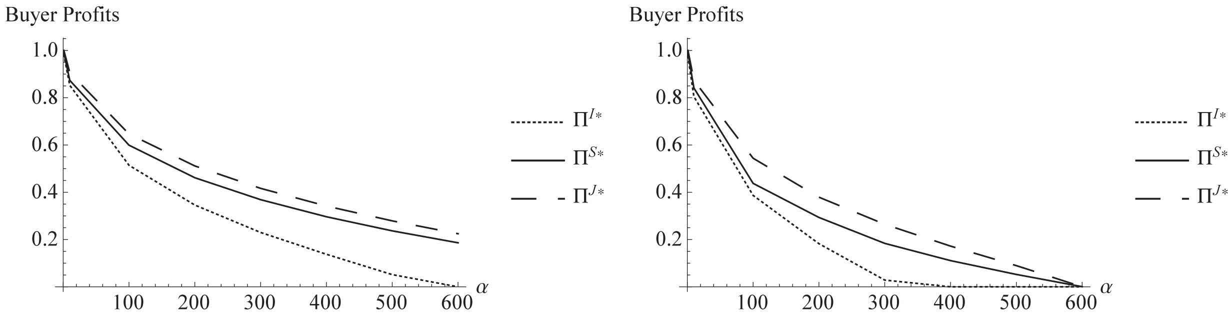
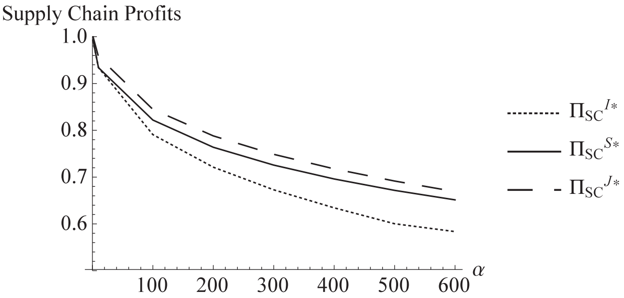


Figure 13 **Buyers' profits when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)**

Finally, Figure 14 is the counterpart of Figure 8 when the wholesale prices are endogenous. We observe the same results as in Proposition 4. In particular, when the buyers' damage costs is higher than the supplier's cost of compliance ($d > \gamma$), the joint and shared mechanisms create supply chain value compared to the independent mechanism for all values of α. This allows for a transfer-payment to compensate the supplier for its higher compliance. In general, a Pareto-improving transfer-payment is always possible when the audit cost α is low enough, as seen in the right plot of Figure 14.

Thus, as demonstrated by the numerical analysis in this section, the key analytical results that we obtained with an exogenous wholesale price in Section 3 continue to hold when the wholesale price is endogenous.



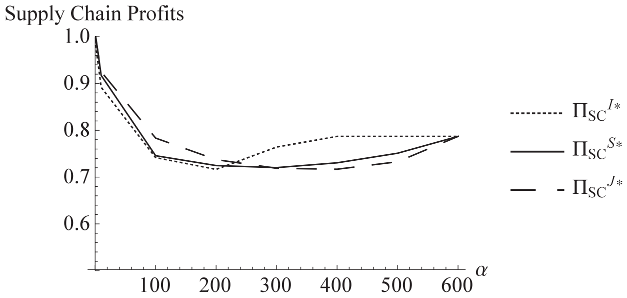


Figure 14 **Supply chain profits when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)**

5. Discussion

In this section we discuss some of our model implications in relation to the Alliance and the Accord. It should be noted that our model by no means fully represents these agreements; instead, it captures various salient features especially the audit-penalty mechanism. Nevertheless, our findings can be relevant for the design of future consortia.

The Alliance and the Accord are fundamentally similar in many aspects (Labowitz and Baumann-Pauly 2014) and both advocate joint audits. However, one important difference is that the Accord is legally binding whereas the Alliance is not (Economist 2013). Specifically, under the Accord, factory workers can take legal action if they believe that the Accord fails to "follow through on their commitment".[3] This can be incorporated in our model through the damage cost d. Assuming a higher damage cost d for the Accord would be consistent with the additional legal costs faced by the Accord when its auditing effort fails to detect non-compliance. A higher damage cost implies higher audit and compliance levels (per Lemma 4), but it also implies lower profits for the buyers. So this would indicate that the Accord might ensure safer factories compared to the Alliance, but at the expense of lower profits due to a higher liability.

The Accord stipulates that a non-compliant factory that fails to eliminate safety hazards must be terminated. This commitment is also legally binding.[4] In contrast, the Alliance is not legally bound to terminate a non-compliant factory. In other words, there is a positive chance that the buyers might continue to do business with a factory that failed the audit. This can be incorporated in our model through the goodwill cost g.[5] Assuming a lower goodwill cost g for the Alliance would be consistent with the fact that the supplier is less

[3] `http://www.cleanclothes.org/resources/background/comparison-safety-accord-and-the-gap-walmart-scheme`

[4] `http://www.just-style.com/news/bangladesh-accord-cuts-ties-with-four-more-factories_id127323.aspx`

[5] Alternatively, one can include an expected payment from the buyer to the supplier that is proportional to $z(1-x)$.

likely to be terminated when non-compliance is detected. If g is lower, then the rate of return on compliance r is lower, and per Equation (10) the audit and compliance levels will decrease.

Aside from being legally binding or not, both agreements stipulate contributions from the buyers toward helping the supplier's compliance. This can be incorporated into the model by assuming that the buyers incur a certain portion δ of the compliance cost γx^2. It can be shown that when $\delta > 0$, for all three mechanisms (I, J, and S): the compliance level is higher, the audit level is lower, and the supplier's profit increases. In contrast, the buyers' profit increases only when the audit cost α is high.[6] Hence, as expected, providing financial assistance benefits the supplier but might not be in the best interest of the buyers.

Finally, we have shown that the joint mechanism effectively increases compliance, so both the Alliance and the Accord should be able to achieve their primary goal. If these consortiums also want to ensure that the suppliers are better off (or at least not worse off), then our results show that some form of transfer-payment is needed.

6. Conclusions and Future Work

In this paper, we presented a unified framework of three different audit-penalty mechanisms (independent, joint, and shared) for improving supplier's compliance in supply chains. By considering a simultaneous move game involving 2 buyers and 1 supplier, we analyzed and compared the equilibrium outcomes (the supplier's compliance level, the buyer's audit level, the supplier's profit, the buyers' profits and the supply chain profit) across all three mechanisms for the case when the wholesale price is exogenously given. We also extended our analysis to the case when the wholesale price associated with each mechanism is endogenously determined by the buyers. We show that the joint mechanism dominates in terms of supplier compliance and the buyers' profit. Moreover, in our numerical analysis we observe that the key structural findings that we made for the case of exogenous wholesale price continued to hold even when the wholesale price is endogenously determined by the buyers.

