SELECTING A SELLING INSTITUTION:  
AUCTIONS VERSUS SEQUENTIAL SEARCH*

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ABSTRACT
Under conditions of asymmetric information, the seller's choice of a selling institution entails sorting of buyers by valuation type in order to price discriminate and maximize profits. We consider the seller's choice between sequential search and an auction in the presence of discounting and transaction costs when selling n homogeneous units of a good. Our analysis shows that the expected return per unit from sequential selling decreases in n. For the auction with suitable restrictions, the expected return per unit is increasing in n. Thus, when choosing between sequential
search and an auction, sequential search is the preferred institution if n is small ($< n^*$) whereas the auction is preferred if n is large ($\geq n^*$). By considering historical details of the evolution of livestock markets, our results explain and comport with empirical reality.
I. Introduction

Sorting customers according to their valuations is the fundamental strategy employed by sellers to exert market power -- and extract consumer surplus. In actual markets an array of selling institutions has evolved, presumably designed to balance the concomitant costs of increasingly refined sorting against the increase in revenue associated with a greater degree of price discrimination. These institutions include auctions, sequential search (in this framework middlemen, such as brokers or consignors, are frequently employed), bargaining (prevalent in the market for automobiles), and posted prices (this can entail a single fixed price or a nonlinear pricing scheme and is common in retail stores).

Although considerable attention has been devoted to the optimal design of each of these institutions, relatively little attention has been devoted to explaining the simultaneous existence of these different institutions or the factors which govern the seller's choice of a specific selling institution. While focusing on the reasons for the widespread use of auctions, Milgrom [1987] addresses this larger issue of mechanism selection. Milgrom considers the "situation in which the seller, for some unspecified reason, has the power to select any institution he likes for conducting trade" and notes that in a deterministic (full information) setting "a take-it-or-leave-it offer to the highest valuation buyer" extracts all the surplus. When the buyers' valuations are private information, then (see his Proposition 5) a sealed-bid auction with a minimum price maximizes the seller's payoff amongst all possible exchange games.

Our goal is more modest than Milgrom's in that we limit the seller's choice to one of two mechanisms. On the other hand, as evidenced by our detailed description and analysis of livestock markets, our model is more attuned to the realities of the marketplace, and it reflects the institutional choices sellers actually face. Furthermore, by applying our results to livestock markets, additional insight into the use of different selling mechanisms is gained. This application reveals the coincidence between the predictions of our theory and actual market institutions.

In the context of private information we compare a sequential mechanism to an auction in which the seller incurs an advertising cost which increases with the number of bidders. This comparison highlights two closely related features of selling mechanisms: the timing in which offers are received and the seller's ability to compare these offers. Sequential selling requires that the seller meet individually with prospective buyers to discuss terms; it presents one extreme in which the seller receives only one offer per period and the offer must be accepted or rejected before another is received: no comparison of offers is possible. The auction mechanism represents the opposite extreme in which multiple buyers are convened in a centralized marketplace where their bids all are solicited in a single
period: an explicit comparison of offers is effected.

The contributions of our analysis are threefold. First, we extend the standard sequential search model to allow for the sale of \( n > 1 \) units. We verify the representation \( S_n = \xi_n + S_{n-1} \) where \( S_n \) and \( \xi_n \) are, respectively, the optimal return and the optimal reservation price when \( n \) units remain to be sold (Lemma 1). Lemma 2 demonstrates that the optimal policy entails an increasing sequence of reservation prices. Although the expected return from following this policy is increasing in \( n \), the expected return per unit is decreasing in \( n \) (Theorem 1). Second, we analyze the auction regime with positive bid solicitation costs. Several properties of the optimal return are established (Lemmas 3 - 5) which enable us to show that the expected return per unit is increasing when bidder valuations are distributed according to a uniform or an exponential distribution (Theorem 2).

Third, a comparison of the two mechanisms is effected. The number \( n \) of units offered for sale determines which of the two mechanisms is preferable. As shown in Theorem 3, there is a crossing point \( n^* \) of the two return functions such that sequential search is preferred when \( n \) is small (\(< n^* \)) and the auction is preferred when \( n \) is large (\( \geq n^* \)). This result has a strong intuitive justification. Sequential search allows the seller to evaluate each offer individually and solicit additional offers only if the initial offers are unsatisfactory. This sequential aspect gives rise to delays in selling the items. While the auction mechanism eliminates delays, it entails soliciting more offers (on average) than sequential search. As the number of units to be sold increases, the auction mechanism's advantage of selling all \( n \) units in the first period dominates the cost saving due to the smaller sample associated with the search mechanism.

We also characterize movements in the crossing point as the solicitation costs, the discount factor, and the mean of the distribution of customer valuations change (Lemmas 6, 7, and 8). The analysis also reveals that centralized auctions are preferable to decentralized auctions: expected revenue is maximized by selling all \( n \) items at one large auction rather than at two or more smaller auctions. We prove this result for all perfectly discriminating auctions (Theorem 5) as well as for uniform and exponential customer valuations in the open, ascending bid auction (Theorem 4).

In the next section we provide a background discussion of the theoretical development of several selling mechanisms and the market use of these mechanisms -- especially sequential mechanisms and auctions. The framework for our analysis is presented in section III where we extend the single unit sequential search problem to the case of \( n \) homogeneous units, solve for the optimal expected return from an auction with positive seller transaction costs, and prove the single crossing point theorem. In the final section we provide a detailed history and analysis of the evolution of livestock
markets which illustrates the pertinence of our results to actual market settings.

II. Background

A substantial literature has developed a variety of sophisticated selling schemes which sort consumers and extract the greatest amount of consumer surplus possible, given specific market conditions. In this literature markets are typically characterized by a seller who knows the distribution of customer valuations but is unable to observe the valuation of any specific customer. The seller utilizes this "aggregate" information to design a selling mechanism which sorts customers by valuation type. More effective sorting results in a greater degree of price discrimination.

Self-selection mechanisms induce individual customers to reveal information regarding their own valuations, thus allowing the seller to price discriminate. Self-selection typically is accomplished by the seller's commitment to a fixed price, a price-quantity schedule, or a price-quality schedule.\(^1\) Such mechanisms are commonly implemented at the retail level. For example, virtually all grocery and department stores post a nonnegotiable price for each item in the store. They also offer product lines of quality differentiated products sold at different fixed prices; this is seen in goods ranging from men's suits to snow skis. Finally, in grocery stores multi-packs and/or several container sizes represent an attempt to price discriminate via a price-quantity schedule. In all three of these posted price frameworks the customer purchases according to the seller's terms or does without the product. The customer's choice to purchase at the posted price or a particular price-quantity or price-quality selection reflects his valuation type.

Auctions constitute another institutional framework in which customers self-select by valuation type. Auctions are particularly well suited to selling an object in a market characterized by informational asymmetry. Several authors\(^2\) have shown that, in the absence of transaction costs, an auction with a reserve price is the optimal selling institution in such markets. In an open, ascending-bid auction the sorting process is accomplished by providing a forum in which customers bid competitively

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\(^1\) In the market for insurance Stiglitz [1977] and Rothschild and Stiglitz [1978] showed how non-linear pricing can be used to sort customers according to their differing -- though unknown to the insurer -- accident probabilities. Spence [1977] developed an optimal quantity discount scheme for a monopolist facing customers with different valuations for different quantities of a product. Mussa and Rosen [1978] derived a similar product differentiation policy for markets characterized by customers with different valuations for product quality. All of this work is nicely summarized and extended by Maskin and Riley [1984].

