

Posted Prices versus Reservation Prices with Imperfectly Informed Buyers

Michael A. Arnold and Steven A. Lippman

Department of Economics, University of Delaware

Graduate School of Management, University of California, Los Angeles

August 28, 1992

ABSTRACT

Bargaining is standard in some markets and posted prices are the norm in others. In this paper we construct a model of imperfect information consistent with this empirical reality in order to *predict* when sellers will utilize posted prices. The key feature of our search model is the proportion of informed agents. Whereas buyers' imperfect information confers a degree of monopoly power on firms by enabling them to price discriminate, our analysis confirms that the firm's monopoly power atrophies -- and the firm responds by reducing its reservation price -- as the proportion of informed buyers increases. Furthermore, if the proportion of informed buyers is sufficiently large, their success in policing the market will induce the seller to utilize a posted price. In accord with our analysis, we anticipate bargaining to be the norm in markets where most buyers are infrequent participants (so the proportion of informed buyers is relatively small). Interestingly, social welfare can decrease as the proportion of informed buyers increases.

I. INTRODUCTION

The spirited bargaining of the bazaar has been the predominant price setting mechanism in most producer and consumer markets in the Western world for many centuries whereas the use of posted prices is a relatively recent phenomenon. The ascent and eventual widespread use of posted prices began in the middle of the nineteenth century¹ when Alexander Stewart introduced posted prices in 1823 in his New York dry goods store -- the forerunner of his famed Marble Dry Goods Palace.² Originally considered a gimmick, the advent of this revolution in retail pricing became evident as his store grew to become the largest in the U.S. and other famous merchants followed his lead: Gimbel's in the 1840's, Macy's and Tiffany's in the early 1850's, and Brooks Brothers in the 1860's. In the 1880's Michael Marks' stalls in Great Britain's local markets were easily distinguished by a banner proclaiming "Don't Ask the Price -- It's a Penny."

By the late 1800's posted prices had emerged as the dominant pricing mechanism in most retail markets. The transition from bargaining to posted prices coincided with the transition from markets in which individual merchants rented stalls to sell their wares to large department stores in which sales clerks were employed by the proprietor to conduct trade.³ Bargaining in a department store would require not only clerks skilled in negotiation but also a monitoring system to ensure that the clerks bargain in the proprietor's best interest. Posted prices eliminate both concerns.⁴

Despite the increasing use of posted prices over the last 170 years, the practice of bargaining persists in many markets, particularly markets for durable goods such as new automobiles and fine jewelry. This suggests that additional factors govern the seller's choice of a pricing strategy. Macy's advertisements in the early 1850's espousing the benefits to the buyer of purchasing at a posted price are germane. The advertisements state that prior to the establishment of Macy's posted price policy "there was no regular price for anything and the most ignorant were the most imposed upon," but with posted prices "a child can trade with us as cheap as the shrewdest buyer in the country" (Scull, p. 83). These advertisements identify the presence of imperfectly informed consumers as an important factor in

¹ It is thought that the Mitsukoshi store, opened in 1673 in Tokyo, was the first major retail establishment to utilize posted prices (see Mahoney and Sloane [1966, p. 3]).

² See Mahoney and Sloane (p. 10) and Chandler [1977, p.225].

³ See Chandler (pp. 225-229) for a discussion of the evolution and advantages of department stores.

⁴ The early department store magnates also believed that bargaining dissipated both customer loyalty and clerks' time. The success of their stores supports this assessment. (See Scull [1967, p.81]).

the seller's choice of a pricing strategy and highlight the advantage of bargaining: the seller may be able to extract sufficiently high prices from imperfectly informed consumers so as to offset the benefits (e.g., decreased training and monitoring costs and increased consumer loyalty) of a posted price policy.

The purpose of this paper is to provide a model of imperfect information, consistent with the fact that some markets witness posted prices and others witness price discrimination, in order to **predict** which markets will utilize posted prices.

Our modelling begins with a single seller who seeks to sell a good by using one of two pricing mechanisms -- a posted price or a (concealed) reservation price -- and a population of consumers who possess differential information: α is the proportion of informed consumers and $1-\alpha$ is the proportion of imperfectly informed consumers.⁵ The information possessed by the informed consumers is perfect: they know the seller's reservation price. In addition to the information they possess, consumers are distinguished by their private values for the good. The seller's action in this game entails selecting first a pricing mechanism and then a reservation price. The first choice is revealed to the buyers whereas the second is not. Of course, her reservation price is irrelevant if she elects to post a price.

The use of a reservation price can, in a limited way, be thought of as a proxy for bargaining: if trade occurs with an informed consumer, this consumer offers the seller's reservation price and obtains the maximum surplus possible (given the seller's outside opportunities); if trade occurs with an imperfectly informed consumer, the consumer may receive little or none of the surplus. As it were, ours is a model with a highly stylized bargaining process.

Our analysis investigates the impact of perfect information by focusing upon the proportion α of informed consumers. We find that each informed consumer confers an externality upon the imperfectly informed consumers as well as the other informed consumers: as α increases, the seller's monopoly power diminishes and she lowers her reservation price. At some point $\alpha^* < 1$, however, the seller's monopoly power diminishes to such an extent that she is better served by a posted price than by exerting market power via price discrimination. Thus, our analysis predicts bargaining in markets in which buyers participate infrequently -- the proportion α of informed buyers is relatively small. [The impact of monitoring costs and *ex post* opportunism is also considered.] In addition, the role of

⁵ These informational asymmetries arise in the market for automobiles. While buyers with low costs devote more effort to collecting information than those with high costs, even with the same amount of effort some buyers will gain more information whether due to more ready access to general information (e.g., *Consumer Reports*), luck, or inside connection (informants). In addition, buyers differ in their ability to assimilate and/or act upon the information they do acquire. In the end, some buyers are well-informed whereas others remain poorly informed.

consumer information is seen to be consistent with the aforementioned Macy's ads: if enough buyers are imperfectly informed, then the seller does best by discriminating against these buyers. The social welfare implications are somewhat surprising. We prove that social welfare must decrease as the proportion of informed buyers increases when imperfectly informed buyers are poor at bargaining.

Section 2 reviews the related literature. The analysis and results of section 3 are followed by social welfare implications in section 4. Issues entailed in modelling imperfectly informed buyers are presented in the Appendix.