Overall, we can summarize the key findings for the joint mechanism as follows:

1. The supplier's compliance always improves, and it always results in higher buyer profit under the joint mechanism.
2. The supplier, however, earns the lowest profit under the joint mechanism and earns the highest profit under the independent mechanism.

[6] The details of this analysis is available from the authors upon request.

3. The buyers have to offer a Pareto-improving transfer-payment to the supplier to make the latter better off under the joint mechanism.
4. Such transfer-payment is possible when the audit cost is low or when the buyers' damage cost is higher than the supplier's cost of compliance. When these conditions hold, the supply chain profit under the joint mechanism is higher than the profit under the independent mechanism and this enables the buyers to provide the Pareto-improving transfer-payment.

We find similar results for the shared mechanism, which shows that it is also a viable mechanism to create supply chain value through collective penalty.

Overall, our results enable us to gain a better understanding about the dynamic interactions among the buyers and the supplier under independent, joint and shared mechanisms. Since the joint mechanism captures two salient features (collective penalty and joint audits), our results provide additional justification for the implementation of the Accord and the Alliance in Bangladesh.

Future research could consider alternative audit-penalty mechanisms and settings where our modeling assumptions do not apply. These include settings in which the buyers are non-identical (different price/cost structure, different bargaining power, etc.), scenarios with incomplete information on costs, or an extension in which the retail price p is endogenous. All of this could potentially affect the ordering of the three mechanisms. Given the current concerns over supplier compliance, addressing these questions could be worthwhile avenues for future research.

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Appendix A: Supplemental Results.

LEMMA 4. *Under the joint mechanism J, the buyer's joint audit level z^J and the supplier's compliance level x^J given in* (10) *possess the following properties:*

(i) The supplier's compliance level is always higher than the buyer's audit level (i.e., $x^J = 2rz^J > z^J$).

(ii) Both the supplier's compliance level x^J and the buyer's audit level z^J are increasing in the buyer's damage cost d and are decreasing in the buyer's audit cost α.

(iii) The supplier's compliance level x^J is decreasing in the supplier's compliance cost γ. However, the buyer's audit level z^J is increasing in γ.

(iv) The supplier's compliance level x^J is increasing in the supplier's goodwill cost g. However, the buyer's audit level z^J is decreasing in g.

(v) The supplier's compliance level x^J is increasing in the wholesale price w. However, the buyer's audit level z^J is increasing in w if, and only if, $w < \sqrt{\alpha\gamma} - (d - p)$.

Lemma 5. *Under the shared mechanism S, the buyer's joint audit level z^S and the supplier's compliance level x^S given in Lemma 2 possess the following properties:*

(i) The supplier's compliance level is higher than the buyer's audit level (i.e., $x^S > z^S$).

(ii) Both the supplier's compliance level x^S and the buyer's audit level z^S are increasing in the buyer's damage cost d and are decreasing in the buyer's audit cost α.

(iii) The supplier's compliance level x^S is decreasing in the supplier's compliance cost γ. However, the buyer's audit level z^S is increasing in γ.

(iv) The supplier's compliance level x^S is increasing in the supplier's goodwill cost g. However, the buyer's audit level z^S is decreasing in g.

(v) The supplier's compliance level x^S is increasing in the wholesale price w. The buyer's audit level z^S is decreasing in w when w is sufficiently large.

Proposition 5. *The total supply chain profit under the shared mechanism is higher than that under the independent mechanism if the total cost of non-compliance for both buyers is larger than the cost of non-compliance for the supplier (i.e., $2(d-m) > g + w$) and $\alpha \geq \widetilde{\alpha}$, where $\widetilde{\alpha}$ is defined as in Proposition 1*

Appendix B: Proportional Sharing of Joint Audit Cost under the Joint Mechanism.

B.1. Exogenous Wholesale Prices

Here we provide the details of the non-cooperative game under the joint mechanism. To ensure that there is an implementable joint audit, we assume that the consortium will agree to adopt the "minimum-audit-level rule" that we describe shortly. This rule embodies the notion of the weakest link or minimum effort that underpins many coordination problems that are modeled as non-cooperative games, see Camerer (2003). Though this is one particular rule to reach consensus, it should be noted that the same results shown here below can be obtained by formulating the joint mechanism as a unanimous game, see Caro et al. (2015).

The buyers have to agree on the joint audit level and the audit cost sharing. In a non-cooperative setting, buyer i would have to propose an audit level z_i and a share θ_i of the audit cost. Hence, each buyer has a two-dimensional strategy space. Analyzing such kind of game is complex. Moreover, without additional structure the profit of buyer i might not be jointly concave in z_i and θ_i. To avoid these problems, recall that

Equation (9) provides a one-to-one mapping between the share θ_i and buyer i's "ideal" joint audit level. We use this relation to reduce buyer i's strategy space to $\theta_i \in [0,1]$ as shown next.

We now introduce the audit level selection process that is agreed upon by both buyers a priori. Specifically, the buyers play a game in which they simultaneously propose the share of the auditing cost each one of them would like to pay. In other words, buyer i proposes θ_i and buyer j proposes θ_j.[7] The outcome of the game is determined according to the following rules:

1. If $\theta_i \neq \theta_j$, then the audit level adopted by the consortium is $z = \min\{z_i(\theta_i), z_j(\theta_j)\}$, where $z_i(\theta_i)$ is given in Equation (9), and the total audit cost will be shared according to the proportion that is proposed by the buyer whose audit level is adopted.
2. If $\theta_i = \theta_j = \theta \geq \frac{1}{2}$, then the joint audit level is $z = z_i(\theta) = z_j(\theta)$ and each buyer pays a proportion θ of the auditing cost.
3. If $\theta_i = \theta_j < \frac{1}{2}$, then the consortium is not formed and the independent mechanism takes place.