\(^2\) See Riley and Samuelson [1981] and Milgrom and Weber [1982]; for an excellent survey see McAfee and McMillan [1987].
for the item. Because it is advantageous for each customer to bid up to his valuation, individual valuations are revealed during the bidding process, and the good is sold to the customer with the highest valuation. The revenue equivalence theorem demonstrates that (with risk neutral customers) bidders' strategic behavior leads to the same expected revenue as would be garnered from other auction formats such as the sealed-bid second-price and Dutch auctions. Harris and Raviv [1981] extended the revenue equivalence result to auctions with multiple units when valuations are distributed uniformly and each bidder demands only one unit. Maskin and Riley [1989] generalized this result by eliminating the restriction to a uniform distribution.

The sequential search literature\(^3\) presents an alternative approach to the seller's problem under asymmetric information. In this framework the seller does not rely on a self-selection mechanism. Rather, he expends an information gathering cost (called a search cost), and he meets one customer each period. Each customer encountered has his own private valuation for the product, these customer values are selected randomly from a known distribution, and the customer's offer may be less than his private value. The seller can accept this offer, or he can reject it and continue the search process. The optimal solution to the search problem is the well known reservation price strategy.

The theoretical analysis of each of the above mechanisms typically has been conducted as if this mechanism were the only one the seller might consider. However, frequently two or more mechanisms are employed concurrently in the same market. Kohls [1961 pp.286-288] lists eight different selling institutions for livestock. These include terminal markets, direct sales, and country dealers (all sequential mechanisms) and auctions (in which all units are sold at once). A similar spectrum of selling institutions is found in most agricultural product markets. Cassady [1967] discusses products ranging from antiques to industrial machinery which are sold at auction as well as through brokers and/or dealers. Artists may sell their works to art dealers, on consignment through art galleries, or at exhibitions they organize themselves. On another front, in addition to the traditional real estate broker, auctions are being used with increasing frequency in the California housing market.\(^4\) These examples illustrate empirical reality: in many differing markets sellers employ a range of mechanisms, and their decision problems are more complex than optimization within the confines of a given mechanism.

Despite the plethora of markets exhibiting several selling institutions, the literature dealing with

\(^3\) See Lippman and McCall [1976] for a comprehensive survey.

\(^4\) During a two hour auction on a Sunday in May, 1991, 52 new tract homes in Torrance California were sold for just over $29 million by auction Marketing Services. The \(n = 52\) homes are situated on similar small lots and range in size from 2723 to 3536 square feet.
this phenomenon is sparse, and the number of papers in which a direct comparison of selling institutions is affected is yet smaller. Harris and Raviv [1981] derive optimal selling mechanisms for a seller with a production capacity constraint in a market characterized by asymmetric information and zero communication costs. If the number of potential buyers exceeds the capacity constraint, both a priority pricing scheme and a modified competitive auction are shown to be optimal. The priority pricing scheme takes advantage of the fact that higher valuation consumers are willing to pay a higher price in order to increase their probability of receiving the item. The optimal policy changes to posting a single price when the capacity constraint is not binding. In a similar market setting, without a capacity constraint, Riley and Zeckhauser [1983] provide another justification for posted prices; they demonstrate that, relative to any haggling strategy, a posted price strategy maximizes seller profits. Maskin and Riley [1989] demonstrate that a nonlinear posted price mechanism is preferable to an auction when a seller is selling multiple units and bidders have downward sloping demand curves (instead of demand for just one unit).

An interesting connection between sequential search and auctions arises in the work of McAfee and McMillan [1988]. In addition to sequential search they permit other selling institutions in the search framework. In an economy with positive search costs, no discounting, n customers, and a single unit being exchanged, they demonstrate that an optimal mechanism specifies that the seller first attempts to sell the good sequentially; he then resorts to an auction only if all n customers have been queried and no offers which exceed the reservation price have been received. This mechanism reduces to sequential search if the time horizon is infinite, and it reduces to an auction if the search cost is zero.

The analysis in our paper is closest in spirit to that of De Vany [1987] who considers a seller with one unit of a commodity choosing between three mechanisms. Customers with unknown valuations arrive according to a Poisson process. The first mechanism entails holding an auction after a fixed time T has elapsed; the second entails holding an auction after a fixed number of customers has arrived. Both of these mechanisms impact the transaction costs and the expected winning bid. Under posted prices, the third mechanism considered, each customer decides whether or not to buy at the posted price immediately upon entering the market. De Vany shows that the higher expected price achieved by competitive bidding in the auction mechanisms can be offset by the higher transaction costs involved in implementing these mechanisms. Thus, the lower transaction costs associated with

\footnote{For continuous uniform valuation distributions the standard auction with a reserve price is optimal. Harris and Raviv need to modify the standard optimal auction because they assume the distribution of bidder valuations is discrete uniform.}
transmitting information via posted prices provides an explanation for the widespread use of posted price mechanisms.
III. The Model

A seller with n homogeneous units of a good seeks to maximize his profit by selling these n units to customers each of whom seek to purchase exactly one unit. Whereas the seller is unable to elicit the specific valuation of any customer, he does know the distribution function $F$ of buyer valuations. The seller must commit to selling his product using either a sequential mechanism or an auction.

III. A. THE SEQUENTIAL SEARCH REGIME

Here, as in the standard search model, the seller can solicit one offer per period at a constant marginal cost of $c$ dollars per offer. The offer $X_i$ received in period $i$ is a random variable drawn from the cumulative distribution function $F$ of buyer valuations. It is assumed that the $X_i$ are independent and $E[X_1] < \infty$ where $X_1$ has distribution $F$. We abstract from the strategic nature of buyer and seller interaction by assuming that the seller has total hegemony over the bargaining process -- to wit, the buyer not only reveals his true valuation, but offers to pay it. This assumption is innocuous: any bargaining assumption which results in a distribution $G$ of offers -- presumably with $F$ stochastically larger than $G$ -- leaves the model, and therefore our results, qualitatively unaltered.

The seller pays the search cost at the beginning of the period and receives the offer (and payment if the offer is accepted) at the end of the period. Payments received in future periods are discounted at the rate $\beta < 1$ per period.\(^6\)

It is a well known result from the search literature that the expected return $\xi$ from selling one unit in this setting when the seller follows an optimal stopping policy is

where $\xi$ is the reservation price: the item is sold for any offer greater than or equal to $\xi$. Search continues if the offer is less than $\xi$. The optimal reservation price is the solution to

$$\xi = -c + \beta \left[ E\left[ \max(\xi, X_i) \right] \right] = -c + \beta \left[ \xi F(\xi) + \int_\xi^\infty x dF(x) \right], \quad (1)$$

$$c = \beta H(x) - (1 - \beta)x, \quad (2)$$

where $H(x) = \int_x^\infty (y - x) dF(y)$.\(^3\)

We now extend this result to the case of selling $n$ units. Let $S_i$ denote the expected discounted value of selling $i$ units using the optimal sequential search policy, and let $\xi_i$ denote the optimal reservation price when the seller has $i$ units remaining to sell.

\(^6\)If $\beta = 1$ (and the solicitation cost is the same for both mechanisms), then for each $n$ the seller's return is larger when using sequential search as opposed to using an auction.
LEMMA 1: For i ≥ 2, S_i = ξ_i + S_{i-1}; also, S_1 = ξ_1.

Proof: Suppose a seller with i units to sell is given the option of selling the first unit at a price of ξ_i (without searching) and then selling the remaining i-1 units by sequential search. The expected discounted value of this option to the seller is ξ_i + S_{i-1}, and, from the definition of ξ_i, it will always be accepted. Thus, S_i ≤ ξ_i + S_{i-1}. If S_i < ξ_i + S_{i-1}, then S_i = ξ_i - ε + S_{i-1} so the seller will accept an offer of ξ_i - ε/2. This violates the definition of ξ_i. Thus, S_i = ξ_i + S_{i-1}. When one unit remains to be sold, the seller's problem reduces to the standard search problem so S_1 = ξ_1.