II. RELATED LITERATURE

As per the literature referenced in this paragraph, analyses of the seller's choice of a pricing mechanism (as well as the related issues of the evolution of selling institutions) recently has received increased attention. Arnold and Lippman [1991] investigate the seller's choice between auctions and sequential selling of multiple units of a homogeneous good in the presence of asymmetric information and transaction costs. Highlighting the relevance of the theoretical results to the actual use of auctions and sequential selling in livestock markets, they demonstrate how the choice between these two mechanisms depends upon the number of units being sold. DeVany [1987] and Wang [1991] consider the seller's choice between a posted price and an auction. Spier [1991] analyzes an economy with high and low valuation buyers who search across sellers for the lowest price. Assuming that the buyer and the seller split the difference between the seller's cost (zero) and the buyer's endogenously determined reservation price when the seller haggles, there exists a competitive equilibrium in which some sellers post a price and others haggle: low valuation customers purchase only from hagglers, high valuation customers purchase from either hagglers or posters, and all sellers earn the same profit.

Riley and Zeckhauser [1983] consider a market with one seller, capable of committing to any selling strategy, who only knows the distribution of buyer valuations and incurs a search cost to attract buyers sequentially. All buyers have perfect information; in particular, they can assess the outcome of any randomization effected by the seller. Consequently, each buyer holds out for a low price when the seller haggles whence posting a price is the optimal (rational expectations) equilibrium strategy for the seller. The seller wields even less market power and the competitive price will result if she is unable to commit to a selling strategy. This follows from an application of the logic of Diamond [1971] to a market in which the seller (rather than the buyer) incurs a search cost: buyers deduce that it is in the seller's best interest to accept any offer greater than or equal to marginal cost because the seller's alternative is to pay the search cost in order to solicit another buyer who is equally savvy and also

offers a price equal to marginal cost. In short, selling to each buyer at the competitive price is a unique subgame perfect equilibrium. Note that this result is closely related to the Coase Conjecture regarding the inability of durable goods monopolists to commit to the monopoly price.

The seller's ability to price discriminate in the above models is restricted by the buyer's extensive knowledge of the market. In actual markets, however, buyers do not possess perfect information about the seller or about the other buyers. If a buyer initially is uninformed, then he must gather information in order to formulate an optimal response. The price dispersion literature presents several scenarios in which the information acquisition process leads to differences in buyers' information at the time of purchase. We now discuss several of these scenarios.

Salop [1977] considers a product market in which consumers know the distribution of (posted) prices charged by a monopolist who operates several different outlets, but they do not know the price charged at any particular outlet. This information can be acquired by shopping -- going to different outlets and observing the posted price -- but shopping is costly, and the cost varies across consumers. Based on the distribution of prices and his cost of shopping, each buyer computes his optimal reservation price strategy. As a result, buyers acquire different amounts of specific price information prior to making a purchase: on average a buyer with low shopping costs solicits more outlets and concludes his buying decision with more information than a buyer with high shopping costs. Salop and Stiglitz [1977] characterize a similar equilibrium with price dispersion and buyers acting upon differential information in a market in which buyers incur either a high or low search cost to gather perfect price information from a fixed number of monopolistically competitive sellers. Burdett and Judd [1983] generalize this result to the case in which buyers have the same search cost but receive a random number of price quotes. Butters' [1977] model in which consumers are endowed with different information via the costless receipt of randomly disseminated ads also exhibits price dispersion in equilibrium.

In the Salop and Salop and Stiglitz models some buyers never collect information, they simply purchase the item from the first store they enter. The results of Wilde and Schwartz [1979] pinpoint the importance of these imperfectly informed buyers to the existence of price dispersion. In their model some buyers solicit only one store while others, called shoppers, solicit $n \geq 2$ stores. If a sufficiently large proportion of the buyers are shoppers, then equilibrium entails all sellers charging the competitive price. In our model the proportion of informed buyers also plays a central role in determining the seller's choice of a selling mechanism

These equilibrium price dispersion models are distinguished by three distinct mechanisms --

different buyer search costs, random endowments of market information, and different propensities to shop -- which induce a population of consumers to come to possess and act upon differential information. They emphasize the importance of information in determining market prices, and they incorporate an empirical reality of the marketplace: different buyers often pay different prices for the same product. The present paper builds upon this theme by focusing upon the seller's choice of a selling mechanism in reaction to the buyers' differential information.

III. MODEL AND ANALYSIS

A monopolist (she) who has one unit of a good to sell must decide between a posted price strategy in which all buyers are charged the same price and a reservation price strategy in which buyers make an offer which the seller can accept or reject. Under either strategy, at a search cost of $c > 0$ per unit time, the monopolist solicits a stream of buyers who arrive according to a Poisson process with rate λ . It is helpful to interpret c as the cost of advertising the product and the arrival stream as the random response to this advertising. The seller discounts payments received in the future at the rate $\delta \geq 0$.

Each buyer (he) desires to purchase one unit of the product. The valuation v of a particular buyer reflects his preferences and (his knowledge about) the price of substitutes for the monopolist's product. From the seller's perspective the valuation of any particular buyer is a random variable V drawn from a cumulative distribution function F with continuous density function f .

In addition to the buyer's valuation, the monopolist's selling strategy is relevant to the buyer's beliefs and his optimal response. If the seller posts a price, then each buyer who enters the store immediately observes the posted price (and therefore is fully informed of the seller's strategy) and purchases the item if his valuation exceeds the posted price. However, if the seller adopts a reservation price, then the buyer's beliefs change in response to this observation, and he must (re)formulate an optimal response based upon his (re)assessment of the seller's strategy. Rather than specifying the mechanism (luck, insider information, superior information endowment, low information acquisition cost, etc.) which endows some buyers with perfect information about the seller's strategy, we simply posit that the proportion of informed buyers is α . The remaining proportion $1 - \alpha$ is imperfectly informed. A discussion of the issues involved in modelling imperfectly informed buyers as well as examples illustrating how buyer beliefs influence the seller's choice of a pricing mechanism are given in the Appendix.

In addition to his valuation v , each imperfectly informed buyer has beliefs about the minimum price at which he can purchase the item (equivalently, beliefs about the seller's strategy) when the seller does not post a price. These beliefs do not change with α . The interaction of the imperfectly informed buyer's beliefs with his valuation v induce him to make an offer b which is optimal in light of his beliefs. From the seller's perspective the offer b which she receives when she solicits an imperfectly informed buyer is a random variable B drawn from the cumulative distribution function G with density g .⁶ Of

⁶ Although a general framework for analyzing the information acquisition and decision processes of the buyers is not developed, we appeal to stylized facts of the marketplace, the various mechanisms in the aforementioned

course, $G \geq F$: the bid b of an imperfectly informed buyer will not exceed his private valuation v .