Since $z_i(\theta_i) < z_j(\theta_j)$ if and only if $\theta_i > \theta_j$, the minimum-audit-level rule reduces to verifying which buyer is willing to pay a higher share of the auditing cost. With this audit selection process, buyer i's profit can be written as:

$$\Pi_i^J(\theta_i;\theta_j,x) = \begin{cases} m(1-z_i(\theta_i)(1-x)) - d(1-z_i(\theta_i))(1-x) - \theta_i\,\alpha\, z_i(\theta_i)^2 & \text{if } \theta_i > \theta_j \\ m(1-z_i(\theta)(1-x)) - d(1-z_i(\theta))(1-x) - \theta\,\alpha\, z_i(\theta)^2 & \text{if } \theta_i = \theta_j = \theta \geqslant \frac{1}{2} \\ m(1-z_j(\theta_j)(1-x)) - d(1-z_j(\theta_j))(1-x) - (1-\theta_j)\,\alpha\, z_j(\theta_j)^2 & \text{if } \theta_i < \theta_j \\ \Pi^I(z^I) & \text{if } \theta_i = \theta_j < \frac{1}{2}. \end{cases} \tag{32}$$

The buyers' simultaneous actions θ_i and θ_j are essentially a coordination game and as such there are multiple equilibria (Fudenberg and Tirole 1991). In fact, any $\theta \in [0,1]$ such that $\theta_i = \theta_j = \theta$ corresponds to an equilibrium. To select one equilibrium point, we adopt the payoff dominance refinement proposed by Harsanyi and Selten (1988). Specifically, we show that the equilibrium $\theta_1 = \theta_2 = \frac{1}{2}$ in which the buyers equally share the joint audit cost is payoff dominant. This is formalized in Lemmas 6 and 7.

LEMMA 6. *Under the minimum-audit-level rule, each buyer will agree to share the joint audit cost equally, i.e., $\theta_1 = \theta_2$ in equilibrium.*

LEMMA 7. *The payoff dominant equilibrium of the joint mechanism game is given by $\theta_1 = \theta_2 = \theta = \frac{1}{2}$.*

B.2. Endogenous Wholesale Prices

Note that since $z_i(\theta_i) = \frac{(d-m_i)(1-x)}{2\alpha\theta_i}$, we have $z_i(\theta_i) < z_j(\theta_j)$ if, and only if, $\frac{\theta_i}{d-m_i} > \frac{\theta_j}{d-m_j}$. Thus, the profit of buyer i under J with unequal wholesale prices and "minimum-audit-level rule" is given by

$$\Pi_i^J(\theta_i;\theta_j,x) = \begin{cases} m_i(1-z_i(\theta_i)(1-x)) - d(1-z_i(\theta_i))(1-x) - \theta_i\alpha z_i(\theta_i)^2 & \text{if } \frac{\theta_i}{d-m_i} > \frac{\theta_j}{d-m_j} \\ m_i(1-z_i(\theta)(1-x)) - d(1-z_i(\theta))(1-x) - \theta\alpha z_i(\theta)^2 & \text{if } \frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} = \frac{1}{2d-m_1-m_2} \\ m(1-z_j(\theta_j)(1-x)) - d(1-z_j(\theta_j))(1-x) - (1-\theta_j)\alpha z_j(\theta_j)^2 & \text{if } \frac{\theta_i}{d-m_i} < \frac{\theta_j}{d-m_j} \\ \Pi^I(z^I) & \text{if } \frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} < \frac{1}{2d-m_1-m_2} \end{cases} \tag{33}$$

where $m_i = p - w_i$. In the last case, when $\frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} < \frac{1}{2d-m_1-m_2}$, the consortium is not formed and each buyer resorts to an independent audit. The following lemmas are equivalent to Lemmas 6 and 7.

[7] The supplier also participates in the game by simultaneously choosing the compliance level x.

LEMMA 8. *For a given wholesale prices w_1 and w_2, the buyers' equilibrium choice of θ_1 and θ_2 satisfy the condition $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$. Hence, the buyers choose the same audit level in equilibrium.*

LEMMA 9. *The equilibrium given by $\theta_i = \frac{d-m_i}{2d-m_1-m_2}$, $i = 1, 2$, is payoff dominant.*

Appendix C: Numerical Study with $d >> g$

Here we present numerical results when the collateral penalty of the buyers d is much larger than the goodwill cost g experienced by the supplier. This scenario is arguably more realistic because in cases of non-compliance the market tends to punish more the buyers and put less blame on the supplier (due to the fact it is located in developing country). The following figures assume $d = 2g = 2000$. All the other parameters remain the same as in Sections 3 and 4.

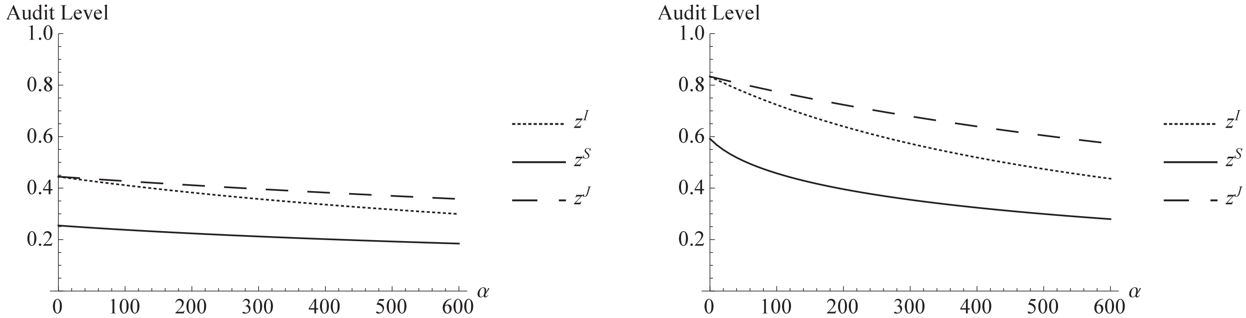

Figure 15 **Audit levels when $d = 2g = 2000$. Left plot has $\gamma = 800$ and right plot has $\gamma = 1500$.**

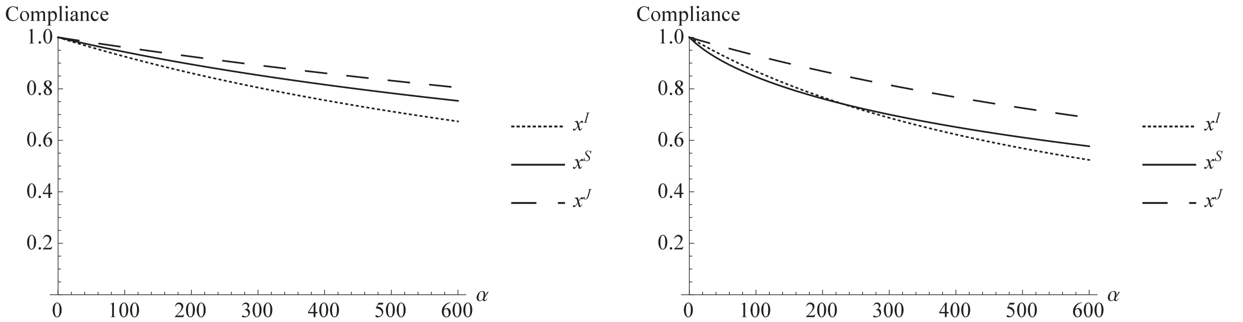

Figure 16 **Compliance levels when $d = 2g = 2000$. Left plot has $\gamma = 800$ and right plot has $\gamma = 1500$.**