LEMMA 2: The reservation prices decrease with the number of units which remain to be sold: ξ_n < ξ_{n-1} < ... < ξ_1. Furthermore, ξ_i solves

\[ c = \beta H(x) - (1 - \beta) (x + S_{i-1}). \]  

Proof: When i units remain to be sold, the expected return from following the optimal policy is

\[ S_i = -c + \beta (E[\max(\xi_i + S_{i-1}, X_i + S_{i-1})])] \]  

so ξ_i is the solution to

\[ \begin{align*}
S_i &= -c + \beta \int \frac{F(\xi_i)}{F(\xi_i + S_{i-1})} f(y)dy + \int_{\xi_i}^{\infty} \frac{f(y)}{F(\xi_i + S_{i-1})} dy \\
&= -c + \beta S_{i-1} + \beta \int \frac{F(\xi_i)}{F(\xi_i + S_{i-1})} f(y)dy + \int_{\xi_i}^{\infty} \frac{f(y)dy}{F(\xi_i + S_{i-1})} \\
&= -c + \beta S_{i-1} + \beta \xi_i + \beta H(\xi_i) \]  

where (x) = 1 - F(x). Thus, ξ_i solves (3). Monotonicity of the reservation prices follows from (3) and the facts that S_{i+1} > S_i and βH(x) is a decreasing function of x.

Lemma 2, essentially an extension of Theorem 4 of Lippman and McCall [1986], provides a simple algorithm -- namely (3) -- for computing the reservation prices. Because the operative reservation price is non-decreasing in time (for ξ_{i+1} < ξ_i), the issue of recall is moot: the seller does not seek to utilize past offers.

To illustrate Lemmas 1 and 2, consider a seller with two units to sell and customers with valuations distributed uniformly on the interval [0,1]. Assume that the offer solicitation cost is c = 1/4 per offer and the discount factor β = 0.9 so (3) and H(x) = \int_x^\infty (y - x)dy = x^2/2 - x + \frac{1}{2} yield ξ_1 ≈ 0.22222 and ξ_2 ≈ 0.19487. Furthermore, letting τ denote the number of periods until an offer of ξ_2 or better is received and noting that τ is a geometric random variable with parameter 1 - F(ξ_2), we have
The implicit assumption of constant stochastic demand -- namely, F is independent of n -- appropriately represents markets for homogeneous products in which random fluctuations in demand conditions facing individual firms are common (such as the market for livestock) so that the valuation of a buyer solicited at any given time is a random draw from a distribution F. It is also relevant in markets in which there is a limited supply (relative to market demand) of a product which does not go out of style or become obsolete over time (such as the market for limited edition art works). On the other hand, the policy in Lemma 2 conflicts with pricing practices observed in many retail markets (such as the market for fashion goods) where a policy of scheduled price reductions typically is employed. These markets, however, are characterized by product obsolescence as new styles or models replace existing products.\footnote{Karlin [1962] analyzed such markets using a search model in which the mean of the offer distribution decreases each period. As per Karlin's model and analysis, the results of Lemma 2 may be reversed in these markets; nevertheless, the results of Theorem 1 below continue to apply.}

Theorem 1: The function $S_n$ is concave; in particular, $S_n/n$ is decreasing in n.

The proof follows easily from $S_n = \sum_{i=0}^{n} \xi_i$ \footnote{If the salespeople are not independent, then one such variant is equivalent to conducting a series of perfectly discriminating auctions in which a reserve price is established, k offers are solicited, and anywhere from zero to k units are sold. Modelling this specification would involve extending the work on optimal sample size in the sequential search setting (Morgan and Manning [1985] and Morgan [1983]) to the case of $n>1$ units. While the}.
salespeople each of whom solicits one offer per period. However, it is our explicit intention to contrast an auction with a search mechanism which exhibits this decreasing returns effect. Allowing for a wider range of search technologies (such as k salespeople) does not significantly alter the nature of the results regarding the seller's choice between sequential selling and an auction provided that the search technology considered suffers from decreasing returns. Any mechanism which exhibits decreasing returns serves our purpose. We focus upon sequential search because it is a prominent selling institution and, as shown in Theorem 1, it exhibits decreasing returns.

III. B. THE AUCTION REGIME

Unlike standard auction models where the seller faces a fixed number of bidders and benefits from the revelation of bidder information during the bidding process for free, obtaining bidder information is costly in our model. In particular the cost of soliciting m bids is md -- where d>0 need not equal c. As was true in the sequential search regime, we assume that the bids $X_1, X_2, \ldots, X_m$ are independently and identically distributed draws from F and $E[X_1]<\infty$. At the beginning of the first period the solicitation cost is paid and the m bids are received. Payment from the winning bidders is received timelessly (immediately) thereafter. Think of the search cost as an advertising expenditure resulting mechanism would allow for either sequential search or an auction as a special case and therefore would dominate either of the mechanisms considered in this paper, we focus on the choice between sequential search and auctions because of the frequently observed use of these two mechanisms in actual markets. Furthermore, if the seller is unable to employ a reserve price in this setting, then Theorem 5 below demonstrates that a single, large auction dominates a series of smaller auctions.

The impact of altering the search technology in this way is depicted in Figure 1. Given a fixed number of salespeople, Theorem 1 implies that an increase in $\nu$ causes $S_\nu/n$ to decline. However, if the number of salespeople is allowed to vary with $\nu$, then economies of scale can lead to a jump in $S_\nu/n$. This would be followed by another decline as $\nu$ increases until it is optimal to employ yet another salesperson. In the extreme the seller can employ one salesperson for each unit to be sold (set $k=\nu$) in which case $S_\nu/n = \nu S_1/n = \xi_\nu$ is constant. Figure 1 also depicts the likely impact of a substantial increase in $\nu$ on the rate at which a single seller is able to solicit offers. While a small increase in $\nu$ (say from $n=1$ to $n=2$) is not likely to have a noticeable impact on the rate at which the seller receives offers, a substantial increase (say from $n=1$ to $n=1000$) significantly increases the seller’s visibility and makes it easier to attract offers. This, in turn, could lead to an increase in $S_\nu/n$. However, a large increase in the arrival rate is required to generate this result. An increase in the arrival rate of offers can be incorporated into the model as an increase in the discount factor $\beta$. In the example presented above if $\beta=0.9$ and $n=1$, then $S_1/n = 0.22222$. If the increase in the arrival rate when $\nu$ increases from $n=1$ to $n=2$ corresponds to an increase (of slightly more than 11%) in $\beta$ to $\beta = 0.91$, then $S_2/n = 0.22882$, and $S_2/n > S_1/n$. An alternative illustration might show $S_\nu/n$ as a smooth function first decreasing and then increasing when visibility increased sufficiently. However, because sequential interactions with customers require some minimum amount of time, there is a maximal arrival rate, and once the maximum arrival rate is achieved, further increases in $\nu$ cause $S_\nu/n$ to decline.

This assumption is used to highlight the reduced delay associated with the auction mechanism. The results
necessary to attract bidders: an increase in advertising increases the number of bidders. This is essentially an auction interpretation of Stigler's [1961] original search article.