3.1 Posted Price Strategy

If the monopolist posts a price p , then each buyer with private value $v \geq p$ who enters the store will purchase the item. Because each buyer's willingness to pay is viewed by the seller as a random variable V drawn from F , the seller's expected return $R(p)$ associated with the posted price p is

$$R(p) = -c / (d + 1) + 1 / (d + 1) \{ R(p) F(p) + p \bar{F}(p) \} \quad .1$$

To see this, define τ to be the (random) time at which the first offer arrives so that τ is an exponential random variable with parameter λ . Then $E e^{-d\tau}$, the expected discounted time of receipt of the first offer, satisfies $E(e^{-d\tau}) = 1 / (d + 1)$.³ Also, the discounted cost of soliciting the first offer is

$$c E \int_0^{\tau} e^{-dt} dt = c E(1 - e^{-d\tau}) / d = c / (d + 1) \quad .4$$

Simplifying, the displayed expression yields

$$R(p) = [-c + 1 p \bar{F}(p)] / [d + 1 \bar{F}(p)] \quad , \quad (1)$$

where $\bar{F}(p) = 1 - F(p)$. From the first order condition associated with (1) we see that the optimal posted price p^* solves

$$d x = -c + (d + 1) \bar{F}(x) / h(x) \quad , \quad (2)$$

where $h(x) = f(x) / F(x)$ is the hazard rate function.⁷ A unique solution to (2) exists provided that the hazard rate is non-decreasing and continuous and $h(0)$ is sufficiently small.⁸

3.2 Reservation Price Strategy

Suppose the seller elects to employ a reservation price strategy in which she requires each solicited buyer to make an offer. Whether the seller accepts or rejects the offer depends upon her expected return from rejecting it and soliciting another buyer. To begin, suppose the proportion α of

price dispersion literature, and the examples in the appendix to motivate the assumption that imperfect buyer information presents the seller with a distribution of possible payoffs when she employs a concealed reservation price: each imperfectly informed buyer represents a random draw from this distribution.

⁷ Note that profitability with a posted price requires $p[1 - F(p)] > c / \lambda$. We assume this condition holds throughout.

⁸ More specifically, a unique solution to (3) exists if $h'(p) > -[\delta(h(p))^2 + \lambda f(p)h(p)] / (\delta + \lambda F(p))$. Because the right-hand-side of this expression is negative, a unique solution exists provided the hazard rate does not decrease too quickly.

informed buyers is 0. Then offers which can be obtained by soliciting successive buyers are independent draws from $G(\cdot)$. Following well known arguments from the search literature, the seller's optimal strategy is a reservation price policy, and the seller's expected return equals the optimal reservation price. Let ξ denote both the optimal reservation price and the seller's expected return. Because the seller accepts any offer in excess of ξ and rejects the offer and solicits another buyer if the offer is less than ξ , her expected return from following the optimal policy is

$$x = -c / (d + 1) + 1 / (d + 1) \left\{ \int_0^x x dG(b) + \int_x^\infty b dG(b) \right\} \quad (7)$$

$$= \{-c + 1 \cdot x + \int_x^\infty (b - x) dG(b)\} / (d + 1) \quad (8)$$

and the optimal reservation price ξ is the solution to

$$d \cdot x = -c + 1 \cdot H(x) \quad (3)$$

where $H(x) = \int_x^\infty (b - x) dG(b)$ ¹⁰.

When $\alpha > 0$, the seller can no longer treat a buyer's offer as a random draw from $G(\cdot)$. Let ξ_α denote the seller's expected return when α is the proportion of informed buyers. Clearly, the optimal strategy of an informed buyer entails offering the smaller of ξ_α and his private valuation v .⁹ As evidenced in Theorem 1, this strategic behavior of informed buyers drives the reservation price toward the competitive level. Nevertheless, the seller can, without loss, reject all offers from informed buyers¹⁰ so that

$$x_a = -c / (d + 1) + (1 - a) \left\{ \int_0^{x_a} x_a dG(b) + \int_{x_a}^\infty b dG(b) \right\} / (d + 1) + a \cdot x_a / (d + 1) \quad (11)$$

$$= \{-c + (1 - a) \cdot x_a + a \cdot x_a + (1 - a) \cdot H(x_a)\} / (d + 1) \quad (12)$$

and the seller's optimal reservation price ξ_α is the solution to¹¹

$$d \cdot x = -c + (1 - a) \cdot H(x) \quad (4)$$

⁹ Because the seller incurs a search cost, informed buyers will offer $\min\{v, \xi_\alpha\}$. Consequently, attempting to enforce a reservation price in excess of ξ_α is not subgame perfect for the seller.

¹⁰ If the seller solicits an informed buyer, then (i) with probability $F(p)$ the buyer's valuation exceeds ξ_α , and the seller receives an offer of exactly ξ_α which leaves her indifferent between accepting and rejecting and (ii) with probability $F(p)$ the buyer offers less than ξ_α , and the seller rejects the offer because the expected return from soliciting another buyer is ξ_α . In either case it is *as if* the seller rejects all offers from informed buyers. Consequently, the distribution F does not enter into either (3) or (4): the introduction of informed buyers has the effect of decreasing the arrival rate from λ to $(1 - \alpha)\lambda$.

¹¹ The strategies just characterized constitute an equilibrium: the offer of each incompletely informed buyer maximizes his expected payoff given his beliefs.

The following theorem establishes the fact that the ability of the seller to exert monopoly power hinges on the proportion of informed buyers in the market.

Theorem 1: The optimal reservation price ξ_α is strictly decreasing on $[0, \alpha_0]$ where $\alpha_0 = 1 - c/\lambda E[B]$. The proof follows directly from differentiation of (4).¹²

Theorem 1 is relevant to Diamond's model in which the buyer cannot exert market power because he incurs a search cost and to the Coase Conjecture¹³ regarding a durable goods monopolist's ability to extract monopoly profits. Theorem 1 implies that the seller can earn positive monopoly rents provided that the proportion of informed consumers is not too large ($\alpha < \alpha_0$). The level of monopoly rents which the seller can extract decreases as the proportion of informed consumers increases until α reaches α_0 ; at this point the seller earns no monopoly rents. When $\alpha \geq \alpha_0$, the seller still is able to price discriminate against imperfectly informed buyers, but her expected return is negative. The market will not open with a reservation price strategy if $\alpha > \alpha_0$: all buyers paying the competitive price cannot occur.