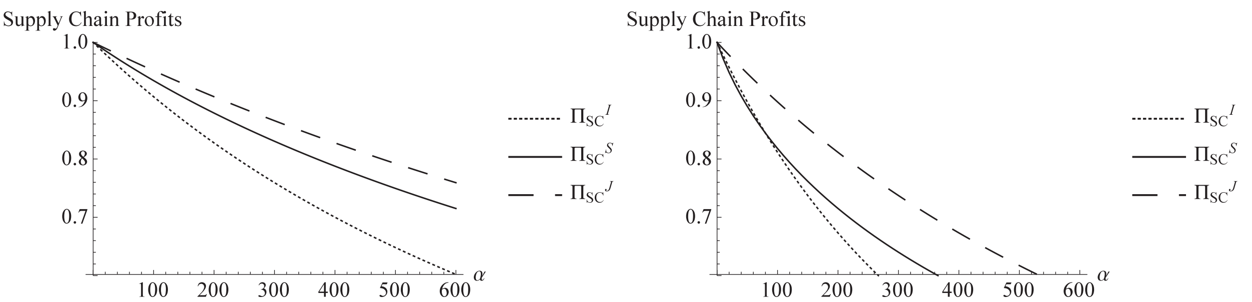

Figure 17 **Supply chain profits (normalized) when $d = 2g = 2000$. Left plot has $\gamma = 800$ and right plot has $\gamma = 1500$.**

Figures 16 and 15 show that the audit and compliance levels are higher compared to the scenarios with $d = g = 1000$, especially for high values of the audit cost α. This follows from Lemmas 1, 4, and 5. Figure 17 shows the supply chain profits. Note that $d > \gamma$ so from Proposition 4(ii) it follows that the joint mechanism achieves higher supply chain profits for all values of α. We omit the figures when the wholesale price is endogenous because they look very similar to the exogenous case. In contrast to Figure 9, when $d = 2000$ the wholesale price in equilibrium is constant for all relevant values of α. The reason is that a high penalty d pushes the buyers to audit more, which in turn increases the compliance level, so they do not have to use the wholesale price to incentivize the supplier. Consequently, the buyers lower the wholesale price as much as possible and the constraint $w_1 + w_2 - 2c \geq \gamma$ becomes active.

Appendix D: Proofs

Proof of Lemma 1: The first statement follows immediately from the fact that $2r > 1$. All other statements can be obtained from differentiating x^I and z^I given in (3) with respect to the corresponding parameter. However, to prove the last statement, one needs to account for the fact that $r = \frac{g+w}{2\gamma}$ and $m = (p - w)$. We omit the details. ■

Proof of Lemma 2: Observe from (15) that $z_1 = z_2 = z$ by symmetry and apply (15) to show that $x^S = 2rz^S(2 - z^S)$. This proves the second statement. Next, by substituting $x = 2r(2z - z^2)$ into (15) and by rearranging the terms, the buyer's audit level z is the solution to $V(z) = 0$. By showing that $V(0) < 0, V(1 - \sqrt{\frac{2r-1}{2r}}) > 0$ and $V(z)$ is concave over $[0, 1 - \sqrt{\frac{2r-1}{2r}}]$, we prove the first statement and that $z^S \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$. Next, observe that $x^S(z) = 2rz(2 - z)$ is increasing in z when $z \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$, that $x^S(0) = 0$ and that $x^S(1 - \sqrt{\frac{2r-1}{2r}}) = 1$, we can use the fact that $z^S \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$ to show that $x^S \in (0, 1)$. ■

Proof of Lemma 3: We have that $S^J - S^I = x^I(1 - x^I) - x^J(1 - x^J) = \dfrac{\alpha r(d-m)\,(2r^2(d-m)^2 - \alpha^2)}{(\alpha + 2r(d-m))^2(\alpha + r(d-m))^2}$. Hence, $S^J > S^I$ if and only if $\sqrt{2}\,r(d-m) > \alpha$. Similarly, $S^S - S^J = x^J(1 - x^J) - x^S(1 - x^S) = (x^J - x^S)(1 - x^J - x^S)$, so that $S^S - S^J \to 0^-$ as $\alpha \to 0^+$. Further, we know: (i) from Lemmas 4 and 5 that $\dfrac{dx^J}{d\alpha} < 0$ and $\dfrac{dx^S}{d\alpha} < 0$, (ii) from Equation (10) that $\lim_{\alpha\to\infty} x^J = 0$, and (iii) from Proposition 1 that $0 \leqslant x^S < x^J$, which indicates that $\lim_{\alpha\to\infty} x^S = 0$. Hence, we conclude that there exists a threshold α_J such that $S^S - S^J < 0$ if and only if $\alpha < \alpha_J$.

When comparing S^I and S^S, we have $S^S - S^I = x^I(1 - x^I) - x^S(1 - x^S) = (x^I - x^S)(1 - x^I - x^S)$. By noting that $\dfrac{dx^I}{d\alpha} < 0$, $\dfrac{dx^S}{d\alpha} < 0$, $\lim_{\alpha\to\infty} x^I = 0$, and $\lim_{\alpha\to\infty} x^S = 0$, we conclude that there exists a threshold α_I such that $(1 - x^I - x^S) < 0$ if and only if $\alpha < \alpha_I$. Further, by Proposition 1, $x^S > x^I$ if and only if $\alpha > \widetilde{\alpha} = \max\{(d-m)(\widetilde{r} - r), 0\}$. Therefore, if $\widetilde{r} \leqslant r$, then $x^S > x^I$ for all $\alpha \geqslant 0$, and hence $S^S - S^I > 0$ if and only if $\alpha < \alpha_I$.