As the $X_i$ are independent, the setting is one of private values in which no individual's valuation is affected by knowledge of others' valuations. We also assume buyers are risk neutral and $F$ is strictly increasing and continuously differentiable. Maskin and Riley [1989] verify that these assumptions ensure revenue equivalence for multiple unit auctions. Consequently, we limit our analysis to an open, ascending-bid auction. With $m>n$ bidders, all $n$ units will be sold for the $(n+1)^{st}$ lowest bid.\textsuperscript{11} If the optimal sample size is positive, then it is strictly greater than $n$, for otherwise the goods would be sold at a price of 0.\textsuperscript{12}

For many multiple unit auctions such as tulip and livestock (but not treasury bill) auctions, the $n$ units are not sold simultaneously. Rather, they are sold one (batch) at a time in a sequence of $n$ auctions: each successive auction finds one less unit for sale and one less bidder. Bulow and Klemperer [1991] extend Maskin and Riley's revenue equivalence theorem, demonstrating that the expected revenue generated is the same whether the units are sold one at a time or simultaneously. Thus, we do not depart from the reality of multiple unit auctions by proceeding as if the $n$ units are sold simultaneously.

Because bidders bid up to their true valuations in an open, ascending-bid auction, the seller's expected revenue is just the expected value of the $(n+1)^{st}$ lowest bidder's valuation multiplied by the number of units sold. Define the order statistics $Y_1 \geq Y_2 \geq \ldots \geq Y_m$ as the values of $\{X_1, \ldots, X_m\}$ ranked in

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\textsuperscript{11} The results also hold if we assume a perfectly discriminating auction in which each bidder bids and pays his valuation.

\textsuperscript{12} Implementing a reserve price would eliminate this problem. However, coupling a reserve price with a bid solicitation cost would introduce enormous algebraic complexity. Consequently, we limit our analysis to auctions without a reserve price.
descending order, and observe that

$$E[Y_{n+1}] = \int_0^\infty P(Y_{n+1} > y) dy = \int_0^\infty \sum_{j=n+1}^m \left( \frac{m}{j} \right) F(y) \cdot F(y)^{m-j} dy. \quad (4)$$

Because the cost of soliciting each bid is d, the expected value $V_n(m)$ of auctioning n items to m bidders is

$$V_n(m) = n \int_0^\infty \sum_{j=n+1}^m \left( \frac{m}{j} \right) F(y) \cdot F(y)^{m-j} dy - md. \quad (5)$$

Computing the expectation in (4) is, in general, quite complex. Henceforth, all discussion is limited to uniformly and exponentially distributed bidder valuations because the expected value of the order statistics can be presented in analytic form.\(^{13}\)

If a sample of size m is drawn from a uniform distribution on $[a,b]$ with $a \geq 0$ (without loss of generality we set $a=0$ and $b=1$), then (4) yields

$$E[Y_n] = \int_0^\infty \sum_{j=n}^m \left( \frac{m}{j} \right) \left( 1 - \frac{y}{m} \right)^{m-j} dy = 1 - \frac{n}{m+1}. \quad (6)$$

Likewise, if the offers are drawn from an exponential distribution with mean $\theta$ (without loss of generality we set $\theta=1$), then

$$E[Y_n] = \int_0^\infty \sum_{j=n}^m \left( \frac{m}{j} \right) e^{-y} \cdot (1 - e^{-y})^{m-j} dy = \sum_{j=n}^m \frac{1}{j}. \quad (7)$$

Substituting (6) and (7) into (5) produces the return functions for the auction mechanism when $F$ is uniform and exponential, respectively:

$$V_n(m) = n \left( 1 - \frac{n+1}{m+1} \right) - md \quad (8)$$

\(^{13}\) For homogeneous products such as tulips and treasury bills, the existence of resale markets may render a common values setting more appropriate than the private values setting. For other homogeneous products such as "fed cattle", the primary buyers at the auction are also the end users -- meat packers are the end users of "live" cattle -- so the private value setting is indeed appropriate. In either case, the analysis of Bikhchandani and Riley [1991, p. 123] is easily extended to show that a common value auction can generate uniformly distributed offers.
Using the return functions (8) and (9), we can derive the optimal sample size $m_n$ as a function of the number $n$ of units being sold. To do so, we first establish the fact that $V_n(m)$ is concave.

**Lemma 3:** Assume that $F$ is either uniform or exponential. For fixed $n$, $V_n(m)$ is strictly concave in nonnegative integer values $m$.

**Proof:** If $F$ is uniform, then utilizing (6) we obtain
\[ \Delta = \frac{n}{m} \cdot 2 \left( \frac{n}{m} + \frac{n}{m+1} \right) = \frac{n}{2m(m+1)} > 0. \]

Because $V_n(m)$ is strictly concave, it attains a maximum at no more than two values of $m$, and these must be consecutive integers. Lemma 3 also implies that the seller will maximize the expected return from the auction by selecting the optimal sample size $m_n$, equal to the smallest integer $m$ such that $V_n(m) - V_n(m+1) > 0$. The following lemma provides an explicit formula for $m_n$ in terms of the solicitation cost $d$.

**Lemma 4:** Let $d$ and $n$ be given, and denote the integer part of $x$ by $\lfloor x \rfloor$. If $m_n$ is strictly positive, then for $F$ uniform and $F$ exponential $m_n$ satisfies, respectively,

\[ m_n = \left\lfloor \frac{-1 + \sqrt{(1+4n(n+1))/d}}{2} \right\rfloor \quad (10a) \]

\[ m_n = \left\lfloor \frac{n}{d} \right\rfloor. \quad (10b) \]

**Proof:** If $F$ is uniform, then
\[ V_n(m) - V_n(m+1) = \frac{n(n+1)}{m+2} - \frac{n(n+1)}{m+1} + d = \frac{-n(n+1)}{(m+1)(m+2)} + d. \]

Thus, $V_n(m) - V_n(m+1) > 0$ implies $m^2 + 3m + 2 - n(n+1)/d > 0$. From the quadratic formula, $\frac{-3 + \sqrt{(1+4n(n+1))/d}}{2}$ is the positive root of this expression, so

\[ m_n = \left\lfloor \frac{-1 + \sqrt{1+4n(n+1)/d}}{2} \right\rfloor. \]

If $F$ is exponential, then $V_n(m) - V_n(m+1) = d - n/(m+1)$. Thus, $V_n(m) - V_n(m+1) > 0$ implies $m > n/d - 1$ and $m_n = \lfloor n/d \rfloor$.

Treating $m_n$ as a continuous variable, it is easily seen from (10) that $m_n/n$ is a nonincreasing
function. On this account, the auction mechanism becomes more efficient as \( n \) increases.

The optimal sample \( m_n \) maximizes \( V_n(m) \), but it does not guarantee a positive return from the auction. To simplify notation define \( V_n \equiv V_n(m_n) \). In general, \( V_n \) will be negative for all values of \( d \) above some critical value \( d^* \) which depends on \( F \) and \( n \). If \( d > d^* \), the seller maximizes expected revenue by not soliciting any bids. (Presumably this implies selecting a selling mechanism other than an auction). Whereas the critical value \( d^* \) can be readily calculated as a function of \( n \) when \( F \) is uniform, we only are able to place bounds on \( d^* \) when \( F \) is exponential.

**Lemma 5:** Fix \( n \). If \( F \) is uniform, then \( d^* = n/2(2n+1) \). If \( F \) is exponential, then \( n/(ne^e) \leq d^* \leq 1/e \).

The proof is given in the appendix.