3.3 Selecting a Mechanism

Now consider a market in which the seller has the option of either employing a reservation price strategy or committing to a posted price. While a reservation price strategy allows the seller to price discriminate, it also subjects her to opportunistic behavior on the part of informed buyers. Committing to a posted price, on the other hand, prevents opportunistic behavior by the informed buyers but eliminates potential gains from price discrimination.

If the seller adopts a reservation price strategy, her expected return is the solution to (4). If she posts a price, her expected return is found by solving (2) for the optimal posted price and substituting

¹² As $(1-\alpha)\lambda$ is the effective arrival rate of offers, (4) implies that a necessary condition for search to be profitable is $c < (1-\alpha)\lambda E[B]$. Note that (4) has no positive solution for $\alpha \geq \alpha_0$ so the seller will not employ a reservation price if $\alpha \geq \alpha_0$. The seller's ability to exert monopoly power increases (α_0 increases) as the difference between the cost $c/(\delta+\lambda)$ of soliciting an offer and the expected gain $\lambda E[B]/(\delta+\lambda)$ from soliciting an imperfectly informed buyer increases. However, $c > 0$ implies $\alpha_0 < 1$ so the seller is prevented from exerting monopoly power without all buyers being informed.

¹³ Although the model presented here differs from Coase's model in that the seller incurs positive transaction costs and encounters consumers at random over time rather than all within "the twinkling of an eye," the intuition is similar. In fact, because transaction costs are zero in Coase's model, the monopolist could always extract monopoly rents from imperfectly informed consumers, and only when $\alpha=1$ are the monopolist's profits are reduced to the competitive level. See Ausubel and Deneckere [1989] for an analysis of the conditions under which the Coase Conjecture fails to obtain when all buyers are informed.

this value into (1). She selects the strategy which yields the higher expected return. Of course, given the presence of imperfectly informed buyers, this strategy is Nash.

If the imperfectly informed buyers are overly optimistic (and/or good at bargaining), that is, if the offers B are much smaller than private values V , then the seller will not utilize a reservation price, not even if $\alpha=0$. Lemma 1 quantifies how far B can drop below V while maintaining the seller's strict preference for a reservation price when $\alpha=0$.¹⁴

Lemma 1: The expected return ξ_0 from the optimal reservation price when no buyers are perfectly informed is strictly greater than the expected return from the optimal posted price policy if and only if

$$\overline{F}(p^*)/h(p^*) < H(p^* - 1/h(p^*)) \quad (5)$$

where p^* is the solution to (2).

Proof: We claim $R(p^*) = p^* - 1/h(p^*)$. To see this, note from (1) that $p - R(p) = (c + \delta p)/(\delta + \lambda F(p))$ and from (2) that $(c + \delta p^*)/(\delta + \lambda F(p^*)) = 1/h(p^*)$. Thus, $p^* - R(p^*) = 1/h(p^*)$, so $R(p^*) = p^* - 1/h(p^*)$, which verifies the claim.

The seller's expected return ξ_0 from a reservation price when $\alpha=0$ is given by the solution to (3). In order for the seller to prefer the reservation price strategy over posting a price, the above claim implies that we must have $\xi_0 > p^* - 1/h(p^*)$. Thus, because $H(x)$ is strictly decreasing in x and δx is strictly increasing in x , (3) implies that $\xi_0 > p^* - 1/h(p^*)$ if and only if

$$c + d [p^* - 1/h(p^*)] < H(p^* - 1/h(p^*)). \quad (5')$$

Condition (5) results from rearranging (2) to get $c + \delta(p^* - 1/h(p^*)) = \lambda F(p^*)/h(p^*)$ and substituting into (5'). ■

We now establish the fact that it will be optimal for the seller to employ a reservation price provided (5) holds and there are enough imperfectly informed buyers.

Theorem 2: If (5) holds, there exists a critical value $\alpha^* < 1$ such that the seller employs a reservation price strategy if $\alpha < \alpha^*$ and a posted price if $\alpha > \alpha^*$.

Proof: If (5) holds, Lemma 1 asserts that $\xi_0 > R(p^*) > 0$ (see footnote 10), while Theorem 1 asserts that ξ_α is strictly decreasing in α and reaches zero at $\alpha_0 < 1$. Because $R(p^*)$ is not a function of α , there is a value α^* such that $R(p^*) \geq \xi_\alpha$ if $\alpha \geq \alpha^*$ and $R(p^*) < \xi_\alpha$ if $\alpha < \alpha^*$. ■

¹⁴ When viewing (5), recall that $H(x) = E\{(B-x)^+\}$ and the distribution of B is G (not F).

Theorem 2 illustrates the role of informed buyers. If the proportion of informed buyers is large enough, then these buyers play a role analogous to the role of shoppers in the Wilde and Schwartz model: they effectively police the market and prevent the seller from price discriminating. However, if the proportion of informed buyers is less than α^* , then price discrimination is optimal.

Because an increase in the *frequency* of buyer participation in a market enlarges the pool of social information available to consumers and increases the information acquired through market experience, Theorem 2 suggests that the frequency of purchase will impact the seller's strategy: α is likely to increase with the frequency of purchase, whence the seller is more likely to post a price. High frequency purchases characterize retail markets for non-durable goods. In these markets consumers interact with sellers on a regular basis and are likely to be (well) informed. In contrast, markets for durable goods favor bargaining because consumers participate in these markets on an infrequent basis. The markets for new automobiles and fine jewelry are two examples. *Monitoring* costs also influence the seller's strategy: posted prices are more likely to occur with low price goods (both durables and non-durables), and price discrimination is more likely with "big ticket" items because gross profits are sufficiently large to justify monitoring. A third consideration which affects the buyers' willingness to bargain is the ability to determine the *quality* of the product or service being purchased. The seller cannot alter the quality of an automobile if the customer secures a low price and drives the car off the lot. In purchasing services, however, *ex post* opportunism is possible: after agreeing upon a low price, the seller can reduce the quality. Thus, we anticipate the absence of bargaining in a market in which the consumer cannot monitor quality easily.

In addition to the impact that the proportion α of informed buyers has on the seller's choice of a selling mechanism, the level of dispersion in buyers' private values also is important. Greater dispersion in buyer information fosters the use of a reservation price strategy.