When $\widetilde{r} > r$, as $\alpha \to 0^+$ we have $(x^I - x^S) \to 0^+$ and $(1 - x^I - x^S) \to -1$. Thus, $S^S - S^I < 0$ when α is sufficiently small (i.e., when $\alpha < \min\{\alpha_I, \widetilde{\alpha}\}$) and sufficiently large (i.e., when $\alpha > \max\{\alpha_I, \widetilde{\alpha}\}$). On the other hand, when α is moderate (i.e., when $\min\{\alpha_I, \widetilde{\alpha}\} < \alpha < \max\{\alpha_I, \widetilde{\alpha}\}$) then $S^S - S^I > 0$. ■

Proof of Lemma 4: The proof follows the same approach as the proof for Lemma 1. We omit the details. ■

Proof of Lemma 5: To prove the first statement, we use the fact that $2r > 1$ and the fact that $z^S \in (0,1)$ to show that $x^S = 2rz^S(2-z^S) > z^s + z^S(1-z^S) > z^S$. To prove the second statement, we differentiate (16) with respect to $k \equiv \frac{2\alpha}{d-m}$ and apply the implicit function theorem, getting: $U(z^S)\cdot\frac{dz^S}{dk} + z^S = 0$, where $U(z) = [6rz^2 - 12rz + (1+4r+k)]$. By noting that $U(z)$ is increasing in z and that $U(0) > 0$ and $U(1-\sqrt{\frac{2r-1}{2r}}) > 0$, we can conclude that $U(z^S) > 0$. Hence, we can conclude that $\frac{dz^S}{dk} < 0$. Also, observe that $\frac{dx^S}{dk} = 4r(1-z^S)\cdot\frac{dz^S}{dk} < 0$. Combine these results with the fact that k in increasing in α and decreasing in d, we obtain the desirable properties about z^S and x^S as stated in the second statement.

To prove the third statement, differentiate (16) with respect to r and apply the implicit function theorem, getting: $U(z^S)\cdot\frac{dz^S}{dr} + W(z^S) = 0$, where $U(z)$ is defined above and $W(z) = 2z(z^2-3z+2)$. By using the fact that both $U(z) > 0$ and $W(z) > 0$ for any $z \in (0,1)$, we can conclude that $\frac{dz^S}{dr} = -\frac{W(z^S)}{U(z^S)} < 0$. Next, observe that $\frac{dx^S}{dr} = 2z^S(2-z^S) + 4r(1-z^S)\cdot\frac{dz^S}{dr}$. By substituting $\frac{dz^S}{dr} = -\frac{W(z^S)}{U(z^S)}$ and by rearranging the terms and by using the fact that $V(z^S) = 0$, it can be shown that: $\frac{dx^S}{dr} = \frac{2z^S\cdot(4r(z^S)^2 - 4rz^S + 3 + 2k)}{U(z^S)} = \frac{2z^S\cdot(-2x^S + 4rz^S + 3 + 2k)}{U(z^S)} > 0$, where the last inequality is due to the fact that $x^S < 1$. Finally, by combining the result that $\frac{dz^S}{dr} < 0$ and $\frac{dz^S}{dr} > 0$ and by using the fact that $r = \frac{g+w}{2\gamma}$, we obtain the third statement.

To prove the fourth statement, implicitly differentiating the equation $V(z^S) = 0$ and the expression for x^S with respect to r, we get:

$$
\begin{aligned}
\frac{\partial z^S}{\partial r} &= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha - 6r(d-m)(2-z^S)z^S + (4r+1)(d-m)} \\
&= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha + (d-m)(1-2rz^S(2-z^S)) + 4r(d-m)(1-(2-z^S)z^S)} \\
&= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha + (d-m)(1-x^S) + 4r(d-m)(1-\frac{x^S}{2r})} < 0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial x^S}{\partial r} &= 2(2-z^S)z^S + 4r(1-z^S)\frac{\partial z^S}{\partial r} \\
&= -\frac{2(z^S-2)z^S(2\alpha + 2r(d-m)(z^S-2)z^S + d - m)}{2\alpha + (d-m)(1-2rz^S(2-z^S)) + 4r(d-m)(1-(2-z^S)z^S)} \\
&= \frac{2(2-z^S)z^S(2\alpha + (d-m)(1-2r(2-z^S)z^S))}{2\alpha + (d-m)(1-x^S) + 4r(d-m)(1-\frac{x^S}{2r})} \\
&= \frac{2(2-z^S)z^S(2\alpha + (d-m)(1-x^S))}{2\alpha + (d-m)(1-x^S) + 4r(d-m)(1-\frac{x^S}{2r})} > 0.
\end{aligned}
$$

It remains to prove the last statement. By noting that $r = \frac{g+w}{2\gamma}$ and $m = (p-w)$, we differentiate (16) with respect to w and apply the implicit function theorem to get:

$$
\begin{aligned}
&\frac{dz^S}{dw} = \frac{z^S\,(2\gamma\alpha - (2-z^S)(1-z^S)(d-p+w)^2)}{(d-p+w)\,[2\gamma\alpha + (d-p+w)(\gamma + 2(g+w) - 3(g+w)(2-z^S)z^S)]} \\
&\Rightarrow \frac{dz^S}{dw} \geqslant 0 \Leftrightarrow 2\gamma\alpha - (2-z^S)(1-z^S)(d-p+w)^2 \geqslant 0
\end{aligned}
$$

because, by using the fact that $x^S = 2rz^S(2 - z^S) \leqslant 1$ it can be easily verified that the denominator of the above expression is positive. Now,

$$
\begin{aligned}
&2\gamma\alpha - (2 - z^S)(1 - z^S)(d - p + w)^2 \geqslant 0 \Leftrightarrow (2 - z^S)(1 - z^S) \leqslant \frac{2\gamma\alpha}{(d - p + w)^2} \\
&\Leftrightarrow 2rz^S(2 - z^S)(1 - z^S) \leqslant \frac{4r\gamma\alpha z^S}{(d - p + w)^2} = \frac{2\alpha z^S(g + w)}{(d - p + w)^2} \\
&\Leftrightarrow 2rz^{S3} - 6rz^{S2} + 4rz^S + z^S\left[1 + \frac{2\alpha}{d - p + w}\right] \leqslant \frac{2\alpha z^S(g + w)}{(d - p + w)^2} + z^S\left[1 + \frac{2\alpha}{d - p + w}\right] \\
&\Leftrightarrow 1 \leqslant z^S\left[1 + \frac{2\alpha}{d - p + w} + \frac{2\alpha(g + w)}{(d - p + w)^2}\right] \quad \text{by using (16)} \\
&z^S \geqslant \left[1 + \frac{2\alpha}{d - p + w} + \frac{2\alpha(g + w)}{(d - p + w)^2}\right]^{-1} = [f(w)]^{-1} \text{ (say)}
\end{aligned}
$$

By noting that $\alpha > 0$, $f'(w) < 0$, $\lim_{w\to\infty} f(w) = 1$, and $\lim_{w\to\infty} z^S = 0$ (from (16)) we infer that there exists a threshold value of w above which $z^S < f(w)^{-1}$, that is, $\frac{dz^S}{dw} < 0$.