**Theorem 2:** If \( F \) is either uniform or exponential, then the per unit return \( V_n/n \) is increasing.\(^{14} \)

**Proof:** Fix \( n \geq 1 \), and define \( D_n \equiv V_{n+1}/(n+1) - V_n/n \). Suppose \( F \) is uniform, fix \( d \in (0, n/2(2n+1)] \), and pick \( i > 0 \) such that

\[
d = \left( \frac{4n(n+1)}{(2(i+1)+1)^2 - 1}, \frac{4n(n+1)}{(2(i+1)+1)^2 - 1} \right)
\]

whence (10a) implies \( m_n = i \). Employing (8) we have

\[
D_n \geq \frac{V_{n+1}(i+1) - V_n(i)}{n+1} - \frac{V_{n+1}(i+1) - V_n(i)}{n+1} = \frac{(n+1)(1 - \frac{n+i}{n+1}) - (i+1)d - \frac{n(1 - \frac{n+i}{n+1}) - id}{n}}{27}
\]

Employing (9) we have

\[
d \in \left( \frac{n}{n+i+1}, \frac{n}{n+i+1} \right)
\]

for some finite integer \( i \geq 0 \) and \( m_n = \lfloor n/d \rfloor = n+i \). Employing (9) we have

\[
D_n \geq \frac{(n+1) \sum_{j=n+2}^{n+i} \frac{1}{j} - (n+i+1)d}{n+1} - \frac{n \sum_{j=n+i+1}^{n+i} - (n+i) d}{n} = \frac{i[(n+i+1)d - n]}{n(n+1)(n+i+1)} > 0
\]

where the first inequality follows from (9) and from \( n+i+1 \) being a suboptimal sample size for \( n+1 \), the equality from straightforward algebraic simplification, and the final inequality from the facts that \( i > 0 \) (which follows from Lemma 4) and \( (n+i+1)d > n \).

\(^{14}\) Due to the impossibility of deriving an analytically tractable expression for the expected value of order statistics for most distributions, this result cannot be generalized to all distributions.
In the auction regime with transaction costs, the seller determines the optimal number of bidders to solicit and then sells the n units to the n highest bidders. The solicitation and comparison of all offers in one period results in less refined sorting than sequential search achieves. However, the auction sells all units in the initial period. Theorem 2 demonstrates that in addition to avoiding delays in selling, the auction becomes an increasingly efficient means of price discriminating as n increases. Furthermore, Theorem 2 continues to apply if there are economies of scale in soliciting buyers (as discussed in Section III. A.). In fact, such economies of scale cause \( V_n/n \) to increase more rapidly.
III. C. THE PREFERRED MECHANISM

Given the parameter values $c$, $d$, $F$, and $\beta$, the number $n$ of units to be sold determines which of the two mechanisms yields the higher return. As made explicit in Theorem 3, the auction is superior for all values of $n$ sufficiently large.

**Theorem 3:** If $F$ is either uniform or exponential, there is a (critical) number $n^*$, $0 \leq n^* \leq \infty$, such that $V_n - S_n < 0$ for $n < n^*$ and $V_n - S_n \geq 0$ for $n \geq n^*$: sequential search is preferred if and only if $n < n^*$.

The proof follows directly from Theorems 1 and 2.

Theorem 3 is illustrated in Figure 2a: as $n$ increases the auction mechanism becomes relatively more efficient for exerting market power and extracting consumer surplus. There may be no crossing point ($n^* = 0$ or $n^* = \infty$). If $d > d^*$, then $n^* = \infty$: the sequential mechanism is used for all values of $n$.\(^{15}\) If $\beta$, $c$ and $d$ are such that $V_1 > S_1$, then $n^* = 0$: the auction is always preferred. However, if $d < d^*$ and $V_1 < S_1$, then $n^*$ is neither 0 nor $\infty$. Finally, note that if there are economies of scale in searching for buyers as discussed in the last paragraph of section III. A, the essence of Theorem 3 continues to apply. As illustrated in Figure 2b, economies of scale may result in more than one switching point, but sequential search remains preferable when $n$ is small and auctions remain preferable when $n$ is large.\(^{16}\)

--- Figures 2a and 2b Here ---

III. D. COMPARATIVE STATIC RESULTS

The single crossing point theorem manifests how the simple structure of the preferred mechanism is selected as a function of $n$, holding $c$, $d$, $\beta$, and $F$ fixed. We now examine this crossing point as a function of these other parameters. For mathematical simplicity, our analysis is limited to uniformly distributed customer valuations.

The first comparative static result asserts that $n^*$ is nondecreasing in $c$ if the solicitation cost is

\(^{15}\) Of course, we assume $\beta E(X) > c$.

\(^{16}\) If we allow for a broader class of mechanisms incorporating both sequential selling and auctions, by stationarity and Theorems 1 and 2 it is clear that if the seller chooses sequential search, it will not be optimal to switch to an auction as the number of units remaining to be sold decreases. However, it may be optimal to hold an auction and accept only very high prices and then sell the remaining items sequentially or at another auction, but implementing such a mechanism could prove very difficult and costly. For example, selling livestock at auction typically entails a substantial cost to transport the livestock to the auction site as well as yardage fees for feed and upkeep while the livestock are at the auction yard. If all of the livestock are not sold at the initial auction, the seller would incur additional yardage fees and transportation costs. Also, the seller's commitment to the auction mechanism may lose credibility amongst buyers if units are consistently withdrawn from the auction block, making it exceedingly difficult to attract bids. Such barriers to implementing more complex selling schemes further highlight the relevance of a direct comparison of the straightforward sequential selling and auction mechanisms analyzed here.
the same for both mechanisms (c=d). The principal advantage of the sequential mechanism is its efficiency in terms of the number of offers that are solicited; compared to the sequential mechanism, the auction mechanism (in effect) wastes offers. As c increases, this advantage is magnified and n* increases.

**Lemma 6:** If the cost of soliciting offers is the same for both mechanisms (c=d), then n* is nondecreasing in c.

Proof: Because H(x) is convex decreasing and S, increases in i, inspection of (3) reveals
\[-dξ_i /dc < -dξ_i /dc 32\] so that \[dS_n /dc = -Σ_{i=1}^n dξ_i /dc < -ndξ_i /dc , 33\] and, as
\[ξ_i = 1/(-1-\beta^2 + 2βc)/β,-dξ_i /dc 34\] reaches a maximum of \[1/\sqrt{2c} 35\] when β=1. Furthermore,
\[dV_n /dc = -m_n 17\] Upon verifying our claim that \[m_n > n/\sqrt{2c} , 36\] we have
\[d(V_n - S_n )/dc < -n/\sqrt{2c} - ndξ_i /dc ≤ 0 37\] whence n* cannot decrease if c increases. To verify our
claim note that
\[m_n = \left[ 1 + \left( \frac{4n^2 + 4n + 1}{c} \right) /c \right] \geq \left[ \frac{\frac{4n^2 + 4n + 1}{c} /c}{c} \right] \geq \left( \frac{\frac{3}{2}}{\sqrt{2c}} \right) \geq \frac{n}{\sqrt{2c}} 38\] for
\[n≥4, \ as \ c<1/2 \ is \ required \ for \ a \ nonnegative \ expected \ return \ from \ the \ auction. \ The \ restrictions \ imposed \ on \ c \ by \ Lemma \ 5 \ ensure \ that \ \sqrt{\frac{4n^2 + 4n + 1}{c} /2} - 3/2 > n/\sqrt{2c} 39 \ for \ n<3.\]

**Lemma 7:** The crossing point n* is nondecreasing in β.

Proof: Merely notice that the expected return from the search mechanism is increasing in β and the return from the auction does not depend upon β.

Lemma 7 suggests that auctions should be more prevalent in markets in which sellers are impatient. This is one explanation for the high incidence of estate sale auctions. This result does not generalize if discounting is applied to the auction mechanism -- not even if we assume that β is the same for the auction and the sequential mechanisms.