Corollary 1: Let α be the proportion of perfectly informed buyers such that the seller is indifferent between a posted price and a reservation price when the offer made by an imperfectly informed buyer is B_1 , and suppose B_1 is riskier than B (i.e., B dominates B_1 in the sense of second-order stochastic dominance) with $E[B_1]=E[B]$. Then $\alpha > \alpha^*$.

Proof: Because B_1 is riskier than B , $E[B_1]=E[B]$, and $\phi(y) = (y-x)^+$ is a convex function, $H_1(x) = E\phi(B_1) > E\phi(B) = H(x)$ (see Lippman and McCall [1981], p. 216). Thus, $H_1 > H$. So (4) implies that $\xi > \xi_\alpha$, where ξ is the optimal reservation price associated with B_1 . Hence, $x_{a^*}^1 > x_{a^*} = R(p^*)$. Because ξ is strictly decreasing in α , it follows that $\alpha > \alpha^*$. ■

IV. WELFARE IMPLICATIONS

Because the seller is indifferent to offers from the informed buyers and the cost of searching for an imperfectly informed buyer increases with α , the monopolist becomes less discriminating and lowers her reservation price as α increases. In turn, the (average) private value of an informed buyer who purchases the item decreases in α : $E[V | V \geq \xi_{\alpha}]$ decreases in α . Furthermore, assuming that imperfectly informed buyers' bids are increasing in v , the (average) private value $E[V | B \geq \xi_{\alpha}]$ of an imperfectly informed buyer whose bid is accepted also decreases in α . On average the misallocation of the good is exacerbated by increased information. On the other hand, the seller's expenditure on search decreases as α increases. Although the net effect of an increase in information on total surplus appears to be ambiguous, we are able to sign it.

To simplify the analysis we focus upon the case in which imperfectly informed buyers are poor at bargaining, $G=F$.¹⁵ Let $S(p)$ denote the seller's return from setting a reservation price of p when $\alpha=0$, then

$$S(p) = -c/(\delta + \lambda F(p)) + \{\lambda F(p)/(\delta + \lambda F(p))\}E(V | V \geq p). \quad (6)$$

The social welfare under either strategy is the expected valuation of the buyer who purchases the item minus the search cost incurred in locating that buyer, both appropriately discounted. As the buyer's payment is merely a transfer from the buyer to the seller, it does not affect social welfare. With $G=F$, the social welfare from either a reservation price or a posted price of x is $S(x)$: the first term is the (discounted) cost of locating a buyer who offers at least x , and the second term is the (discounted) expected private value of the purchaser. Thus, when α is the proportion of informed buyers, the social welfare W_{α} is given by

$$W_a = \begin{cases} S(x_a), & a < a^* \\ S(p^*), & a \geq a^*. \end{cases} \quad (7)$$

Because the seller is not perfectly informed, she cannot ensure that the item is sold to the highest valuation buyer. In this sense the allocation of the good is second best. Theorem 3 characterizes social

¹⁵ Whereas imperfectly informed buyers do not even know α , the fact that they are optimizing agents implies that their bids will be less than their private values so the case $G = F$ can not occur. Nevertheless, recalling that the reservation price strategy can be interpreted as a proxy for bargaining, the case $G = F$ corresponds to imperfectly informed agents who are so hopelessly poor at bargaining that the outcome of the (unmodeled) bargaining process leaves these buyers with the worst possible split of the available surplus: each pays his valuation and the seller captures all of the surplus. In short, the case $G = F$ represents a model in which the proportion α of the buyers are expert bargainers and the remaining buyers are totally inept.

welfare for α less than α^* ; its proof is immediate from Lemma 2 (which generalizes the result on page 219 of Lippman and McCall).

Lemma 2: The seller's return $S(p)$ is unimodal and reaches a maximum at ξ_0 . In addition, $S(p) > p$ for $p < \xi_0$, and $S(p) < p$ for $p > \xi_0$.

Proof: Differentiating $S(p)$ we have

$$S'(p) = [d + 1 \bar{F}(p)]^2 \{ f(p) \{-c - p(d + 1 \bar{F}(p)) + \int_p^\infty x dF(x)\} \}. \quad (18)$$

By assumption, the term in braces is positive for $p=0$. Furthermore, the derivative of the term in braces is $-(\delta + \lambda F(p)) < 0$. Thus, $S'(p)$ changes sign (from positive to negative) exactly once: $S(p)$ is unimodal. Furthermore, $S(p)$ reaches a maximum at ξ_0 as $S'(p)=0$ is equivalent to the first-order condition (3).

To verify the second statement of the Lemma, rearrange (6) to produce

$$c = \int_p^\infty \{x - S(p)[d + 1 \bar{F}(p)] / \bar{F}(p)\} f(x) dx. \quad (8)$$

By definition of ξ_0 , $p < \xi_0$ implies that soliciting one more buyer is preferred to stopping so utilizing (8) produces

$$p < -c / (d + 1) + \{p + H(p)\} / (d + 1) \quad (20)$$

$$= -\left\{ \int_p^\infty [x - S(p)(d + 1 \bar{F}(p)) \wedge \bar{F}(p)] f(x) dx \right\} / (d + 1) + \{p F(p) + \int_p^\infty x f(x) dx\} / (d + 1) \quad (21)$$

$$= (d + 1 \bar{F}(p)) S(p) / (d + 1) + p F(p) / (d + 1), \quad (22)$$

whence $p < S(p)$. ■

Theorem 3: For $\alpha < \alpha^*$, social welfare W_α is decreasing in α .

Proof: Lemma 2 and Theorem 1 (ξ_α is decreasing in α) imply that $S(\xi_\alpha)$ is decreasing in α whence (7) implies W_α is decreasing on $[0, \alpha^*]$. ■

Interestingly, social welfare decreases as the proportion of informed buyers increases. The reason is simple: because the seller becomes less discriminating by lowering her reservation price as more buyers are informed, the probability that a low valuation buyer purchases the item increases. This negative effect exceeds the reduction in search costs.

When α increases to $\alpha^+ < \alpha^*$, the imperfectly informed buyers are indifferent (because they continue to receive zero surplus) and the seller is worse off (for ξ_α falls to x_{a^+} 23). At first blush it appears that the informed buyers strictly prefer α^+ . This intuition emanates from the fact that an informed buyer who purchases the item pays $x_{a^+} < x_a$ 24, but it fails to consider the fact that lower

valuation buyers -- those with valuations between x_{a+25} and ξ_α -- might purchase the item when the reservation price drops. Just as the elasticity of search depends upon whether the hazard function h is increasing or decreasing, the change in the welfare of informed buyers also depends upon the monotonicity of h . This fact is nearly equivalent to the next lemma about the expected surplus of an informed buyer who purchases the item.