Next, by noting that $x^S = 2rz^S(2 - z^S)$ and by using the expression for $\frac{dz^S}{dw}$, we get:

$$
\begin{aligned}
\frac{dx^S}{dw} &= \frac{2(g + w)(1 - z^S)\left(\frac{dz^S}{dw}\right) + (2 - z^S)z^S}{\gamma}, \\
&= \frac{z^S}{\gamma}\left\{(2 - z^S) + \frac{2(g + w)(1 - z^S)\left[2\alpha\gamma - (2 - z^S)(1 - z^S)(d - p + w)^2\right]}{(d - p + w)\left[2\alpha\gamma + (d - p + w)\left(\gamma - 3(g + w)(2 - z^S)z^S + 2(g + w)\right)\right]}\right\}
\end{aligned}
$$

It follows from Assumptions 2 and the fact that $r = \frac{g+w}{2\gamma} > \frac{1}{2}$ and $(d - m) = (d - p + w) > 0$, the denominator of the second term is also positive. Hence, the sign of the above term depends on the sign of the numerator alone. By expanding and rearranging the terms, the numerator can be simplified as: $\gamma\left(2\alpha\left[(d - m)(2 - z^S) + 4r\gamma(1 - z^S)\right] + (d - m)^2(2 - z^S)(1 - x^S)\right) > 0$, where the last inequality is due to the fact that both x^S and z^S are bounded above by 1. This completes our proof. ■

Proof of Lemma 6: For a given θ_2 of buyer 2, we show by contradiction that buyer 1's best response must satisfy $\theta_1 \leqslant \theta_2$. Suppose buyer 1's best response has $\theta_1 > \theta_2$. Then for every fixed compliance level x,

$$
\begin{aligned}
&\Pi_1^J(\theta_1; \theta_2, x) = m(1 - z_1(\theta_1)(1 - x)) - d(1 - z_1(\theta_1))(1 - x) - \theta_1\,\alpha\, z_1(\theta_1)^2 \\
&\Rightarrow \frac{\partial \Pi_1^J(\theta_1; \theta_2, x)}{\partial \theta_1} = -\alpha\, z_1(\theta_1)^2 < 0.
\end{aligned}
$$

Hence, buyer 1 sets θ_1 such that $\theta_1 \leqslant \theta_2$. Similarly, buyer 2 sets θ_2 such that $\theta_2 \leqslant \theta_1$. Hence, $\theta_1 = \theta_2$ in equilibrium. ■

Proof of Lemma 7: Given the symmetry of the buyers, we drop the indexes in this proof. By Lemma 6 it is true that the equilibrium under J comprises of symmetric cost sharing. Let $\Pi^J(\theta; \theta, x)$ be the profit under J when $\theta_i = \theta_j = \theta$ as obtained from from (32) and let x^J be the equilibrium compliance level when $\theta = \frac{1}{2}$. We prove that $\theta = \frac{1}{2}$ is the payoff dominant equilibrium.

Suppose $\theta < \frac{1}{2}$, then by (32) the profit of each buyer under J is given by (4). Let z^I and $x^I = 2rz^I$ be the equilibrium audit and compliance levels in the independent mechanism. Then,

$$
\Pi^I(z^I) < \Pi^I(z^I) + \frac{1}{2}\alpha {z^I}^2 = \Pi^J(\theta = \frac{1}{2}; \theta = \frac{1}{2}, x^I) \leqslant \Pi^J(\theta = \frac{1}{2}; \theta = \frac{1}{2}, x^J)
$$

where x^J is the equilibrium compliance level in the joint mechanism with $\theta = \frac{1}{2}$, and the last inequality follows by noting that $\frac{\partial \Pi^J}{\partial x} = mz + d(1-z) > 0$ (obtained from using Envelope theorem on (8)) and $x^J \geqslant x^I$. Hence, the equilibrium with $\theta < \frac{1}{2}$ is dominated by $\theta = \frac{1}{2}$.

Now, suppose $\theta > \frac{1}{2}$. Let $z^J(\theta)$ and $x^J(\theta) = 2rz^J(\theta)$ be the equilibrium audit and compliance levels. Then,

$$
\begin{aligned}
& z^J(\theta) = \frac{(d-m)}{2\alpha\theta + 2r(d-m)} \Rightarrow \frac{dz^J(\theta)}{d\theta} < 0 \text{ and} \\
& \Pi^J(\theta;\theta,x) = m(1-z(\theta)(1-x)) - d(1-z(\theta))(1-x) - \theta\alpha z(\theta)^2 \\
& \Rightarrow \frac{d\Pi^J(\theta;\theta,x^J(\theta))}{d\theta} = -\alpha z^J(\theta)^2 + \frac{\partial \Pi^J}{\partial x} \cdot 2r \frac{dz^J(\theta)}{d\theta} < 0 \text{ since } \frac{\partial \Pi^J}{\partial x} > 0 \text{ and } \frac{dz^J(\theta)}{d\theta} < 0.
\end{aligned}
$$

Hence, the equilibrium with $\theta > \frac{1}{2}$ is dominated by $\theta = \frac{1}{2}$. ■

Proof of Lemma 8: For a given θ_2 of buyer 2, we show by contradiction that buyer 1's best response must satisfy $\theta_1 \leqslant \theta_2 \left(\frac{d-m_1}{d-m_2}\right)$. Suppose buyer 1's best response has $\theta_1 > \theta_2 \left(\frac{d-m_1}{d-m_2}\right)$. Then,

$$
\begin{aligned}
& \Pi_1^J(\theta_1;\theta_2,x) = m_1(1-z_1(\theta_1)(1-x)) - d(1-z_1(\theta_1))(1-x) - \theta_1\alpha z_1(\theta_1)^2 \\
& \Rightarrow \frac{\partial \Pi_1^J(\theta_1;\theta_2,x)}{\partial \theta_1} = -\alpha z_1(\theta_1)^2 < 0.
\end{aligned}
$$

Hence, buyer 1 sets θ_1 such that $\theta_1 \leqslant \theta_2 \left(\frac{d-m_1}{d-m_2}\right)$. Similarly, buyer 2 sets θ_2 such that $\theta_2 \leqslant \theta_1 \left(\frac{d-m_2}{d-m_1}\right)$. Hence, $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$ in equilibrium.