**Lemma 8:** The crossing point n* is nonincreasing in the mean of the distribution F of buyer valuations.

Proof: Consider a translation of X by a positive constant δ, and define ΔS_{n,δ} and ΔV_{n,δ} as the increase in the expected return resulting from such a translation from the sequential and auction mechanisms, respectively. In the sequential framework the most the seller will receive is an increase of δ in the price

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\[17\] This is true as long as c is not at an endpoint of an interval such that a change in c causes a change in \[m_n\] -- i.e., there is not a positive integer i such that \[c=4n(n+1)/(2i+1)^2-1}\] (see proof of Lemma 7). Because there are only a countable number of such endpoints, they occur with probability zero.
of each unit: $\Delta S_n, \delta \leq \beta \delta + \beta^2 \delta + \cdots + \beta^n \delta$. In the auction framework the translation has no impact on $m_n$, and it increases the expected return from the auction mechanism by $n\delta$ (or $\beta n\delta$ if discounting is involved). Thus, $\Delta S_{n,\delta} < \Delta V_{n,\delta}$.

This result is intuitive: the greater the object's value, the greater the cost of delay in selling, and the auction mechanism entails a delay of no more than one period.

IV. Evolution of Selling Mechanisms in Livestock Markets

In addition to highlighting the trade-offs between the sequential search and auction mechanisms, the implications of our theory coincide to a remarkable degree with the choice of different selling mechanisms actually observed in markets for agricultural products. We examine the livestock market in detail.

IV. A. EVOLUTION OF THE TRADING ENVIRONMENT

Four selling mechanisms -- terminal markets, direct sales, country dealers, and auctions -- have dominated the market for all types of livestock in the United States. Terminal markets, direct sales, and country dealers function sequentially with a broker, farmer, or dealer searching for customers offering an acceptable price, while auctions result in the sale of all of the livestock at once. Analysis of the evolution of these selling mechanisms in livestock markets manifests interesting extensions and

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18 In fact, Lippman and McCall [1986] show that the increase in the expected return is less than $\delta$.

19 Much of this discussion is based on studies by Engelman and Pence [1958], Skaggs [1986], Kohls [1961], Yeager [1981], and Wade [1987]. Statistical data were taken from various issues of the USDA publication Packers and Stockyards Statistical Report.

20 Terminal markets -- more commonly known as stockyards -- are large centralized markets open to all customers. The farmer pays a "yardage fee" for the use of facilities to display his livestock. He may be present at the site to sell his own livestock, but more frequently a commission broker is employed to represent the farmer. Such brokers interact with customers sequentially and are paid a percentage of the sale price. In direct sales the farmer contacts prospective buyers (usually meat packers) directly by phone to arrange a trade. No middleman organization is involved. Once a trade is agreed upon, the livestock is transported from the farm to the meat packing plant without an intermediate stop at a terminal market location. Country dealers typically travel to the farm to purchase livestock and then transport it themselves to buyers and resell it for a profit. They may also have holding facilities to maintain a small inventory.

Kohls lists four other livestock selling mechanisms. They are local cooperative associations, concentration yards (small, local versions of terminal markets), retail meat dealers (who purchase livestock, slaughter and dress it, and then sell it to retail customers), and sales to other farmers (which are limited primarily to feeding and breeding stock). The first three of these account for a very small portion of the market, and their importance has been decreasing over the last 40 years.
applications of the theory.

SEQUENTIAL SELLING MECHANISMS

For a number of economic and technological reasons\textsuperscript{21} highly centralized terminal markets located along railroad lines in major population centers were the primary market outlet for livestock in the late 1800's and early 1900's. These highly centralized markets provided the seller with lower solicitation costs $c$, a shorter epoch between offers (higher $\beta$), and lower transportation costs than any other sequential selling mechanism available at that time.\textsuperscript{22} As a result, the use of other sequential selling mechanisms was confined to geographic regions located close to the livestock producing areas and isolated from terminal markets.

This situation began to change in the 1920's following the Packers Consent Decree of 1920 which effectively ended oligopoly control of the meat packing industry, facilitating entry by new meat packing companies.\textsuperscript{23} At the same time, increased access to producing locations provided by the

\textsuperscript{21}First, rail transport provided the most feasible means of moving livestock from rural producing areas to potential buyers in urban areas during this period. Second, due to considerable shrinkage (i.e. weight loss) which occurred during transportation of livestock, economic efficiency entailed slaughtering livestock very close to the point of purchase. Third, \textit{economies of scale} in slaughtering and dressing livestock entailed a highly specialized subdivision of labor, for "in 1904, the disassembling of a single steer involved 157 men ... whose work was divided into no less than seventy-eight distinct processes" (Chandler [1977], p. 392). Thus, the minimum efficient scale was large. Terminal markets provided the requisite quantities of livestock for sale necessary to supply the large packing plants. Fourth, managerial practices at Swift and Armour, the two largest integrated meat packers, had developed cost accounting procedures which, though not sophisticated, were sufficient to provide accurate margins between costs and sales prices on a daily basis: slaughtering and purchases at the terminal markets were diminished in response to a decrease in margins (Chandler, p. 396). Consequently, in view of shrinkage and the absence of refrigeration equipment adequate for storing dressed meat for more than a few weeks, terminal markets -- by virtue of possessing "beef on the hoof" (i.e. live cattle) -- provided the only feasible method of \textit{inventory control}.

\textsuperscript{22}Although transportation cost is not explicitly included in our model, given the number of units to be sold it easily could be added as a fixed cost. The implications of incorporating such a cost are clear. A decline in transportation costs for direct sales relative to terminal market sales would result in a decline in the use of terminal markets and an increase in the use of direct sales.

\textsuperscript{23}By the 1880's oligopoly/oligopsony was established in the meat packing industry (see Yeager [1981] for an excellent study) by the "Big Four" companies (Swift, Armour, Hammond, and Morris). The oligopoly exerted substantial control over methods by which livestock were traded; it impeded decentralization in the industry. In particular, given losses due to shrinkage and death of livestock during transportation, it is surprising that the introduction of refrigeration (refrigerated rail cars, in particular) did not lead to the relocation of meat packing plants to livestock growing regions more quickly than actually occurred.

The transport of refrigerated meat on a large scale began in the 1880's following the invention of the Swift-Chase refrigerated car in 1879 (Skaggs, pp. 90-95), and mechanical refrigeration was employed in meat packing plants in the 1890's (although natural refrigeration and some primitive mechanical refrigeration systems were used in the 1870's) (Wade [1987, pp. 198-201]). However, the facts that the packing industry was controlled by oligopoly (which deterred entry to the industry) when refrigeration was being developed as well as the fixed investment required to build new plants provide an explanation for the delay.
expansion of the highway system in rural areas and technological advances in truck transportation (e.g., improvements in the pneumatic tire and engines which allowed for the transportation of heavier loads) enabled farmers to transport livestock directly to buyers. In addition, improvements in refrigerated trucks and changes in transportation rate regulations that favored the transportation of livestock by truck instead of rail also favored the relocation of packing plants to producing areas. These changes effectively reduced transportation costs for direct sales and sales to country dealers relative to terminal markets and provided the impetus for the large scale decentralization of livestock markets: terminal markets were replaced by direct sales and sales to country dealers. By 1955 terminal markets accounted for only thirty-four percent of all livestock sales while direct sales and sales to country dealers each accounted for fifteen percent.