Lemma 3: Define $\Delta(x)$ by $\Delta(x) \equiv E(X | X \geq x) - x$, and let $X \geq 0$ be a random variable with density f and cumulative distribution function F so that the hazard function $h = f/F$ exists. If h is non-decreasing [non-increasing], then $\Delta(x)$ is non-increasing [non-decreasing].

The proof is presented in the Appendix.

Theorem 4: Let $I(\alpha)$ be the expected surplus of a representative informed buyer. If h is non-decreasing [non-increasing], then $I(\alpha^+) \geq I(\alpha)$ [$I(\alpha^+) \leq I(\alpha)$] so an informed buyer would [would not] elect to have α increase to $\alpha^+ < \alpha^*$.

Proof: The (undiscounted) surplus of an informed buyer with valuation v who purchases the item is $v - \xi_\alpha$ so the expected surplus of an informed buyer, conditional upon purchasing the item, is $E[V | V \geq \xi_\alpha] - \xi_\alpha = \Delta(\xi_\alpha)$. As α is the probability that the item is sold to an informed buyer, the expected surplus accruing to the set of all informed buyers is $\alpha \Delta(\xi_\alpha)$ whence $I(\alpha) = \Delta(\xi_\alpha)$. The proof now follows by coupling this fact with Lemma 3 and Theorem 1.¹⁶ ■

The change in the expected return of a specific informed buyer caused by an increase in α will depend upon that buyer's private value v . It is clear upon reflection that there is a number $\eta_\alpha > \xi_\alpha$ such that for $v < \eta_\alpha$ the buyer's surplus is so small that he prefers to see α increase -- and he will attempt to increase the proportion of informed buyers by the age old method, word-of-mouth. However, if $v > \eta_\alpha$, then the increase in surplus the buyer receives if he is solicited is not large enough to justify the reduction in the probability that he is solicited,¹⁷ and he prefers that α not increase to α^+ .

Finally, we turn to the question of whether social welfare is greater if the seller employs a posted price or a reservation price when $\alpha = \alpha^*$. Toward this end let u denote the unique value of

¹⁶ We have chosen to ignore the impact of the time of sale upon the expected surplus. If the effect of this timing is taken into account, then note that β_α , the discounted time of sale, is given by $\beta_\alpha = \lambda F(\xi_\alpha) / \{\delta + \lambda F(\xi_\alpha)\}$, that β_α is increasing in α , and that $I(\alpha) = \beta_\alpha \Delta(\xi_\alpha)$. As above, $I(\alpha^+) > I(\alpha)$ continues to apply when h is non-decreasing. When h is non-increasing, however, we can no longer assert that $I(\alpha^+) \leq I(\alpha)$.

¹⁷ As α increases, the probability that a given buyer is solicited decreases because the set of buyers willing to offer the reservation price (or more) increases.

$p \neq x_a^*$.²⁶ such that $S(p) = S(x_a^*)$.²⁷ As per Lemma 2, $u > \xi_0$. Furthermore, if $F(p) < 1$ and $\alpha = 0$, the seller prefers using the reservation price p to the posted price p : $S(p) > R(p)$.¹⁸

(*** Figure 1 here ***)

Figure 1 illustrates Lemma 2, $S(p) > R(p)$, $u > \xi_0$, and reveals that social welfare under the posted price exceeds that from the reservation price if and only if $p^* < u$. Necessarily,

$p^* > x_a^*$.²⁸ ¹⁹ Therefore, when $\alpha = \alpha^*$, the posted price regime yields greater social welfare than the reservation price regime if $p^* < u$: $S(p^*) > S(x_a^*)$ ²⁹ is equivalent to $p^* < u$. Furthermore, if $p^* < u$, then social welfare is *strictly* greater under the posted price than the reservation price for some $\alpha < \alpha^*$: the seller may elect to employ a reservation price even though society would be better off if she employed a posted price.

In general, it is difficult to determine whether or not p^* exceeds u . As drawn in Figure 1, R is unimodal and p^* exceeds ξ_0 . While this is not true in general, if the hazard function h is non-decreasing and $h(0)$ is sufficiently small, then indeed R is unimodal and $p^* > \xi_0$.

The impact of a switch to posted prices is ambiguous: social welfare increases only if p^* is less than u . If h is non-increasing and $\delta = 0$, Lemma 4 asserts that social welfare is greater under a posted price regime.

Lemma 4: If $h(x)$ is non-decreasing and $h(0) < (\delta + \lambda)/c$, then $R(p)$ is unimodal and $p^* \geq \xi_0$. If h is non-increasing and $\delta = 0$, then $p^* \leq \xi_0$.

Proof: Unimodality follows directly from differentiation of (1). Lemmas 2 and 3 imply that $R(p)$ is non-decreasing on $[0, \xi_0]$ so $p^* \geq \xi_0$.

By Lemma 3 h non-increasing implies Δ is non-increasing. If $\delta = 0$, then $S(p) - R(p) = \Delta(p)$ whence R is non-increasing on $[\xi_0, \infty)$. ■

In addition, when h is non-decreasing, $p^* \leq \xi_0$ holds for small values of δ , but it need not hold

¹⁸ To see this observe that $S(p) - R(p) = \{E[V | V \geq p] - p\} \lambda F(p) / (\delta + \lambda F(p)) > 0$.

¹⁹ The claim in the proof of Lemma 1 implies [Install Equation Editor and double-click here to view equation.](#) **Error! Main Document Only.**

for all values of δ .²⁰

²⁰ Suppose F is the Weibull distribution with parameter γ .
Install Equation Editor and double-click here to view equation. **Error! Main Document Only.** and $h(t) = \gamma t^{\gamma-1}$. When $\gamma=1/2$,
Install Equation Editor and double-click here to view equation. **Error! Main Document Only..** From (2), p^* solves
Install Equation Editor and double-click here to view equation. **Error! Main Document Only.** and from (3), ξ_0
solves click here to view equation. **Error! Main Document Only..** If $\delta=0$, then $p^* < \xi_0$. If $\delta \gg \lambda$, then $p^* > \xi_0$.