Clearly, for every given compliance level x of the supplier,

$$
\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2} \Rightarrow z_1(\theta_1) = \frac{(d-m_1)(1-x)}{2\alpha\theta_1} = \frac{(d-m_2)(1-x)}{2\alpha\theta_2} = z_2(\theta_2). \quad ■
$$

Proof of Lemma 9: For ease of notation, let $\hat{\theta}_i = \frac{d-m_i}{2d-m_1-m_2}$. By Lemma 8 that in equilibrium $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$. Let $\Pi_i^J(\theta_i;\theta_j,x)$ be the profit of buyer i and x^J be the compliance level when $\theta_i = \hat{\theta}_i$ and $\theta_j = \hat{\theta}_j$. We prove that $\theta_i = \hat{\theta}_i$, $i = 1,2$ is the payoff dominant equilibrium. We argue for buyer i and the argument for buyer j is similar.

Suppose $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2} < \frac{1}{2d-m_1-m_2}$, then by (33) the profit of buyer i under J is given by

$$
\Pi_i^I(z_i^I;z_j^I,x^I) = m_i(1-z_i^I(1-2rz_i^I)) - d(1-z_i^I)(1-2rz_i^I) - \alpha {z_i^I}^2. \tag{34}
$$

Let z_i^I and $x^I = r_1 z_1^I + r_2 z_2^I$ be the equilibrium audit and compliance levels under independent audits. Then for buyer i we have

$$
\begin{aligned}
& \Pi_i^J(z^J) = m_i(1-z^J(1-x^J)) - d(1-z^J)(1-x^J) - \hat{\theta}_i\alpha {z^J}^2 \\
& \geqslant m_i(1-z_i^I(1-x^J)) - d(1-z_i^I)(1-x^J) - \hat{\theta}_i\alpha {z_i^I}^2 \text{ since } z^J \text{ maximizes (25) for every fixed value of } x \\
& \geqslant m_i(1-z_i^I(1-x^I)) - d(1-z_i^I)(1-x^I) - \hat{\theta}_i\alpha {z_i^I}^2 \text{ because } x^I < x^J \text{ and } \frac{\partial \Pi_i^J}{\partial x} = mz + d(1-z) > 0 \\
& \geqslant m_i(1-z_i^I(1-x^I)) - d(1-z_i^I)(1-x^I) - \alpha {z_i^I}^2 \text{ because } \hat{\theta}_i \in [0,1] \\
& = \Pi_i^I(z_i^I),
\end{aligned}
$$

and $x^J \geqslant x^I$ because

$$\begin{aligned}
\theta_i = \hat{\theta}_i \Rightarrow x^J &= \frac{(r_1+r_2)(2d-m_1-m_2)}{2\alpha+(r_1+r_2)(2d-m_1-m_2)} \\
\Rightarrow x^J - x^I &= \frac{(r_1+r_2)(2d-m_1-m_2)}{2\alpha+(r_1+r_2)(2d-m_1-m_2)} - \frac{r_1(d-m_1)+r_2(d-m_2)}{2\alpha+r_1(d-m_1)+r_2(d-m_2)} \\
&= \frac{2\alpha((d-m_1)r_2+(d-m_2)r_1)}{(2\alpha+(r_1+r_2)(2d-m_1-m_2))(2\alpha+r_1(d-m_1)+r_2(d-m_2))} \geqslant 0.
\end{aligned}$$

Hence, the equilibrium with $\theta_i < \hat{\theta}_i (\Leftrightarrow \theta_j < \hat{\theta}_j)$ is dominated by $\theta_i = \hat{\theta}_i (\Leftrightarrow \theta_j = \hat{\theta}_j)$. Now, suppose $\theta_i > \hat{\theta}_i (\Leftrightarrow \theta_j > \hat{\theta}_j)$. Then

$$\begin{aligned}
&z^J(\theta_i) = \frac{(d-m_i)}{2\alpha\theta_i+(d-m_i)(r_1+r_2)} \Rightarrow \frac{dz^J(\theta_i)}{d\theta_i} < 0 \text{ and} \\
&\Pi_i^J(\theta_i;\theta_j,x) = m_i(1-z_i(\theta_i)(1-x)) - d(1-z_i(\theta_i))(1-x) - \theta_i\alpha z_i(\theta_i)^2 \\
&\Rightarrow \frac{d\Pi_i^J(\theta_i;\theta_j,x^J(\theta_i))}{d\theta_i} = -\alpha z^J(\theta_i)^2 + \frac{\partial \Pi_i^J}{\partial x}\cdot(r_1+r_2)\frac{dz^J(\theta_i)}{d\theta_i} < 0 \text{ since } \frac{\partial \Pi_i^J}{\partial x} > 0 \text{ and } \frac{dz^J(\theta_i)}{d\theta_i} < 0.
\end{aligned}$$

Hence, the equilibrium with $\theta_i > \hat{\theta}_i$ is dominated by $\theta_i = \hat{\theta}_i$. ■

Proof of Proposition 1: Observe from (3) and (10) that $z^I = \frac{d-m}{2(\alpha+r(d-m))} < \frac{d-m}{\alpha+2r(d-m)} = z^J$. Next, by sustituting $z^I = \frac{d-m}{2(\alpha+r(d-m))}$ into (16) and by rearranging the terms, one can show that $V(z^I) = 2(2r+1)\alpha^2 + 2r(d-m)(4r-1)\alpha + r(1-2r)^2(d-m)^2 > 0 = V(z^S)$. By using the fact that $V(z)$ is increasing in z, we can conclude that $z^I > z^S$. Therefore, we prove the first statement: $z^S < z^I < z^J$.

Noting that $x^J = 2rz^J$ and $x^I = 2rz^I$, it follows that $x^J > x^I$.

Before we proceed further, we define the function $L(z) = z(2-z)$, which is an inverted parabola with roots 0 and 2, and mode at 1, for better exposition and shorthand notation.

In the region $[0,1]$, we note that $L(z) > z^J \Leftrightarrow z > \overline{z}$ where $\overline{z}$ is the solution of $L(z) = z^J$. The solution is given by $\overline{z} = 1 - \frac{\sqrt{\alpha+(2r-1)(d-m)}}{\sqrt{\alpha+2r(d-m)}}$ and, hence $\frac{(d-m)V(\overline{z})}{\alpha} = 2 - 2\sqrt{1-z^J} - z^J\sqrt{1-z^J} = 2 - (2 - z^J)\sqrt{1-z^J} > 0$. Thus, $\overline{z} > z^S \Leftrightarrow z^J = L(\overline{z}) > L(z^S) \Leftrightarrow x^J > x^S$.