Advances in telecommunications technology and the introduction of uniform government livestock grading standards also had a significant impact on the development of sequential selling mechanisms in livestock markets. As the telephone continued to replace the telegraph, farmers were able to contact prospective buyers more quickly and at a lower cost. This reduced the cost of soliciting offers and shortened the epoch between offers, effectively increasing the discount rate $\beta$ for direct sales. Uniform government grading standards for live animals furthered the farmers' ability to conduct trades directly by eliminating the need for buyer observation of the livestock prior to agreeing upon a trade. These developments greatly diminished the need for terminal markets as a centralized market where sellers and buyers could interact, and they also limited the role of dealers as middlemen. As a result, the decentralization of livestock markets continued, and direct sales emerged as the dominant sequential selling mechanism. By 1987 nearly ninety percent of the sequential sales of livestock were direct sales.

**AUCTIONS**

The first known livestock auction to operate on a regular basis in the United States was

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24 By 1946 the red-meat industry was utilizing approximately 67,000 mechanically refrigerated trucks. Furthermore, trucks significantly reduced delivery times relative to rail: the trip from Detroit to Toledo was reduced from two and one half days to six hours (Skaggs pp. 152-153).

25 Following the Transportation Act of 1920, the Interstate Commerce Commission (ICC) imposed 25 to 40 percent increases in railroad freight rates. ICC interpretations of the Motor Carrier Act of 1935 also favored truck rates relative to rail. See Chapter 1 of Hilton [1969] for the history of ICC rate regulation.

26 Legislation requiring livestock inspection (for disease only) was implemented in 1890. Grading was not introduced until 1926, and an official livestock grading vocabulary arose by 1940. Furthermore, grading standards were revised several times between 1939 and 1981 (Skaggs, pp. 156, 169).
established in London, Ohio in 1856 by the Madison County Importing Company (Engelman and Pence, p. 3), but the number of livestock auctions remained small until the 1930's. This number expanded rapidly in the 1930's and 1940's, reaching its peak at just over 2500 in 1952. The factors responsible for the transition from terminal markets to direct sales were also the primary catalysts responsible for the rapid expansion of livestock auction markets. First, the Packer Consent Decree of 1920 ended the oligopsonistic control of the market by the Big Four and enabled the sale of large quantities of livestock by competitive bidding. Furthermore, the expansion of roads in rural areas, the decline in the cost of transporting livestock by truck, and the development of refrigerated trucks made it more cost effective for meat packers to relocate away from terminal markets and closer to producing areas. As a result, local auction markets developed to serve the growing demand in producing areas caused by these changes. Auctions also offered farmers a particularly efficient means of selling "odd lots" because the freight rate for shipping odd lots was considerably more expensive per animal. In many cases, due to the proximity of auction markets, the farmer would transport odd lots to the auction himself. By 1955 auctions accounted for twenty-six percent of livestock sales.

IV. B. AUCTIONS VS. SEQUENTIAL SEARCH

The Packer Consent Decree of 1920 and changes in transportation costs provide an explanation for the decentralization of livestock markets which occurred as meat packing was moved to producing regions and terminal markets were gradually replaced by direct sales and local auction markets. However, analysis of the impact of these factors alone provides little insight to the relationship between the sequential and auction mechanisms or movements in the crossing point \( n^* \). This relationship is more clearly reflected in the developments in telecommunications technology and the introduction of livestock grading standards. As discussed above, these factors led to the emergence of direct sales as the dominant sequential selling mechanism. Although these factors had a significant impact on both \( c \) and \( \beta \) for direct sales, their impact on auction markets was minimal. Because auctions were held on a fixed schedule (typically once or twice a week, depending on the size of the

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27 Livestock is typically shipped to market in either railroad car loads or truck loads. An odd lot is a shipment too small to fill a railroad car or truck.

28 In 1955 56 percent of cattle and 64 percent of calves sold at auction were transported less than 25 miles to the auction market (Engelman and Pence, p. 18).

29 In 1955 terminal markets accounted for 34 percent of all sales, direct sales for 15 percent, country dealers for 15 percent, and auctions for 26 percent (Kohls p. 288).
local market), decreasing the time between offers (increasing $\beta$) for auction markets was not a viable option. The consequence of a reduction in $c$ and an increase in $\beta$ for the sequential mechanism only is an increase in $S_n/n$ and no change in $V_n/n$. The curve labelled $S/n$ in Figure 3 depicts this increased return. Clearly, this increased return produces an increase in the crossing point (from $n^*$ to $n^+$).

-- Figure 3 here --

Because many ranchers sell their cattle at auction, the choice for each rancher is slightly different. For an individual rancher who seeks to market $n_R$ cattle and who anticipates that there will be a total of $n_A+n_R$ cattle auctioned if he brings his cattle to the auction, the choice between the sequential and the auction mechanisms merely calls for comparing $S_{n_A}/n_R$ 40 and $V_{n_A+n_R}/(n_A+n_R)$ 41. [As depicted in Figure 3, the preferred mechanism depends upon which of the two sequential returns is operative.]

A decline in auctions of relatively few units is certainly an anticipated outcome of the relative improvement in the sequential selling mechanism over the auction. However, this does not preclude the use of auctions altogether. Theorem 3 predicts that the percentage of sales conducted at auction could remain unchanged (or even increase) provided a sufficiently large quantity of livestock is offered for sale at each auction (i.e., $n>n^*$). In point of fact, this actually has occurred in livestock auction markets since 1955.

In 1955 there were 2322 livestock auction markets in operation in the United States. This number was stable through the 1950's, but it began to decline in the 1960's. The decline accelerated in the 1970's and 1980's. In 1985 the number of livestock auction markets was 1590. A corresponding reduction in the number of livestock sold at auction did not occur. In 1955 almost 52 million head of livestock were sold at auction. By 1985 this number was slightly over 60 million. The average annual throughput per auction market increased by more than 70 percent, from 22,164 in 1955 to 37,736 in 1985. Furthermore, although data on the number of days an auction market operates is not published, private communication with USDA analysts indicates that this number has not changed significantly since the 1950's. Whereas the percentage of livestock sold at auction remained fairly stable from 1955 to 1985 (decreasing from 26% to 23%), this evidence implies that both the annual throughput per auction market and the number of units sold on each day the auction is in operation have increased by 70% (i.e., $n$ has increased). Further evidence of a transition to larger livestock auction markets in response to an increase in $n^*$ is presented by Barker [1981 p. 108] in his analysis of agricultural markets.
in Great Britain.\textsuperscript{30}

\textbf{IV. C. CENTRALIZED AUCTIONS}

Lemmas 6 and 7 suggest that changes in the market favoring sequential mechanisms should result in an increase in the size of auction markets (an increase in $n^*$). Theorems 4 and 5 demonstrate that, holding other factors such as transportation and solicitation costs fixed, larger auctions are superior to smaller auctions.\textsuperscript{31} These theorems detail the circumstances under which centralized auctions yield higher expected profits per unit than decentralized auctions.

\textbf{Theorem 4:} If buyer valuations are either uniform or exponential, then $V_{n+m} \geq V_n + V_m$.

Proof: Define $A_n \equiv V_n/n$. Then $V_{n+m} = (n+m)A_{n+m} = nA_n + mA_m \geq nA_n + mA_m = V_n + V_m$, where the inequality follows from the fact that $A_n$ is increasing (Theorem 2). Note that the superadditivity of $V_n$ does not imply Theorem 2.

Theorem 5 generalizes this result to any distribution of buyer valuations for perfectly discriminating auctions in which each bidder bids his true valuation and winning bidders pay their bids.

\textbf{Theorem 5:} For a perfectly discriminating auction, $V_{n+m} \geq V_n + V_m$.