APPENDIX: MODELLING IMPERFECTLY INFORMED BUYERS

A strong implication of the results of Riley and Zeckhauser is that buyers will expect to observe a posted price in a world of unbounded rationality in which the seller can commit to a strategy. Thus, a buyer who does not observe a posted price will reasonably deduce that either the seller cannot commit or a proportion of the buyers (possibly including himself) are imperfectly informed and are being exploited by the seller. However, this deduction does not alter the buyer's information regarding the lowest price the seller is willing to accept. At this point it may be impossible for the buyer to determine optimally whether to delay taking an action and collect more information or take the best action given his current information and beliefs²¹: the buyer must base his decisions on his current information, fully aware that this information may be erroneous. Thus, a model in which the seller can commit to any pricing strategy but chooses not to post a price must specify either the way in which each buyer responds to this unexpected behavior or the limits on buyer rationality which lead him to expect that the seller will not post a price.²²

Lipman [1991] demonstrates that the infinite regress problem of "deciding how to decide" which arises in this context has a fixed point. Consequently, there is no logical inconsistency in modelling agents who behave "optimally" given their current (albeit limited) information and perceptions. While Lipman's results justify modelling the boundedly rational agent as an optimizer and smooth the transition from the agent's initial beliefs to the action he eventually takes, these results offer no guidance regarding how to model the buyers' initial beliefs.

We now offer two simple examples of markets with limitedly rational buyers in which posting a price is not optimal when all buyers are imperfectly informed.²³ In both examples the beliefs of

²¹ For example, MBA students across the country are taught to analyze some investment decisions via decision trees. However, the decision to use a decision tree is problematic. Consider the simple situation in which the net gain to "not investing" is 0, the net gain to investing is π , and the net gain to gathering information and then deciding whether or not to invest is $\pi^+ - k$ where $\pi^+ > \pi$. Finally, the cost of engaging in the decision tree analysis is c . Whether or not the values of k , π , and π^+ are revealed during or before the analysis, the MBA is unable to decide whether or not it is optimal to expend c and draw the decision tree prior to drawing the tree.

²² Even in the Riley and Zeckhauser framework with precommitment and unbounded rationality, market frictions other than imperfect information also can render a posted price suboptimal. For example, consider a seller with one unit of a good which she values at 4 who can solicit risk averse buyers sequentially for a search cost of 2. With probability .6 a randomly selected buyer's valuation is 8 and with probability .4 it is 10; the buyers' utility function is $U(x)=x$ if $0 \leq x \leq 1$, and $U(x)=1+.9(x-1)$ if $x > 1$, where x is the difference between the buyer's valuation and the price he pays. In this case the optimal posted price is 8 and yields an expected return of 2. However, the seller can achieve an expected return of 2.016 by committing to sell the good with probability 1 if the buyer offers 9.52 and with probability .25 if the buyer offers 8.

²³ For additional examples of models of limitedly rational agents in various economic settings see Simon

imperfectly informed buyers lead to a non-degenerate distribution of offers when the seller employs a reservation price strategy. Given this distribution, the seller's ability to discriminate amongst buyers induces her to prefer a reservation price rather than a posted price. Most importantly, the examples illustrate how buyer beliefs influence the seller's choice of a pricing strategy.

Example 1: Suppose risk neutral buyers, with private value b_1 or b_2 , know that the seller's private value for the item is either s_1 or s_2 , where $s_1 < b_1 < s_2 < b_2$. The seller solicits a buyer at a cost of c . If an agreement is not reached with that buyer, the buyer receives a surplus of zero and the seller solicits another buyer.

If the seller can commit to any strategy, then an s_2 seller should post a price of b_2 .²⁴ Thus, if a price of b_2 is not posted, buyers will deduce that the seller's valuation is s_1 . Consequently, the results of Riley and Zeckhauser imply that the s_1 seller is best served by a posted price (of either b_1 or b_2 depending upon the proportion of each type of buyer). Therefore, the absence of a posted price is inconsistent with the buyers' beliefs. Arriving at reasonable assumptions regarding buyer behavior in the absence of a posted price is the fundamental difficulty in addressing this inconsistency.

One possible assumption is that buyers update a non-degenerate prior on the probability that the seller can commit to a posted price while not changing their beliefs about the seller's value. In this case the seller may choose not to post a price. To see this, suppose the seller's valuation is $s=0$, a proportion $\pi_{b_1}=.67$ of the buyers have valuation $b_1=4$, and a proportion $\pi_{b_2}=.33$ have valuation $b_2=8$. The buyers believe that the seller's valuation is $s_1=3$ or $s_2=6.05$ with probability $\pi_{s_1}=.38$ and $\pi_{s_2}=.62$, respectively. Furthermore, the buyers know that it is costly for the seller to solicit another buyer, but they do not know the actual solicitation cost $c=2$. If the buyers believe there is a small probability that the seller cannot commit to a price (so they are not surprised when a price is not posted), then (following the logic of Diamond) if the seller employs a reservation price strategy in which the buyer must make an offer, the buyer will offer either s_1 or s_2 so as to maximize expected surplus. Given the buyer's beliefs, the b_2 buyer's expected return from offering s_1 is $\pi_{s_1}(b_1-s_1)=1.9$ versus 1.95 from offering s_2 : the b_2 buyer will offer s_2 . Clearly, the b_1 buyer will offer s_1 . When posting a price of b_2 , the seller's expected return is $R(b_2) = -c + \pi_{b_2}8 + \pi_{b_1}R(b_2)$, whence $R(b_2) \approx 1.94$, while a posted price of b_1 yields an expected return of 2: b_1 is the optimal posted price. However, if the seller employs a reservation price of s_1 (she accepts an offer of either s_1 or s_2), her expected return is $S(s_1) = -c + \pi_{b_1}s_1 +$

[1982], Williamson [1985], Kreps [1990], and the December, 1990 issue of the *Journal of Institutional and Theoretical Economics*.

²⁴ We assume that c is sufficiently small so that the expected return from selling the object is positive.

$\pi_{b2S2} \approx 2.007$ which exceeds $R(b_1)$. As shown, the seller's optimal strategy is to employ a reservation price (of s_1).