Similarly, to compare x^I and x^S, we need to compare z^I and $z^S(2-z^S)$. To compare z^I and z^S we consider the solution of the equation $L(z) = z^I$ in the region $[0,1]$. On solving, we get $\underline{z} = 1 - \sqrt{\frac{2\alpha+(2r-1)(d-m)}{2(\alpha+r(d-m))}}$. Now, in order to compare $\underline{z}$ and z^S, we consider $V(\underline{z})$. On substituting the value of $\underline{z}$ in $V(z)$ we get $\frac{(d-m)V(\underline{z})}{\alpha} = 1 - (1+z^I)\sqrt{1-z^I}$. Hence,

$$V(\underline{z}) > 0 \Leftrightarrow z^I \geqslant \frac{\sqrt{5}-1}{2} \Leftrightarrow (d-m)\left[1 - r(\sqrt{5}-1)\right] \geqslant (\sqrt{5}-1)\alpha.$$

When $r \geqslant \frac{1}{\sqrt{5}-1}$, then $V(\underline{z}) < 0 \Leftrightarrow \underline{z} < z^S \Leftrightarrow z^I = L(\underline{z}) < L(z^S) = z^S(2-z^S) \Leftrightarrow x^I < x^S$. On the other hand, if r is small (i.e., $r < \frac{1}{\sqrt{5}-1}$) and when α is sufficiently small then $V(\underline{z}) > 0 \Leftrightarrow \underline{z} > z^S \Leftrightarrow z^I = L(\underline{z}) > L(z^S) = z^S(2-z^S) \Leftrightarrow x^I > x^S$. And, when r is small but α is sufficiently large, then $V(\underline{z}) < 0 \Leftrightarrow \underline{z} < z^S \Leftrightarrow z^I = L(\underline{z}) < L(z^S) = z^S(2-z^S) \Leftrightarrow x^I < x^S$. This concludes the proof. ■

Proof of Proposition 2: First, it follows from (5) and (12) that $\pi_s^I(z^I) - \pi_s^J(z^J) = \gamma[(1-2r\cdot z^I) + (1-2r\cdot z^J)]\cdot[2r(z^J - z^I)]$. By applying the first statement of Proposition 1 (i.e., $z^J > z^I$), we prove the first statement. By using the same approach, we obtain the second statement. Finally, observe from (5) and (12) that $\pi_s^I(z^I) - \pi_s^S(z^S) = \gamma[(1-x^I)+(1-x^S)]\cdot(x^S - x^I)$. We prove the third statement by applying (2) and (3) of Proposition 1 (i.e., $x^S > x^I$ when α is sufficiently large). This completes our proof. ■

Proof of Proposition 3: First, we note that from (2), we note that for every fixed audit level z_i of buyer i, the buyer's profit is increasing in the supplier's compliance level x. That is:

$$\frac{\partial \Pi_i(z_i; z_j, x)}{\partial x} = m z_i + d(1-z_i) > 0. \tag{35}$$

Now, the joint mechanism profits at the payoff-maximizing equilibrium $\theta_1 = \theta_2 = \frac{1}{2}$ is

$$\begin{aligned}
\Pi_i^J(z^J) &= m(1 - z^J(1-x^J)) - d(1-z^J)(1-x^J) - \frac{1}{2}\alpha\, {z^J}^2 \\
&\geqslant m(1 - z^I(1-x^J)) - d(1-z^I)(1-x^J) - \frac{1}{2}\alpha\, {z^I}^2 \\
&\qquad\qquad\qquad\qquad \text{since } z^J \text{ maximizes } \Pi_i^J(z; x^J) \\
&\geqslant m(1 - z^I(1-x^I)) - d(1-z^I)(1-x^I) - \frac{1}{2}\alpha\, {z^I}^2 \quad \text{using (35) and } x^J \geq x^I \\
&= \Pi_i^I(z^I) + -\frac{1}{2}\alpha\, {z^I}^2 > \Pi_i^I(z^I)
\end{aligned}$$

Next, it follows from (4) and (17), we get: $\Pi^I(z^I) - \Pi^S(z^S) = \alpha\left({z^S}^2 - {z^I}^2\right) + (z^I - (2-z^S)z^S)\,T_I(z^S)$, where $T_I(z^S) = 2r(d-m)(z^S)^2 - 4r(d-m)z^S + (d-m)(1-2rz^I) + 2dr > 0$. By noting that the term $T_I(z^S) > 0$ for $z^S \in (0,1)$, we can prove our second statement by applying Proposition 1 to show that the terms $((z^S)^2 - (z^I)^2)$ and $(z^I - (2-z^S)z^S)$ are both negative. This proves second statement. ■

Proof of Proposition 4:

$$[2\,\Pi^J + \pi_s^J] - [2\,\Pi^I + \pi_s^I] = \frac{\alpha(d-m)}{2(\alpha + 2r(d-m))^2(\alpha + r(d-m))^2}\, f(\alpha)$$

where $f(\alpha) = (d - m + 4r(d-\gamma))\,\alpha^2 + 6r^2(2d-\gamma)(d-m)\alpha + 2r^2(d-m)^2\,(4dr - (d-m))$, which is a quadratic in α. Note that $f(0) > 0$ always and $f(\alpha)$ is continuous in α. It follows that $f(\alpha) > 0$ for α sufficiently low. This proves the first statement. For the second statement: when $d > \gamma$, we have $f(\alpha) > 0$. Finally, for the third statement: if $2d > \gamma$, then $f'(0) > 0$. Further, if $2d > \gamma$ and $d - m > g + w$, then we get $f''(\alpha) = 2\,[d - m + 4r(d-\gamma)] \geqslant 2\,[d - m - 2r\gamma] = 2\,[(d-m) - (g+w)] > 0$, which indicates that f is convex. Thus, $f > 0$ for all positive values of α. This completes the proof. ■

Proof of Proposition 5:

$$[2\,\Pi^S + \pi_s^S] - [2\,\Pi^I + \pi_s^I] \quad = \left(\frac{d-m}{r} - \gamma\right)(x^S - x^I)(2 - x^I - x^S) + 2\left(d - \frac{d-m}{2r}\right)(x^S - x^I) + 2\alpha(z^I - z^S)(z^I + z^S)$$

From Proposition 1, the last term in the above expression is positive. If $\frac{d-m}{r} > \gamma$ ($\Leftrightarrow d - m > \frac{g+w}{2}$) then the first term in brackets is always positive. Hence, if the compliance of supplier under S is higher than that under I, then $2\,\Pi^S + \pi_s^S > 2\,\Pi^I + \pi_s^I$. From Proposition 1, $x^S > x^I$ if and only if $\alpha \geq \widetilde{\alpha}$. This concludes the proof. ■