Proof: Let $s$ and $t$ be the optimal number of bids to obtain when auctioning $n$ and $m$ items, respectively. Accordingly, let $\{x_1, \ldots, x_s, x_{s+1}, \ldots, x_{s+t}\}$ be the $s+t$ bids for these two auctions. Clearly, $V_{n+m} \geq V_{n+m}(s+t) > V_n + V_m$ as one or more of the $n$ largest values of $\{x_1, \ldots, x_s\}$ will, with positive probability, not be amongst the $n+m$ largest values of $\{x_1, \ldots, x_{s+t}\}$.

Theorems 4 and 5 imply that centralization enhances the ability to price discriminate using the auction mechanism. Although data related to these theorems is limited, it is supported by the data that is available. According to Engelman and Pence [1958, pp. 32-33], in 1955 the value per head was $84.30 for cattle sold at small auctions (less than 10,000 units per year), $89.83 for cattle sold at medium auctions (10,000 - 24,999 units per year), and $99.63 for cattle sold at large auctions (25,000

\textsuperscript{30} In addition to discussing reasons for the increased centralization of livestock auction markets that has occurred over the last forty years, Barker cites analysis which predicts that auctions with an annual throughput of more than 20,000 units of livestock will be successful while those which handle fewer than 10,000 units will diminish.

\textsuperscript{31} These results also raise the question of why auctions were not held at terminal markets, given the highly centralized nature of these markets. However, the oligopsony control exerted by the Big Four would have rendered auctions ineffective.
or more units per year). These price differences held despite the fact that small auctions were heavily concentrated in the Northeast where the value per head was $126.27, while large auctions were concentrated in the South West Central and the West where the value per head was $79.15 and $97.09, respectively. These data coincide with the implicit predictions of Theorems 4 and 5. In view of the increase in \( n^* \) (caused by the advances in the technology of direct sales), centralization was essential to maintaining auctions’ 23 percent share of all livestock sales.

Auctions conducted via satellite video are the most recent development in livestock marketing. In these auctions an auctioneer then conducts a live auction in which buyers submit bids by telephone and videotape of the livestock being sold accompanied by a narrative description using United States Department of Agriculture grading terminology is broadcast via satellite. Because the cattle are transported directly from the seller to the buyer without an intermediate trip to an auction yard or terminal market, transportation costs are reduced, shrinkage is minimized, and yardage fees are eliminated. The video auctions also have the advantage of being highly centralized: the Superior Livestock Auction Company sold an average of 35,000 cattle at each video auction they conducted in 1991.\(^{32}\) This compares with 6,000 to 7,000 cattle sold per auction in the peak season at the largest traditional auctions. Furthermore, a comparison of video and traditional cattle auctions indicates that prices received at video auctions exceed those received at traditional auctions.\(^{33}\) Again, actual results in livestock auctions comport with the predictions of Theorems 4 and 5 that centralized auctions will generate more revenue.

\(^{32}\) Over 90,000 cattle were sold at a single auction in 1991 -- answering the immortal question: Where's the beef?

\(^{33}\) Adjusting for quality, shrink, and transportation and commission charges cost paid by sellers, the regression analysis effected by Bailey, Peterson, and Brorsen [1991] indicates that the average revenue to sellers generated at satellite video auctions was between $.95 and $3.63 per hundred weight higher than at the comparison auction markets.
Figures 2a and 2b
Figure 3
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Appendix: Proof of Lemma 5

Fix n and d.

F is uniform: Pick i such that \( d \in \left( \frac{4n(n+1)}{2(i+1)^2 - 1}, \frac{4n(n+1)}{2(i+1)^2 - 1} \right) \) \( \cup \) \( 2i = i \), and recall that \( m = i \). Thus,

\[ V_n(\frac{m_n}{n}) = n(1 - \frac{n+1}{i+1}) - i, \]

where the first equality follows from (8), the inequality from the fact that \( d \leq 4n(n+1)/{(2i+1)^2 - 1} \), and the final equality from algebraic simplification. If \( n \geq 2n(n+1)/(i+1) \), (this implies \( i \geq 2n+1 \)), then \( V_n(m_n) \geq 0 \). But \( i \geq 2n+1 \) implies that \( d \leq n/2(2n+1) \). Likewise,

\[ V_n(\frac{m_n}{n}) = n\left( 1 + \frac{2n(n+1)}{(2i+2)(2i+3)^2 - 1} \right) - i, \]

where the inequality follows from the fact that \( d > 4n(n+1)/{(2i+3)^2 - 1} \), and the equalities, from algebraic simplification. Thus, if \( n < 2n(n+1)/(i+2) \), (this implies \( i < 2n \)), then \( V_n(m_n) < 0 \). But \( i < 2n \) implies \( d > n/2(2n+1) \).

We now have \( V_n(m_n) \geq 0 \) if \( d \leq n/2(2n+1) \), and \( V_n(m_n) < 0 \) if \( d > n/2(2n+1) \) which proves the result.

F is exponential: The following preliminaries will prove useful in the proof:

(A1) \( \frac{1}{j} \int_j^{j+1} dx > \frac{1}{j+1} \int_j^{j+1} dx = \ln(j+1) - \ln(j) = \ln(\frac{j+1}{j}) \)

(A2) \( \frac{1}{j+1} \int_j^{j+1} dx < \frac{1}{j} \int_j^{j+1} dx = \ln(j+1) - \ln(j) = \ln(\frac{j+1}{j}) \)

(A1) implies

\[ \sum_{i=n+1}^{m} \frac{1}{i} \int_i^{i+1} dx > \sum_{i=n+1}^{m} \frac{1}{i} \int_i^{i+1} dx = \sum_{i=n+1}^{m} (\ln(i+1) - \ln(i)) \]

\[ = \ln(m+1) - \ln(n+1) = \ln(\frac{m+1}{n+1}) \]

and (A2) implies

\[ \sum_{i=n+1}^{m} \frac{1}{i} \int_i^{i+1} dx < \sum_{i=n+1}^{m} \frac{1}{i} \int_i^{i+1} dx = \sum_{i=n+1}^{m} (\ln(i) - \ln(i-1)) = \ln(\frac{m}{n}) \]

Thus,

\[ \ln(\frac{m+1}{n+1}) < \sum_{i=n+1}^{m} \frac{1}{i} < \ln(\frac{m}{n}) \]

(9), and (A1)-(A5) give us the following:

\[ V_n(m_n) > nln(\frac{m+1}{n+1}) - mc. \]

So, \( V_n(m_n) > 0 \) if \( nln(\frac{m+1}{n+1}) > mc \), or if

\[ \ln(\frac{m+1}{n+1}) > (\frac{n}{c})c, \]

since \( m = \left\lfloor \frac{n}{c} \right\rfloor \leq \frac{n}{c} \). This implies \( V_n(m_n) > 0 \) if \( \frac{m+1}{n+1} > e \) or, \( m > ne+e-1 \) which holds if \( (n/d)-1 > ne+e-1 \), or \( d < n/(ne+e) \). Thus, \( d^* \geq n/(ne+e) \). To establish the upper bound, note that \( V_n(m_n) < 0 \) if \( nln(m/n)-dm < 0 \), or \( dm > nln(m/n) \). This holds if \( n > nln(m/n) \) (because \( m \leq n/d \)), or \( e > m/n \), which holds if \( ne > n/d \), or \( d > 1/e \). Thus, \( d^* \leq 1/e \).
\begin{align*}
S_{n_R} &= n_R \text{, } 54 \\
V_{n_A+n_R} &= (n_A + n_R) \text{, } 55 \\
S_{n_R} &= n_R \text{, } 56
\end{align*}

\begin{align*}
n_R & \quad n^* & \quad n^+ & \quad n_A+n_R
\end{align*}

\begin{align*}
V_n = n
\end{align*}

\begin{align*}
S/n \\
S_n/n
\end{align*}