Example 2: Each buyer's search for information reveals a "best" alternative -- with value v -- to purchasing from the seller. Because different buyers acquire different information, v differs across buyers. Each buyer's search provides him not only with an alternative but also with limited information regarding the seller's alternatives. In particular, the buyer's belief about the seller's alternatives is correlated with v : each buyer believes that the seller's reservation price ξ is uniformly distributed over the interval $[v/2, v]$. Furthermore, the seller knows that a randomly solicited buyer's value is a random variable V distributed uniformly over the interval $[0, 10]$. The seller's solicitation cost is 2. The optimal posted price is 5.53, and the expected return from posting this price is 1.06.²⁵

If the buyer must make an offer which the seller either accepts or rejects, then given his beliefs about the seller's (concealed) reservation price, the buyer makes an offer b so as to maximize his expected surplus: b maximizes $(v-b)[P(\xi < b)] = (v-b)[(b-v/2)/(v/2)]$.

Consequently, each buyer's optimal offer is $3v/4$ so the seller receives offers distributed uniformly over the interval $[0, 7.5]$. The optimal reservation price (which equals the seller's expected return from employing a reservation price) is 2.02.²⁶ Thus, the seller elects to employ a reservation price.

²⁵ This follows from (1) and (2).

²⁶ This follows from (3).

Proof of Lemma 3: Integration followed by exponentiation produces

$$\bar{F}(t) = e^{-\int_0^t h(s) ds} \quad (A1)$$

As $m \bar{F}(m) = m \int_m^\infty f(t) dt \leq \int_m^\infty t f(t) dt$ 31, $E[X] < \infty$ implies $m \bar{F}(m) \rightarrow 0$ as $m \rightarrow \infty$. (If $E[X] = \infty$, then $\Delta(x) \equiv \infty$.) Integration by parts yields

$$\int_x^m t f(t) dt = x \bar{F}(x) - m \bar{F}(m) + \int_x^m \bar{F}(t) dt \quad 32$$

so that

$$\int_x^\infty t f(t) dt = x \bar{F}(x) + \int_x^\infty \bar{F}(t) dt . \quad (A2)$$

Consequently, application of (A2) and then (A1) yields

$$\begin{aligned} E[X | X \geq x] &= \int_x^\infty t f(t) dt / \bar{F}(x) = x + e^{\int_0^x h(s) ds} \int_x^\infty e^{-\int_0^t h(s) ds} dt \quad 34 \\ &= x + \int_x^\infty e^{-\int_x^t h(s) ds} dt . \end{aligned} \quad (A3)$$

In view of (A3), $\Delta(x)$ is non-increasing in x if and only if the integral in (A3) is non-decreasing in x . By Leibniz's Formula

$$\frac{d}{dx} \int_x^\infty e^{-\int_x^t h(s) ds} dt = -e^{-\int_x^x h(s) ds} + \int_x^\infty \frac{d}{dx} e^{-\int_x^t h(s) ds} dt \quad 36$$

$$= -1 + \int_x^\infty h(x) e^{-\int_x^t h(s) ds} dt . \quad 37$$

Because h is non-decreasing, $\int_x^t h(s) ds \geq (t-x)h(x)$ 38 so that $e^{-\int_x^t h(s) ds} \leq e^{-xh(x)} e^{-t h(x)}$.39 Thus, h non-decreasing yields

$$\Delta'(x) \leq -1 + e^{xh(x)} \int_x^\infty h(x) e^{-t h(x)} dt = -1 + e^{xh(x)} e^{-xh(x)} = 0 . \quad 40$$

All inequalities are reversed if h is non-increasing. ■

References

- Arnold, Michael A., and Steven A. Lippman. "Optimal Selling Institutions: Auctions Versus Sequential Search," Western Management Science Institute Working Paper No. 387, August 1991.
- Ausubel, Lawrence M., and Raymond J. Deneckere. "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, **57** (1989), 511-531.
- Burdett, Kenneth, and Kenneth L. Judd. "Equilibrium Price Dispersion," *Econometrica*, **51** (1983), 955-969.
- Butters, Gerard R. "Equilibrium Distribution of Sales and Advertising Prices," *The Review of Economic Studies*, **44** (1977), 465-491.
- Chandler, Alfred D. *The Visible Hand*. Cambridge: Harvard University Press, 1977.
- De Vany, Arthur, "Institutions for Stochastic Markets," *Journal of Institutional and Theoretical Economics*, **143** (1987), 91-103.
- Diamond, Peter A. "A Model of Price Adjustment," *Journal of Economic Theory*, **3** (1971), 156-168.
- Kreps, David M. *Game Theory and Economic Modelling*. Oxford: Oxford University Press, 1990.
- Lipman, Barton L. "How to Decide How to Decide How to...: Modeling Limited Rationality," *Econometrica*, **59** (1991), 1105-1125.
- Lippman, Steven A., and John J. McCall. "The Economics of Uncertainty: Selected Topics and Probabilistic Methods," Chapter 6 in Kenneth J. Arrow and Michael D. Intriligator, eds., *Handbook of Mathematical Economics*, 213-284. Amsterdam: North Holland Publishing Co., 1981.
- Mahoney, Tom, and Leonard Sloane. *The Great Merchants*. New York: Harper and Rowe, 1966.
- Riley, John, and Richard Zeckhauser. "Optimal Selling Strategies: When to Haggle, When to Hold Firm," *The Quarterly Journal of Economics*, **98** (1983), 267-289.
- Salop, Steven. "The Noisy Monopolist: Imperfect Information, Price Dispersion and Price Discrimination," *The Review of Economic Studies*, **44** (1977), 393-406.
- Salop, Steven, and Joseph Stiglitz. "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," *The Review of Economic Studies*, **44** (1977), 493-510.
- Scull, Penrose. *From Peddlers to Merchant Princes*. New York: Follet Publishing Co., 1967.

- Simon, Herbert A. *Models of Bounded Rationality Volume I: Economic Analysis and Public Policy*. Cambridge: The MIT Press, 1982.
- Spier, Kathryn E. "Hagglers and Posters: The Coexistence of Flexible and Fixed Prices," Harvard Institute of Economic Research Discussion Paper No. 1517, September 1990.
- Stiglitz, Joseph. "Imperfect Information in the Product Market," Chapter 13 in: R. Schmalensee and R. Willig, eds., *Handbook of Industrial Organization, Volume 1*. Amsterdam: Elsevier Science Publishers B. V., 1989, 769-847.
- Wang, Ruqu. "Auctions Versus Posted-Price Selling," Queen's University Economics Department Discussion Paper No. 812, May 1991.
- Wilde, Louis L., and Alan Schwartz. "Equilibrium Comparison Shopping," *The Review of Economic Studies*, **46** (1979), 543-553.
- Williamson, Oliver E. *The Economic Institutions of Capitalism*. New York: The Free Press, 1985.